Experiment 8: Undriven & Driven RLC Circuits

Answer these questions on a separate sheet of paper and turn them in before the lab

1. RLC Circuits



Consider the circuit at left, consisting of an AC function generator ($V(t) = V_0 \sin(\omega t)$, with $V_0 = 5$ V), an inductor L = 8.5 mH, resistor $R = 5 \Omega$, capacitor $C = 47 \mu$ F and switch S.

The circuit has been running in equilibrium for a long time. We are now going to shut off the function generator (instantaneously replace it with a wire).

- (a) Assuming that our driving frequency ω is not necessarily on resonance, what is the frequency with which the system will ring down (in other words, that current will oscillate at after turning off the function generator)? Feel free to use an approximation if you wish, just make sure you know you are.
- (b) At what (numerical) frequency *f* should we drive to maximize the peak magnetic energy in the inductor?
- (c) In this case, if we time the shut off to occur when the magnetic energy in the inductor peaks, after how long will the electric energy in the capacitor peak?
- (d) Approximately how much energy will the resistor have dissipated during that time?

2. Phase in an RLC Circuit

Using the same circuit as in problem 1, only this time leaving the function generator on and driving below resonance, which in the following pairs leads (if either):

- (a) Voltage across the capacitor or voltage across the resistor
- (b) Voltage across the function generator or voltage measured across the inductor
- (c) Current or voltage across the resistor
- (d) Current or voltage across the function generator

IN-LAB ACTIVITIES

EXPERIMENTAL SETUP

- 1. Download the LabView file from the web and save the file to your desktop. Start LabView by double clicking on this file.
- 2. Set up the circuit pictured in Fig. 5 of the pre-lab reading (no core in the inductor!)
- 3. Connect a voltage probe to channel A of the 750 and connect it across the capacitor.

MEASUREMENTS

Part 1: Free Oscillations in an Undriven RLC Circuit

In this part we turn on a battery long enough to charge the capacitor and then turn it off and watch the current oscillate and decay away.

1. Press the green "Go" button above the graph to perform this process.

Question 1:

What is the period of the oscillations (measure the time between distant zeroes of the current and divide by the number of periods between those zeroes)? What is the frequency?

Question 2:

Is this frequency the same as, larger than or smaller than what you calculated it should be? If it is not the same, why not?

Part 2: Energy Ringdown in an Undriven RLC Circuit

1. Repeat the process of part 1, this time recording the energy stored in the capacitor $(U_c = \frac{1}{2}CV^2)$ and inductor $(U_L = \frac{1}{2}LI^2)$, and the sum of the two.

Question 3:

The circuit is losing energy most rapidly at times when the slope of total energy is steepest. Is the electric (capacitor) or magnetic (inductor) energy a local maximum at those times? Briefly explain why.

Part 3: Driving the RLC Circuit on Resonance

Now we will use the function generator to drive the circuit with a sinusoidal voltage.

- 1. Enter the frequency that you measured in part 1 of the lab as a starting point to find the resonant frequency.
- 2. Press GO to start recording the function generator current and voltage vs. time, as well as a "phase plot" of voltage vs. current.
- 3. Adjust the frequency up and down to find the resonant frequency and observe what happens when driving above and below resonance.

Question 4:

What is the resonant frequency? What are two ways in which you know?

Question 5:

What is the impedance of the circuit when driven on resonance (hint: use the phase plot)?

Question 6:

When driving on resonance, insert the core into the inductor. Are you now driving at, above or below the new resonant frequency of the circuit? How can you tell? Why?

Part 4: What's The Frequency?

For the remainder of the lab you will make some measurements where you are given incomplete information (for example, you won't be shown the frequency or won't be told what is being plotted). From the results you must determine the missing information. If you find this difficult, play with the circuit using the "further questions" tab to get a better feeling for how the circuit behaves.

- 1. Remove the core from the inductor
- 2. Press GO to record the function generator current and voltage

Question 7:

At this frequency is the circuit capacitor- or inductor-like? Are we above or below resonance?

Part 5: What's That Trace?

1. Press GO to record the function generator current and voltage as well as the voltage across the capacitor. Note that you are not told which trace corresponds to which value.

Question 8:

What value is recorded in each of the three traces (I, V_{FG} or V_C)? How do you know?

Question 9:

Are we above, below or on resonance? How do you know?

Further Questions (for experiment, thought, future exam questions...)

- What happens if we insert the coil core in the undriven circuit?
- For a random frequency can you bring the circuit into resonance by slowly inserting the core into the coil? Are there any conditions on the frequency (e.g. does it need to be above or below the resonant frequency of the circuit with the empty coil)?
- Could you do part 5 if you were given only two traces instead of three? Would it matter which two you were given?
- What is the energy doing in the driven case? Is the resistor still dissipating power? If so, where is this power coming from?
- What happens to the resonant frequency of the circuit if a resistor is placed in series with the capacitor and coil? In parallel? NOTE: You can use the variable resistor, called a potentiometer or "pot" (just to the left of the coil, connect to the center and right most contacts, allowing you to adjust the extra resistance from 0Ω to 3.3Ω by simply turning the knob).
- With the resistor in series with the coil and capacitor, at what frequency is the energy dissipation a maximum? How could you verify this experimentally?

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Experiment 8: Microwaves

OBJECTIVES

- 1. To observe the polarization and angular dependence of radiation from a microwave generator
- 2. To measure the wavelength of the microwave radiation by analyzing an interference pattern similar to a standing wave

PRE-LAB READING

INTRODUCTION

Heinrich Hertz first generated electromagnetic waves in 1888, and we replicate Hertz's original experiment here. The method he used was to charge and discharge a capacitor connected to a spark gap and a quarter-wave antenna. When the spark "jumps" across the gap the antenna is excited by this discharge current, and charges oscillate back and forth in the antenna at the antenna's natural resonance frequency. For a brief period around the breakdown ("spark"), the antenna radiates electromagnetic waves at this high frequency. We will detect and measure the wavelength λ of these bursts of radiation. Using the relation $f \lambda = c = 3 \times 10^{10}$ cm/s, we will then deduce the natural resonance frequency of the antenna, and show that this frequency is what we expect on the basis of the very simple considerations given below.



Figure 1 Spark-gap transmitter. The "33" is a 33 pF capacitor. It is responsible for storing energy to be rapidly discharged across a "spark gap," formed by two tungsten cylinders pictured directly above it (one with a vertical axis, one horizontal). Two M Ω resistors limit current off of the capacitor and back out the leads, protecting the user from shocks from the 800 V to which the capacitor will be charged. They also limit radiation at incorrect frequencies.

The 33-pF capacitor shown in fig. 1 is charged by a high-voltage power supply on the circuit board provided. This HVPS voltage is typically 800 V, but this is safe because the current from the supply is limited to a very small value. When the electric field that this voltage generates in the "spark gap" between the tungsten rods is high enough (when it exceeds the breakdown field of air of about 1000 V/mm) the capacitor discharges across the gap (fig. 2a). The voltage on the capacitor then rebuilds, until high enough to cause another spark, resulting in a continuous series of charges followed by rapid bursts of discharge (fig. 2b).



Figure 2 Charging and Discharging the Capacitor. The capacitor is slowly charged (limited by the RC time constant, with $R = 4.5 M\Omega$) and then (a) rapidly discharges across the spark gap, resulting in (b) a series of slow charge/rapid discharge bursts. This is an example of a "relaxation oscillator."

The radiation we are seeking is generated in this discharge.

Resonant Frequency of the Antenna

The frequency of the radiation is determined by the time it takes charge to flow along the antenna. Just before breakdown, the two halves of the antenna are charged positive and negative (+, -) forming an electric dipole. There is an electric field in the vicinity of this dipole. During the short time during which the capacitor discharges, the electric field decays and large currents flow, producing magnetic fields. The currents flow through the spark gap and charge the antenna with the opposite polarity. This process continues on and on for many cycles at the resonance frequency of the antenna. The oscillations damp out as energy is dissipated and some of the energy is radiated away until the antenna is finally discharged.

How fast do these oscillations take place – that is, what is the resulting frequency of the radiated energy? An estimate can be made by thinking about the charge flow in the antenna once a spark in the gap allows charge to flow from one side to the other. If *l* is the length of one of the halves of the antenna (about l = 31 mm in our case), then the distance that the charge oscillation travels going from the (+,-) polarity to the (-,+) polarity and back again to the original (+,-) polarity is 4*l* (from one tip of the antenna to the other tip and back again). The time *T* it takes for this to happen, assuming that information flows at the speed of light *c*, is T = 4l/c, leading to electromagnetic radiation at a frequency of 1/T.

Detecting (Receiving) the Radiation

In addition to generating EM radiation we will want to detect it. For this purpose we will use a receiving antenna through which charge will be driven by the incoming EM radiation. This current is rectified and amplified, and you will read its average value on a multimeter (although the fields come in bursts, the multimeter will show a roughly constant amplitude because the time between bursts is very short, as you will calculate in pre-lab #1).

Creating Standing Waves (sort of) Using a Reflector



If a conducting sheet (blue in the figure at left) is placed in the path of an electromagnetic wave, surface currents will be generated in the conductor that reradiate the wave backwards, as a reflected wave. The phase of the reflected wave is set so as to guarantee that the electric field at the surface of the conductor is always zero. The reflected and incoming waves interfere, summing to create a new standing wave, which has nodes every half wavelength. In the figure at left at the time pictured the waves cancel everywhere. A small time later this will no longer be the case, and the cancellation will only be perfect at those nodes (as you will calculate in pre-lab #3).

Thus in order to measure the wavelength of radiation we can simply move the receiver between the reflector and the antenna and look for nodes. Using the distance between nodes we can then calculate the wavelength.

This ideal picture doesn't perfectly hold in our case. The antenna creates dipole radiation rather than plane waves, so the amplitude of the radiation is not spatially constant, but instead falls with distance from the transmitter. Thus we do not create a perfect standing wave. However, minima in the radiation intensity will still be created where standing wave nodes occur in the ideal case, and you will still be able to measure the wavelength.

APPARATUS

1. Spark Gap Transmitter & Receiver



These have been described in detail above. The spark gap of the transmitter (pictured left) can be adjusted by turning the plastic wing nut (top). It is permanently wired in to the high voltage power supply on the circuit board. The receiver (pictured right) must be plugged in to the circuit board.

2. Circuit Board



This board contains a high voltage power supply for charging the transmitter, as well as an amplifier for boosting the signal from the receiver. It is powered by a small DC transformer that must be plugged in (AC in). When power is on, the green LED (top center) will glow.

3. Science Workshop 750 Interface and Voltage Probe

We read the signal strength from the receiver – proportional to the radiation intensity at the receiver – by connecting the output (lower right of circuit board) to a voltage probe plugged in to channel A of the 750.

GENERALIZED PROCEDURE

In this lab you will turn on the transmitter, and then, using the receiver, measure the intensity of the radiation at various locations and orientations. It consists of three main parts.

Part 1: Polarization of the Emitted Radiation

In this part you will measure to see if the produced radiation is polarized, and if so, along what axis.

Part 2: Angular Dependence of the Emitted Radiation

Next, you will measure the angular dependence of the radiation, determining if your position relative to the transmitter matters.

Part 3: Wavelength of the Emitted Radiation

Finally, you will measure the wavelength (and hence frequency) of the emitted radiation by placing a reflector near the transmitter and creating a standing-wave like pattern. By measuring the distances between minima, you can determine the wavelength (as calculated in pre-lab #3).

END OF PRE-LAB READING

Experiment 8: Microwaves

Answer these questions on a separate sheet of paper and turn them in before the lab

1. Spark Gap Distance and Timing

The time to charge the transmitter capacitor until it discharges depends on the resistance in the charging circuit ($R = 4.5 \text{ M}\Omega$), the capacitance (C = 33 pF) and the voltage required to initiate breakdown. Assume that the power supply supplies 800 V but that breakdown typically occurs at a voltage of about 500 V on the capacitor.

- (a) Thinking of the tungsten electrodes as parallel plates, how far apart must they be in order generate a spark at 500 V?
- (b) In reality, the electrodes aren't parallel plates, but rather cylinders with a fairly small radius of curvature. Given this, will the distance needed between the electrodes to generate sparking be smaller or larger than you calculated in (a)? Why?
- (c) About how much time will it take for the power supply to charge the capacitor from empty to discharge?

2. Wavelength and Frequency of the Radiation

The spark-gap antenna is a quarter-wavelength antenna, radiating as described above. Using l = 31 mm for the length of one of the arms of the antenna, what is

- (a) the wavelength of the emitted radiation?
- (b) the frequency of the emitted radiation?

3. Reflections

Now place the transmitter some distance in front of a perfectly conducting sheet, oriented so that the propagation direction of the waves hitting the reflector is perpendicular to the plane of the reflector (so that they'll reflect straight back out towards the transmitter). For example, place the transmitter at z = -D with the antenna parallel to the x-axis, and have the reflector fill the z=0 (xy-) plane.

- (a) Write an equation for the electric field component of the radiation from the transmitter (the *incident* wave). Treat the field as plane wave, with a constant amplitude E_0 and angular frequency ω_0 .
- (b) What condition must the total electric field satisfy at the surface of the conductor (z=0)?
- (c) What is the direction of propagation of the reflected wave?
- (d) Write an equation for the time dependent amplitude & direction of the reflected wave, making the same assumptions as above.
- (e) Write an equation for the total amplitude of the electric field as a function of position, by adding (c) and (d).
- (f) Nodes are locations (in this case planes) where the electric field is zero at all times. What is the distance between nodes along the z-axis?
- (g) What is the numerical distance you thus expect for our transmitter (i.e. use 2a)?

IN-LAB ACTIVITIES

EXPERIMENTAL SETUP

- 1. Download the LabView file from the web and save the file to your desktop. Start LabView by double clicking on this file.
- 2. Plug the power supply into the circuit board
- 3. Plug the receiver into the input jack on the circuit board
- 4. Turn on the transmitter a LED will light indicating it is on
- 5. Adjust the spark gap using the wing nut on the clothespin antenna. Start with a large gap, and close the gap until a steady spark is observed. You should observe a small, steady bright blue light and hear the hum of sparking.
- 6. Use the receiver to measure the intensity of the radiation as described below

MEASUREMENTS

Part 1: Polarization of the Emitted Radiation

In this part we will measure the polarization of the emitted radiation.

- 1. Press the green "Go" button above the graph to perform this process.
- 2. Rotate the receiver between the two orientations (a & b) pictured at right



Question 1:

Which orientation, if either, results in a larger signal in the receiver?

Question 2:

Is the electric field polarized? That is, is it oscillating along a certain direction, as opposed to being unpolarized in which case it points along a wide variety of directions? If it is polarized, along which axis?

Question 3:

Is the magnetic field polarized? If so, along which axis? How do you know?

Part 2: Angular Dependence of the Emitted Radiation

1. Now measure the angular dependence of the radiation intensity by moving the receiver along the two paths indicated in the below figures.





Angular dependence - Horizontal

Angular dependence - Vertical

Question 4:

Which kind of motion, horizontal or vertical, shows a larger change in radiation intensity over the range of motion?

Part 3: Wavelength of the Emitted Radiation

Now we will use the reflector to create an interference pattern that we can use to measure the wavelength of the radiation. We will do this in two ways.

- 1. Position the reflector so that it is about 30-40cm from the spark gap transmitter (note that you can insert the tube into notch on the bottom of the reflector in order to make a stable base for it to stand on). You may need to change this distance slightly to get good results. Experiment!
- 2. Orient the transmitter so that the propagation direction of the waves hitting the reflector is perpendicular to the plane of the reflector (so that they'll reflect straight back out towards the transmitter).
- 3. Place the receiver between the transmitter and the reflector and then on the other side of the reflector to see that the waves are reflected (not at all transmitted).
- 4. Place the receiver, in the best orientation, between the reflector and the transmitter very near the reflector.

Part 3A: Imprecise Method

- 1. Slowly move the receiver towards the transmitter, looking for a minimum in the radiation intensity. Be careful to keep the receiver at a constant height and in line between the transmitter and the reflector.
- 2. Measure the distance between the first and second minimum that you find.

Question 5:

What is the distance between intensity minima?

Part 3B: More Precise Method

- 1. Slowly move the receiver towards the transmitter, looking for a minimum in the radiation intensity.
- 2. Now, hold the receiver very steady and have another group member slowly pull the reflector away from the receiver. Because the reflector sits on the table it is easier to measure the distance it has traveled when the next minimum is found.
- 3. Repeat the measurement several times to get an accurate average distance reading.

Question 6:

What is the distance between intensity minima?

Question 7:

What is the wavelength of the radiation? What frequency does this correspond to?

Further Questions (for experiment, thought, future exam questions...)

- Is there any radiation intensity of any polarization off the ends of the antenna?
- An antenna similar to this was used by Marconi for his first transatlantic broadcast. What issues would you face to receive such a broadcast?
- What happens if the reflector is tilted relative to the transmitter? How tilted can it be for you to still get a reasonable result?
- If you had a second reflector on the other side of the transmitter, could you create a resonant cavity where the reflected waves from each reflector add constructively? Would it matter where the transmitter was in the cavity?