Hints to Assignment #3 -- 8.022

(10 points) [1] Useful identities

- Assume u is a scalar function of space, i.e., u(x,y,z) and **V** is a vector function of space, i.e., **V**(x,y,z)=v1(x,y,z) **i^** + v2(x,y,z) **j^** + v3(x,y,z) **k^** where v1,v2,v3 are scalar functions.
- Express the divergence and curl in cartesian coordinates.
- Do a bit of regrouping of terms (recall the definition of gradient) and you are done.

(15 points) [2] Charges near a conducting plane (Purcell 3.3)

This is the first of a series of problems for which you will have to digest section 3.4 from Purcell (p.97-103).

- Call R the distance where a horizontal field line lands on the plane. That is of course a circle on the plane that is being defined.
- What is the surface charge density sigma on the plane? (did you read section 3.4?)
- If only you knew what is the total charge q induced on the plane and contained within R then you could calculate the integral of sigma over that circular disk and equate it to q thus finding an equation for R.
- Remember Mr. Gauss: flux is proportial to q (and vice versa). What is the total flux 'hitting' the disk of radius R if measured in units of number of field of lines?

(15 points) [3] More charges near a conducting plane (Purcell 3.4)

- Set up a coordinate system.
- Both the given Q and -Q are in front of a mirror, if you know what I mean. Does this tell you anything about their images?
- Write down the force balance equation. How many solutions did you come up with?

(15 points) [4] And even more charges near a conducting plane (Purcell 3.5)

- Q^2/2h would be the work spent by the external agent to move *both* charges Q and -Q (imagine, symmetrically). But is the second charge 'real' or just an 'image'?

- The charge Q at distance x can not tell the difference between the presence of the plane or the presence of the -Q at position -x (behind the mirror, distance 2x from Q). Find then what is the force Q feels at any distance x and integrate.

(15 points) [5] Spherical capacitor (Purcell 3.10)

- Find **E** in all space.
- If you know **E** you can find the potential.
- Use the definition of capacitance (from Q,phi) and you are done.
- The formal limiting behavior might require some Taylor expansion but you can probably get to right answer by some back of the envelope approximations.

(15 points) [6] Electric Force on a capacitor (Purcell 3.16)

- First of all, you have to appreciate that the plates of the capacitor are held in equilib by some mechanical forces which act against the electrical ones!
- We have proven in class that the (electric) force per unit area dF/da=u=dU/dv is the energy density of the electric field.
- You know the potential, thus you know the electric field, thus you know u. The integration is straighforward (?).
- Now, answers to the following questions will help you guide through the rest of the problem:
  - if q remains the same, does E change as the plates come closer?
  - how does the force you've just calculated change?
  - what is the energy stored in the capacitor initially and finally (plates collapsed)?
  - do you have any reason to believe conservation of energy is violated?

(15 points) [7] Design of a spherical capacitor (Purcell 3.17)

This can be a real life problem if you work in the capacitor industry... But you are well prepared as you've solved by now problem 3.10.

- Armed with the math from 3.10 assume b=(lambda) x a and express the energy stored in the capacitor in terms of lambda, a and E_o
- Find the extrema of the energy function in terms of lambda (a, E_o are fixed).
