Massachusetts Institute of Technology
Department of Physics
Physics 8.022 - Fall 2002

Assignment #11
RLC Circuits
Displacement Current, Electromagnetic Waves

Reading *Purcell:* Chapters 8 and 9

Problem Set #11
Work on **all** problems. Not all problems receive equal points. Total points for this set is 100.

- **(15 points) [1]** Charge in series RLC.
  We have examined in details a series RLC circuit driven by a periodic *Emf* $E = E_0 cos(\omega t)$.

  We will now work on the expression and features of $q(t)$, the charge on the capacitor in this circuit.
  - Derive a differential equation showing the time evolution of the charge q on the capacitor (do not solve it!).
  - Use the known solution for $I(t)$ and the definition $I(t)=dq/dt$ to find an expression for $q(t)$. Show its form in complex number notation, phasor and physical charge (real number).
  - Show that the charge amplitude $q_0 = E_0/\omega Z$ is maximum for frequency

    $\omega = \sqrt{\omega_0^2 - R^2/2L^2}$. Show that for large quality factors Q, that frequency reduces to

    the natural frequency ω_0.

- **(10 points) [2]** Quality factor.
  For the series RLC circuit, show that the frequencies ω_1 and ω_2 for which the average (over a

  cycle) power provided by the source is equal to half the maximum (obtained at resonance) are given by

$$\omega_{1,2} = \pm\frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}$$

  In addition show that the quality factor of the circuit is given by

$$Q = \frac{\omega_0}{\omega_2 - \omega_1}.$$

- **(10 points) [3]** *Purcell* Problem 8.9 (p.320): Quality matters.

- **(15 points) [4]** *Purcell* Problem 8.10 (p.320): RL and C in parallel.
  For the general case of the circuit shown in figure 8.10 (p.320) derive an expression for $I(t)$ assuming it is driven by a periodic *Emf* $E = E_0 cos(\omega t)$. Describe how the circuit behaves as $\omega \to 0$ and $\omega \to \infty$.
- **(10 points) [5]** *Purcell* Problem 9.1 (p.343): Accompanying magnetic field.
- **(15 points) [6]** *Purcell* Problem 9.3 (p.343): Wave hitting a proton.
- **(15 points) [7]** *Purcell* Problem 9.5 (p.343): Electromagnetic wave.
- **(10 points) [8]** *Purcell* Problem 9.10 (p.344): Magnetic field in a capacitor.

Erotokritos Katsavounidis