ESG 8.022 Fall 2006 Exam 2

Instructor: Michael Shaw

November 7, 2006@ 2:00PM

1 Short Answer Questions

a. Find the magnetic field whose vector potential is $\vec{A} = A_0(x-y)\hat{z}$

b. How is it possible that an observer on a moving train sees clocks on the ground ticking slowly, and an observer on the ground see clocks on the moving train tick slowly?

c. Two straight parallel wires carry currents 2I and I that run in opposite directions in them. What is the direction of the magnetic force on the first wire?

d. Explain the meaning of $\nabla \cdot J = \frac{\partial \rho}{\partial t}$. Be sure to distinguish between local and global.

2 Field of a Wire with non-uniform current

Consider a long wire of radius a centered on the z axis with current density $\vec{J} = J_0 \frac{r^2}{a^2}\hat{z}$, where r is the distance from the z axis.

a. Determine the direction of the magnetic field inside and outside of the wire. No calculation is required.

b. What is the strength of the magnetic field inside the wire?

c. What is the strength of the magnetic field outside the wire?

d. What is the divergence and curl of B inside and outside the wire?

e. Consider a new wire with a total current I equal to the current in the old wire, but with all the current constrained to the cylindrical wall of the wire. Without any calculation, find B inside and outside this wire. Do not worry about $\vec{B}(r=a)$
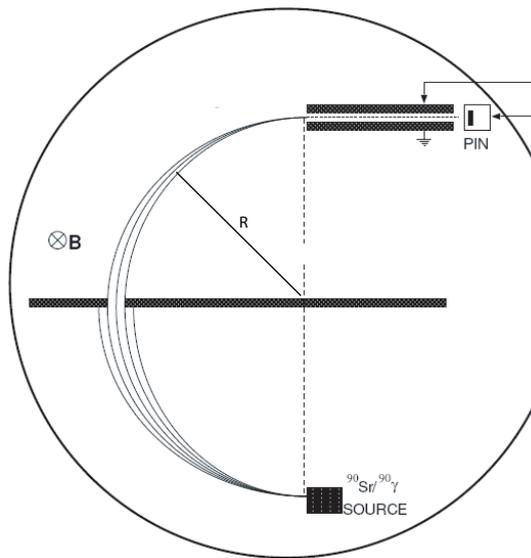
3   Particle Moving in Electric and Magnetic Field

The velocity selector pictured above is taken from the Physics Junior Lab. Junior lab students study relativistic dynamics with this apparatus. We will investigate a simpler (and not dependant on relativity) version of the problem.

    a. The Strontium source at the bottom releases electrons at high velocities into a region of constant magnetic field into the page. The plate in the middle is intended to "select" some electron velocities. Why does that work?

    b. The constant magnetic field has magnitude B. If the gap in that middle plate is of width ΔR, how large a spread in velocities (Δv) can get through the gap? You may leave your answer in terms of B, the electron mass, m_e, and the electron charge q.

    c. At the top of the diagram is a long straight region between two parallel plates. These plates are charged, creating an electric field between them. Junior lab students tune this electric field to cancel the force of the magnetic field, allowing particles to travel through with no forces acting on them. If the magnetic field is of magnitude B into the page, what is the necessary electric field? Leave your answer in terms of B, R, m_e and q.

    d. Extra Credit: Using constant currents, how could one create a uniform magnetic field?