Capacitance & Capacitors, Energy Stored in Capacitors
Challenge Problems

Problem 1:

A parallel-plate capacitor is charged to a potential V_0, charge Q_0 and then disconnected from the battery. The separation of the plates is then halved. What happens to

(a) the charge on the plates?

(b) the electric field?

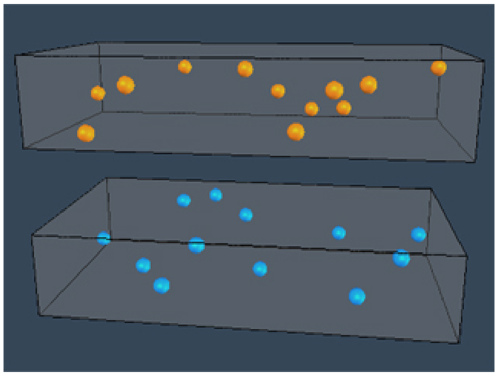
(c) the energy stored in the electric field?

(d) the potential?

(e) How much work did you do in halving the distance between the plates?

Problem 2:
The simulation at
http://web.mit.edu/viz/EM/visualizations/electrostatics/CapacitorsAndCondcutors/capacitor/capacitor.htm

illustrates the interaction of charged particles inside the two plates of a capacitor. Each plate contains twelve charges interacting via the Coulomb and Pauli forces, where one plate contains positive charges and the other contains negative charges.



(a) Before running the simulation, **PREDICT** will happen to the charges (i.e. how will they arrange themselves). Now run the simulation. Describe what you observe.

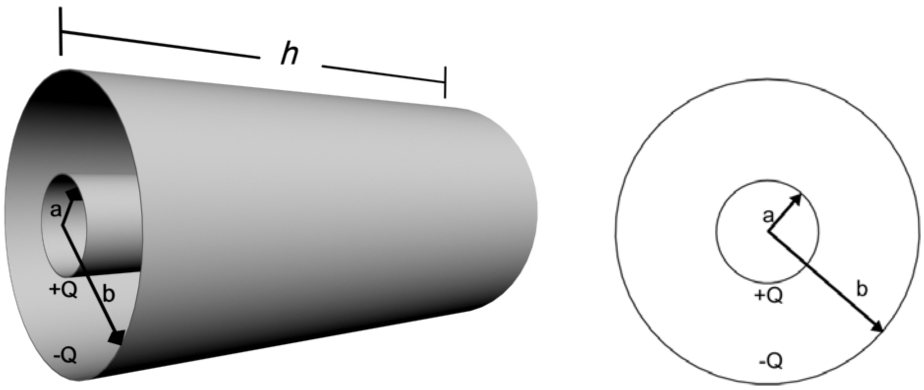
(b) Suppose *both* the top and bottom plates now consist of twelve ***negative*** charges. What do you expect to see and why?

(c) Keeping the number of charges on the ***bottom*** array the same (and negative), what do you suppose would happen if the **top** array had a larger amount of charge (i.e. sixteen positive charges, instead of twelve)? Explain.

(d) Suppose you now have *six positive charges* **AND** *six negative charges* on the top array and further suppose that the bottom array also consists of *six positive charges* **AND** six negative charges. What do you expect will happen and why?

Problem 3:

Part 1 Consider two nested cylindrical conductors of height h and radii a & b respectively. A charge $+Q$ is evenly distributed on the outer surface of the pail (the inner cylinder), $-Q$ on the inner surface of the shield (the outer cylinder).



(a) Calculate the electric field between the two cylinders ($a < r < b$).

(b) Calculate the potential difference between the two cylinders:

(c) Calculate the capacitance of this system, $C = Q/\Delta V$

(d) Numerically evaluate the capacitance for your experimental setup, given:
$h \cong 15$ cm, $a \cong 4.75$ cm and $b \cong 7.25$ cm

(e) Find the electric field energy density at any point between the conducting cylinders. How much energy resides in a cylindrical shell between the conductors of radius r (with $a < r < b$), height h**,** thickness dr, and volume $2\pi rh\,dr$? Integrate your expression to find the total energy stored in the capacitor and compare your result with that obtained using $U_E = (1/2)C(\Delta V)^2$.

Problem 4:

A parallel-plate capacitor is charged to a potential V_0, charge Q_0 and then disconnected from the battery. The separation of the plates is then halved. What happens to

(a) the charge on the plates?

(b) the electric field?

(c) the energy stored in the electric field?

(d) the potential?

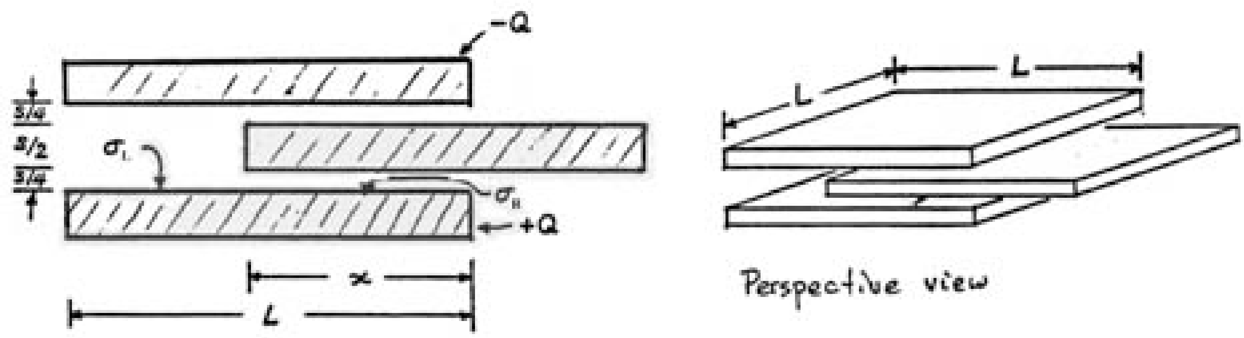
(e) How much work did you do in halving the distance between the plates?

Problem 5:

What, approximately, is the capacitance of a typical MIT student? Check out the exhibit in Strobe Alley (4^{th} floor of building 4) for a hint or just to check your answer.

Problem 6:

Two flat, square metal plates have sides of length L, and thickness $s/2$, are arranged parallel to each other with a separation of s, where $s << L$ so you may ignore fringing fields. A charge Q is moved from the upper plate to the lower plate. Now a force is applied to a third uncharged conducting plate of the same thickness $s/2$ so that it lies between the other two plates to a depth x, maintaining the same spacing $s/4$ between its surface and the surfaces of the other two. You may neglect edge effects.



a) Using the fact that the metals are equipotential surfaces, what are the surface charge densities σ_L on the lower plate adjacent to the wide gap and σ_R on the lower plate adjacent to the narrow gap?

b) What is the electric field in the wide and narrow gaps? Express your answer in terms of L, x, and s.

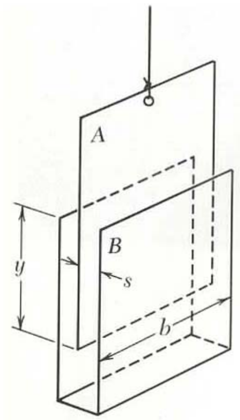
c) What is the potential difference between the lower plate and the upper plate?

d) What is the capacitance of this system?

e) How much energy is stored in the electric field?

Problem 7

A flat conducting sheet A is suspended by an insulating thread between the surfaces formed by the bent conducting sheet B as shown in the figure on the left. The sheets are oppositely charged, the difference in potential, in statvolts, is $\Delta\phi$. This causes a force F, in addition to the weight of A, pulling A downward.



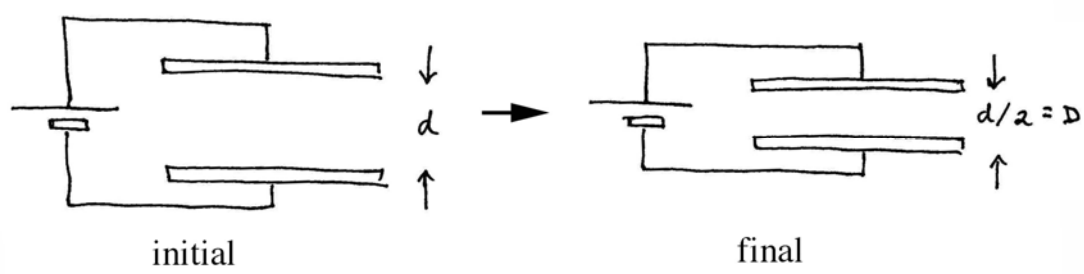
a) What is the capacitance of this arrangement of conductors as a function of y, the distance that plate A is inserted between the sides of plate B?

b) How much energy is needed to increase the inserted distance by Δy?

c) Find an expression for the difference in potential $\Delta\phi$ in terms of F and relevant dimensions shown in the figure.

Problem 8:

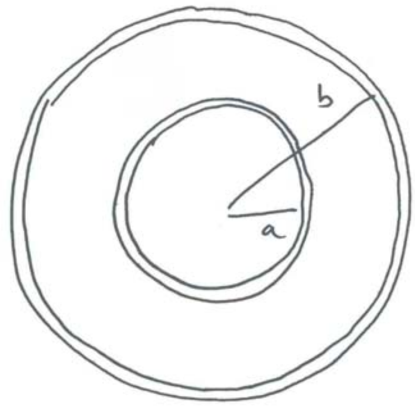
Consider a simple parallel-plate capacitor whose plates are given equal and opposite charges and are separated by a distance d. The capacitor is connected to a battery. Suppose the plates are pushed together until they are separated by a distance D = d/2. How does the final electrostatic energy stored in the capacitor compare to the initial energy?



a) Final is half the initial.
b) Final is one fourth the initial.
c) Final is twice than initial.
d) Final is four times the initial.
e) They are the same.

Problem 9

Consider a spherical vacuum capacitor consisting of inner and outer thin conducting spherical shells with charge $+Q$ on the inner shell of radius a and charge $-Q$ on the outer shell of radius b. You may neglect the thickness of each shell.



a) What are the magnitude and direction of the electric field everywhere in space as a function of r, the distance from the center of the spherical conductors?

b) What is the capacitance of this capacitor?

c) Now consider the case that the dimension of the outer shell is doubled from b to $2b$. Assuming that the charge on the shells is not changed, how does the stored potential energy change? That is, find an expression for $\Delta U \equiv U_{after} - U_{before}$ in terms of Q a, b, and ε_0 as needed.

Problem 10:

A parallel-plate capacitor has fixed charges $+Q$ and $-Q$. The separation of the plates is then doubled.

(a) By what factor does the energy stored in the electric field change?

(b) How much work must be done if the separation of the plates is doubled from d to $2d$? The area of each plate is A.

Consider now a cylindrical capacitor with inner and outer radii a and b, respectively.

(c) Suppose the outer radius b of a cylindrical capacitor is doubled, but the charge is kept constant. By what factor would the stored energy change? Where would the energy come from?

(d) Repeat (c), assuming the voltage remains constant.

8.02SC Physics II: Electricity and Magnetism
Fall 2010