MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.01T   Fall Term 2004

Experiment 03: Modeling Forces

Purpose of the Experiment:
In this experiment you will measure the repulsive force between two ceramic magnets as a function of the distance between them. This experiment has two goals:

1. More practice in using the computer to analyze data. You will take the data by hand and enter them into a table in the computer so that you may then use *DataStudio* to make graphs of the data and to look for a mathematical function that represents your measurements.

2. Finding a theory to explain the forces requires a deeper understanding of magnetism. You will acquire that next term in 8.02, so we will mostly be searching for sensible ways to think about measurements that we do not understand. In an appendix to these notes I will suggest an approximate model to explain the measurements that is based on something you will learn in 8.02.
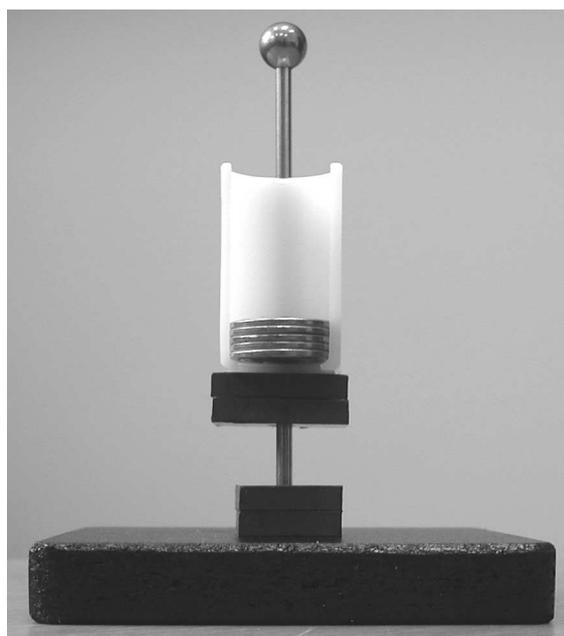
The photo at the left shows the apparatus for this experiment. You will need four rectangular ceramic magnets with a hole in the middle, a plastic coin holder, and about 50 pennies. Stick the magnets together in pairs to make two stronger magnets. Slide the coin holder over the rod, followed by the two paired magnets oriented so they repel, and place the rod into the hole in the center of the wooden base. The upper magnet will be held up by the bottom one and they will align so the long axis of one magnet's rectangle is perpendicular to the long axis of the other magnet. If you add pennies to the coin holder, their weight will force the two magnets closer together. You should add pennies equally to both sides to avoid tilting and friction with the rod.

In the experiment you will make a table of the gap between the magnets for various weights pressing them together and thereby measure the "force law" for the repulsion between the magnets. I suggest that you try to complete the table below, or one very similar.

Weight (pennies)	10	14	18	22	30	40	50	58
Gap (mm)								

It is convenient to measure the force in units of pennies; if you want to convert to a more conventional unit, a penny has a mass of $2.50\,\text{gm}$ so a force of 1 penny is $0.0245\,\text{N}$.

I weighed the other items and found that the plastic coin holder (with no rubber bands) and two magnets together weigh 10 pennies; this is the minimum force you will have.

Measure the gap with your transparent plastic ruler. I found the best way to do it was to measure the distance from the experiment base to the bottom of the upper magnet and subtract from it the distance from the base to the top of the lower magnet. It is convenient to express the gap in mm. (I found it ranged from 3.0 to 18 mm and that I could measure it to ± 0.5 mm) You may use different weights than in the table, but measure the gap for at least eight different weights over the range from 10 to 60 pennies.

Analysis of Your Measurements:
In order to analyse your results you should start up *DataStudio* and enter them into a table. The *DataStudio* "howto" document explains how to do this;

Be sure to type the gap values in the X (left) column of your table and the corresponding force in the Y (right) column. Give the table a useful name, such as "Magnetic Force", label the X and Y columns "Gap" and "Force", respectively, and enter their units as "mm" and "pennies" (don't type any of the quotation marks). Plot the data by dragging the "Magnetic Force" entry in the program's "Data" window onto the "Graph" entry in the "Displays" window. You should get a graph like this one.
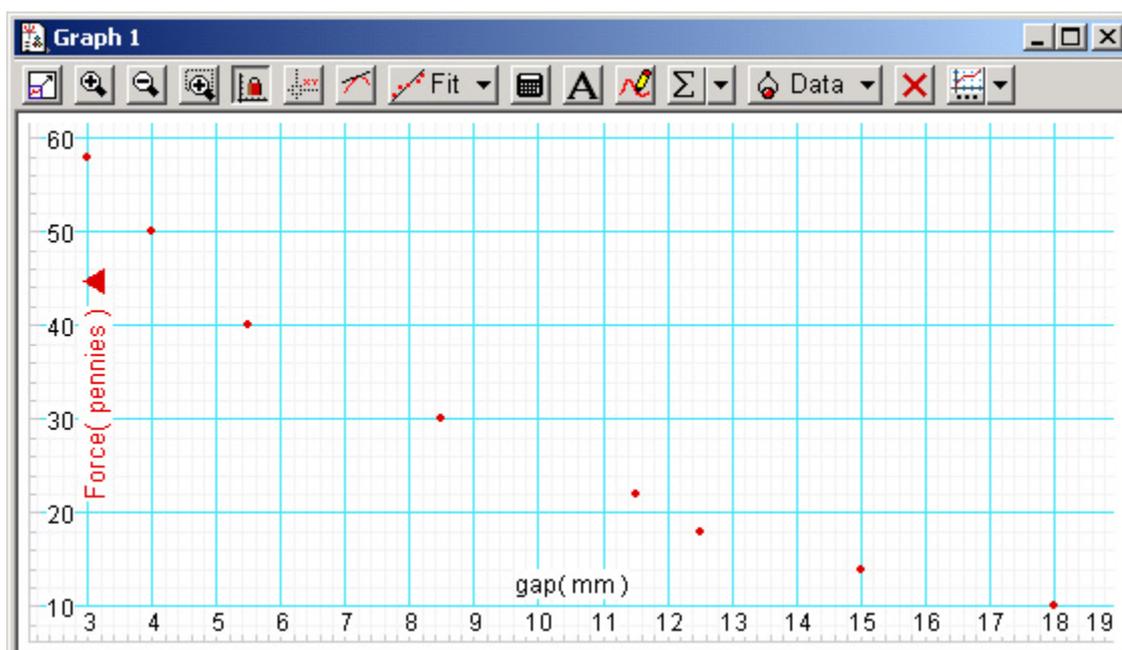
To continue the analysis you will need two more data sets: one with the gap as the X variable and the natural log of the force as the Y variable, and a second data set with the gap as the X variable and the natural log of the gap as the Y variable. I was never able to figure out a way to get *DataStudio* to do any calculations on the independent variable, so you will have to do it by hand, with a calculator. Make a new table, like you did to put your measurements into the computer, and lable it "Log Gap" or something similar. Put your measured gap values in the X column and their natural log in the Y column.

You can make a data table "Log Force" in the same way, but here is how to get *DataStudio* to do it for you. Click the Calculate button on the toolbar, to the right of timing counter window (it should be the one with orange "00:00.0" in it). That should open a calculator window:
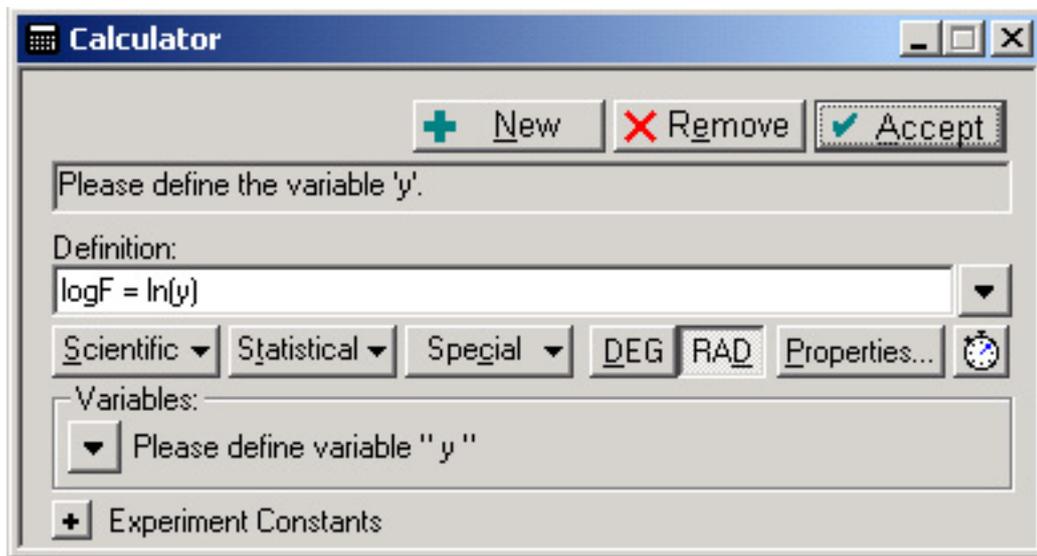
Move the window to a convenient location and type the definition you see here "LogF = ln(y)" and click "Accept". If you get the message 'Please define variable "y"' that you see here, click the pull-down menu (down arrow) in the "Variables" area and choose "Data Measurement". That will open a yellow window that offers a choice among all of the entries currently in the "Data" window. Select your magnetic force measurement, and then click the "Accept" button when you return to the calculator window. That will create a new data set called "LogF = ln(y)" in the "Data" window. This data set will have the gap as the independent variable and the natural log of the force as the dependent variable. You now have the three data sets you need to continue the analysis.

First, plot your "LogF = ln(y)" data by dragging it from the "Data" window onto "Graph" in the "Displays" window. You will get a graph like this:
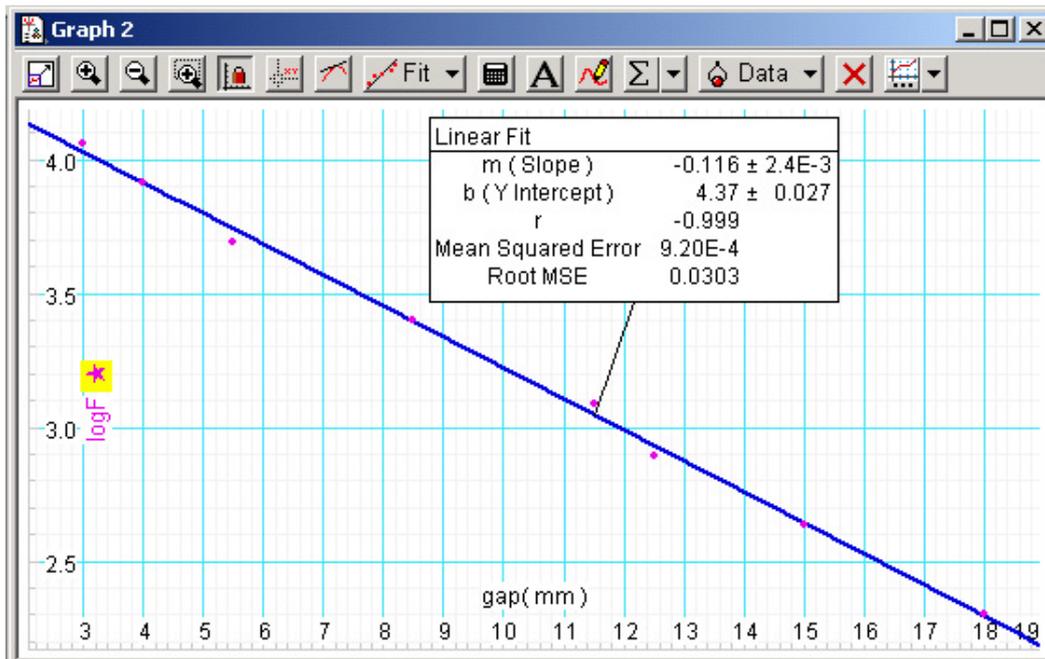
Your graph will not (yet) have the line fit on it, but if you look at the data points you will see they look awfully close to being on a straight line. To test this perception, choose "Linear Fit" under the graph's "Fit" buton and you will get the straight line fit as on my graph. This makes it pretty clear that the force law must be exponential. However, let's see if it resembles a power law. To test that, make a plot of $\ln(force)$ on the Y axis and $\ln(gap)$ on the X axis. *DataStudio* makes it easy. Remove the linear fit from the graph and drag the "Log Gap" data from the "Data" window onto the _X axis_ of the graph. That will give a plot like this one.
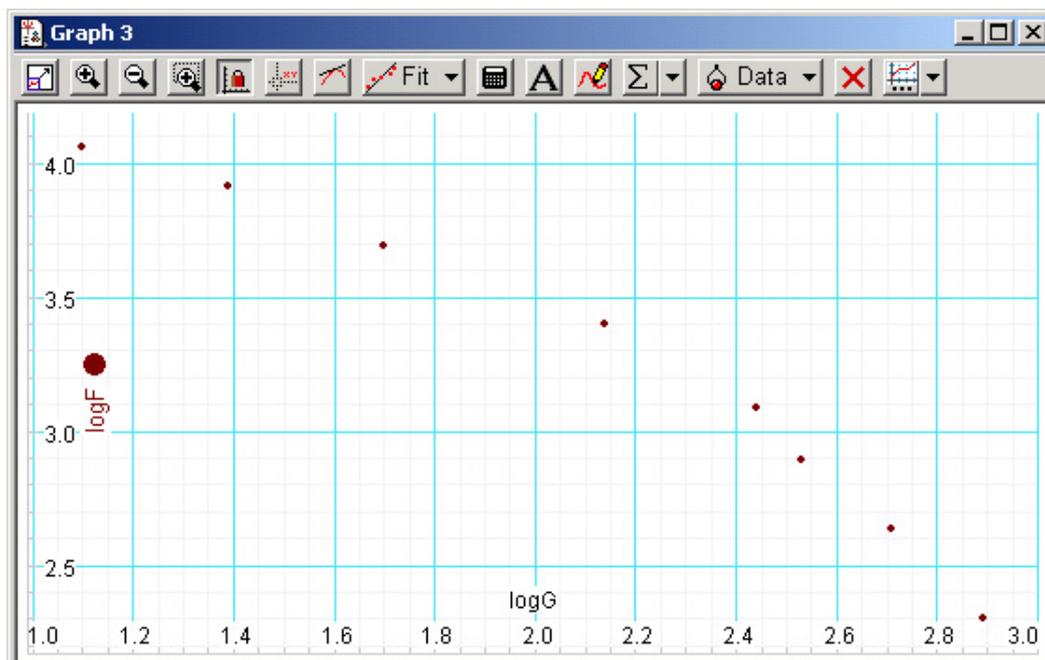
It is obvious that these points do not fall on a straight line, so a power law is not a good representation of the force. You could use the Slope Tool on this graph to see how $d\ln(y)/d\ln(x)$ varies with the gap. If you were going to represent the data as $F = x^n$, where x is the gap, the power n would have to be a function of the gap and this slope tells you what it would be.

It would be correct to conclude that an exponential force law describes the measurements well and would be useful to interpolate to find the force for different gap values than you measured. I would leery of using it to extrapolate too far beyond a gap of 20 mm and, of course, negative gaps do not make sense. Here is a plot of an exponential fit to the data.
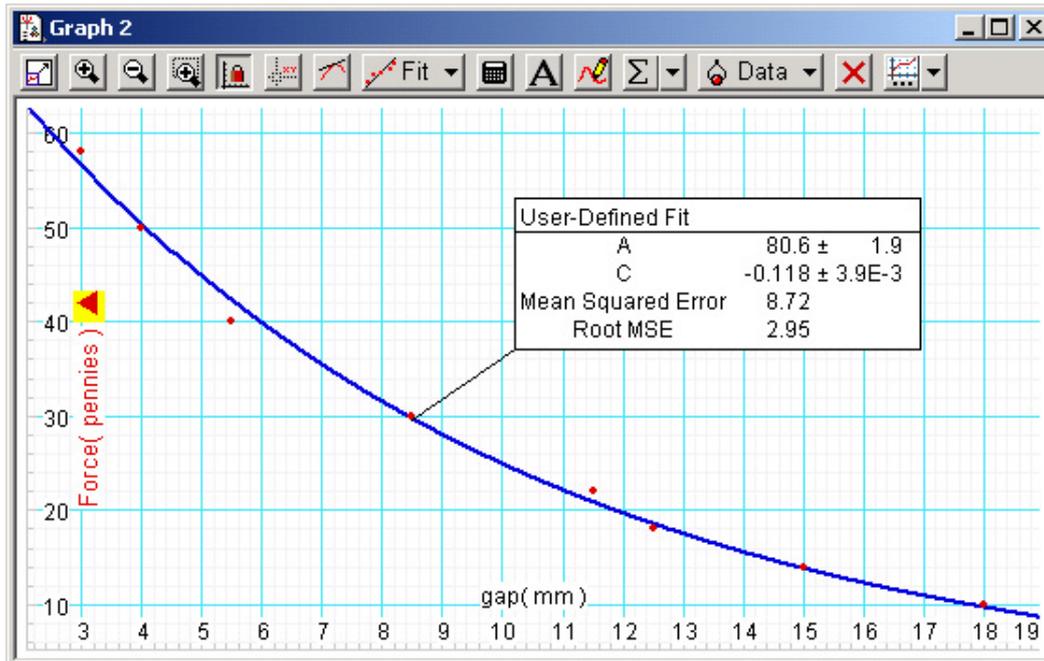
So far, this analysis has not given much insight into what causes the force between the magnets and why it follows an exponential law. You might think that is because you don't know enough about electromagnetism yet, but read the following appendix for the real explanation.

Explanation of the "Exponential" Force Law:

This discussion is not part of what you need to know for 8.01T, but I hope your curiosity has been piqued enough that you will read it. It goes a bit beyond what you will probably learn in 8.02 (but hopefully not 8.022) but you may see it again if you take 8.02 next semester.

The part that goes beyond 8.02 is that the external magnetic field of a flat magnet like those used in this experiment is the same as that of a current loop flowing around the edges of the magnet. That means the two magnets will exert the same force on each other as two rectangular current loops would. So I can solve a different problem and get the answer for the magnets.

Once that is understood (or at least accepted) the rest of the solution could be found by an 8.02 student who knows the Biot-Savart rule for calculating the magnetic field of a wire that carries an electrical current and the Lorentz rule to calculate the force on a wire carrying a current when in a magnetic field.

The goal is to find the force between two rectangular current loops oriented so their long axes are crossed. The two rules mentioned above tell us that the short side of each loop experiences forces only due to the currents flowing in the long sides of the other loop. (If you want to know why, wait until 8.02.) Now I introduce an approximation: that the long sides of the other loop extend infinitely in both directions—but only for purposes of calculating their magnetic fields. This makes the calculation of their magnetic fields so easy that you could do it in your head (if you were taking 8.02!). The force can also be calculated in your head, once you do a little trigonometry that might be better done on paper. Since there is a total of four short sides in the two loops, multiply the result by four. If you were an excellent student in 8.02, you would then come up with this answer for the force:

$$F = \left(\frac{2\mu_0 I^2}{\pi} \right) \frac{w^2 l z}{z^4 + \left(\frac{w^2 + l^2}{2} \right) z^2 + \left(\frac{l^2 - w^2}{4} \right)^2}$$

where l is the length of the long sides of the rectangular loops, w is the length of the short sides, z is their separation, and I is the current flowing in each loop. The quantity μ_0 ($= 4\pi \times 10^{-7}$ in S.I. units) is called the permittivity of free space. This is the kind of expression you <u>love</u> to have as a theoretical model to fit your data to, because it has only two parameters: the current I and the fact that z might not be zero when the gap is zero (because of the thickness of the magnets). Everything else is geometry that can be measured. The fraction involving l, w, and z is dimensionless, so all you need to do is express these three lengths in the same units, say mm, and things will come out right.

Well, for our magnets $l = 1\,\text{in} = 25.4\,\text{mm}$ and $w = 3/4\,\text{in} = 19\,\text{mm}$, and I will assume $z = x + B$, where x is the gap between the magnets. If I put in the numbers and let $A = 2\mu_0 I^2 / \pi$ the expression to fit is

$$F = y = A \frac{9170(x + B)}{(x + B)^4 + 500(x + B)^2 + 5000}$$

I typed this into *DataStudio* and got an excellent fit with $A = 23.6 \pm 6.4$ pennies and $B = 2.6 \pm 2.4$ mm. The Root MSE (correcting for *DataStudio*'s idiosyncracies) was 1.2 pennies. The graph is shown below.
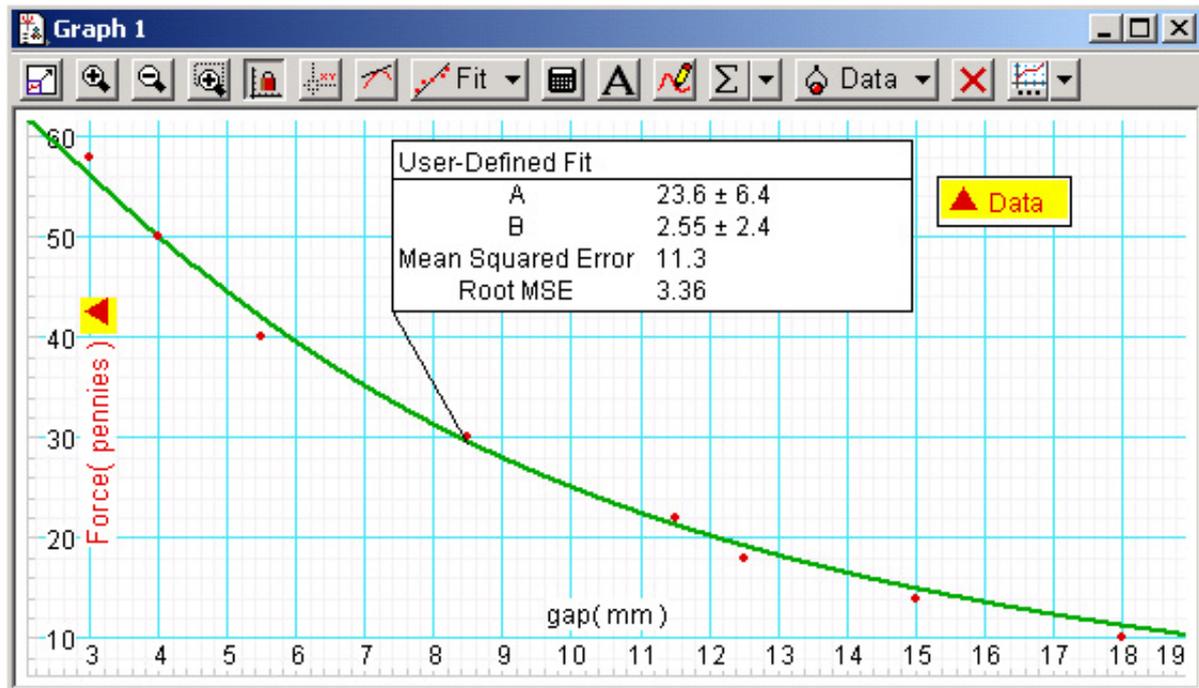
This function does not represent the data any better (statistically speaking) than the exponential, but I like it <u>much</u> better because I understand where it comes from; it has some intellectual content. Furthermore, I am unable to think of any explanation based on electromagnetism that would give an exponential forec law.

After converting A from pennies to Newtons (multiply by 0.0245), I was able to calculate $I = 170 \pm 24$ A. It's impressive to think that that this pair of little permanent magnets is the equivalent of 170 A of current flowing around a loop.

An interesting question is why did the exponential force law represent the data so well? Was this a rare accident? I think not. There is a computational technique in applied mathematics called *Padé Approximants* in which a function that is difficult to evaluate analytically is approximated by the ratio of two polynomials, which are much easier to calculate. I looked up some of the Padé approximants for e^x and the function above is not one of them. However, I can imagine that many functions can be approximated as the ratio of two polynomials that are not a true Padé approximant to them; the approximation will work, just not as well or over as wide a range as a true Padé approximant. This a more subtle example of what we saw in the regression analysis of the first experiment; there may be many functions that fit your data equally well and you simply do not have enough data to distinguish among them. That's where a knowledge of the science that underlies the measurements is essential; here, it's knowledge you won't have until 8.02. Regression analysis should always be combined with an understanding of what you are doing and a healthy scepticism.

If you are ever interested in Padé approximants, I recommend the book *Padé Approximants* by George A Barker, Jr. and Peter Graves-Morris, published by Cambridge University Press in 1996.