

Topics: Capacitors

Related Reading:

Course Notes: Chapter 5

Topic Introduction

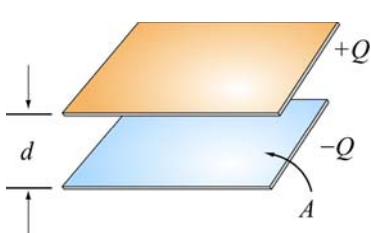
Today we will practice calculating capacitance and energy storage by doing problem solving #4. Below I include a quick summary of capacitance and some notes on calculating it.

Capacitance

Capacitors are devices that store electric charge. They vary in shape and size, but the basic configuration is two conductors carrying equal but opposite charges ($\pm Q$). In order to build up charge on the two plates, a potential difference ΔV must be applied between them. The ability of the system to store charge is quantified in its capacitance: $C \equiv Q/|\Delta V|$. Thus a large capacitance capacitor can store a lot of charge with little “effort” – little potential difference between the two plates.

A simple example of a capacitor is pictured at left – the parallel plate capacitor, consisting of two plates of area A , a distance d apart. To find its capacitance we do the following:

- 1) Arbitrarily place charges $\pm Q$ on the two conductors
- 2) Calculate the electric field between the conductors (using Gauss’s Law)
- 3) Integrate to find the potential difference



Finally we calculate the capacitance, which for the parallel plate is $C = Q/|\Delta V| = \epsilon_0 A/d$. Note that the capacitance depends only on geometrical factors, not on the amount of charge stored (which is why we were justified in starting with an arbitrary amount of charge).

Energy

In the process of storing charge, a capacitor also stores electric energy. The energy is actually stored in the electric field, with a volume energy density given by $u_E = \frac{1}{2} \epsilon_0 E^2$. This means that there are several ways of calculating the energy stored in a capacitor. The first is to deal directly with the electric field. That is, you can integrate the energy density over the volume in which there is an electric field. The second is to calculate the energy in the same way that you charge a capacitor. Imagine that you start with an uncharged capacitor. Carry a small amount of positive charge from one plate to the other (leaving a net negative charge on the first plate). Now a potential difference exists between the two plates, and it will take work to move over subsequent charges. A third method is to use one of the formulae that we can calculate using the second method: $U = \frac{Q^2}{2C} = \frac{1}{2} Q|\Delta V| = \frac{1}{2} C|\Delta V|^2$.

Important Equations

Capacitance:

$$C \equiv Q/|\Delta V|$$

Energy Stored in a Capacitor:

$$U = \frac{Q^2}{2C} = \frac{1}{2}Q|\Delta V| = \frac{1}{2}C|\Delta V|^2$$

Energy Density in Electric Field:

$$u_E = \frac{1}{2}\epsilon_0 E^2$$