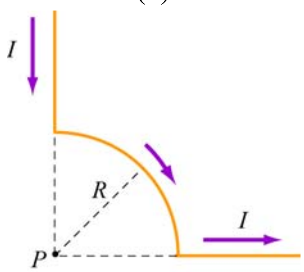
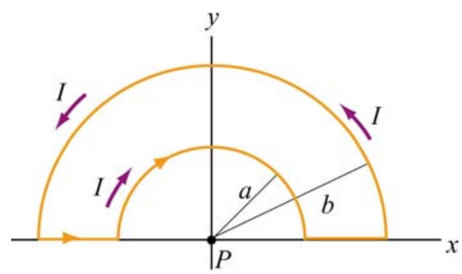
Creating Fields: Biot-Savart Law
Challenge Problems

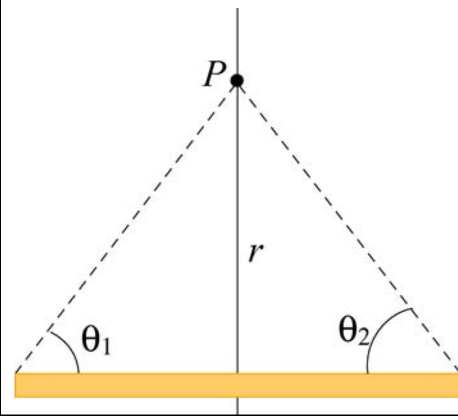
Problem 1:
Find the magnetic field at point *P* due to the following current distributions:

(a)	(b)
	

Problem 2:

A conductor in the shape of a square loop of edge length $\ell = 0.400$ m carries a current $I =$ 10.0 A as in the figure.

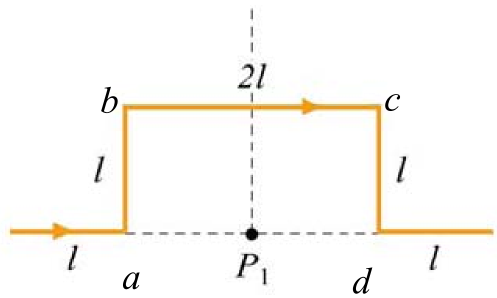
(a) Calculate the magnitude and direction of the magnetic field at the center of the square.



(b) If this conductor is formed into a single circular turn and carries the same current, what is the value of the magnetic field at the center?

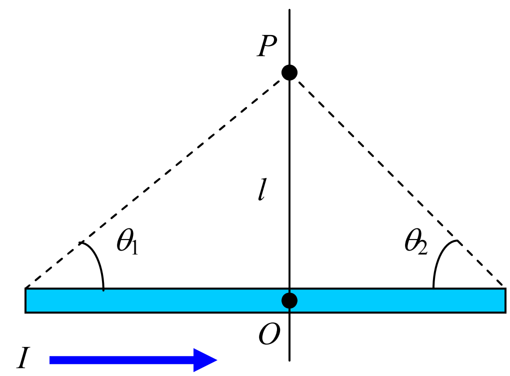
Problem 3:

A wire is bent into the shape shown on the right, and the magnetic field is measured at P_1 when the current in the wire is I.



From the discussion given in Example 9.1 The magnetic field is calculated as

$$B = \frac{\mu_0 I}{4\pi l}(\cos\theta_2 + \cos\theta_1)$$



For $a \to b$, $\theta_1 = \frac{\pi}{2}$ and $\theta_2 = \frac{\pi}{4}$

$$B_{ab} = \frac{\mu_0 I}{4\pi l}\left(\frac{1}{\sqrt{2}} + 0\right) = \frac{\sqrt{2}\mu_0 I}{8\pi R}$$

For $b \to c$, $\theta_1 = \frac{\pi}{4}$ and $\theta_2 = \frac{\pi}{4}$

$$B_{bc} = \frac{\mu_0 I}{4\pi l}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \frac{\sqrt{2}\mu_0 I}{4\pi R}$$

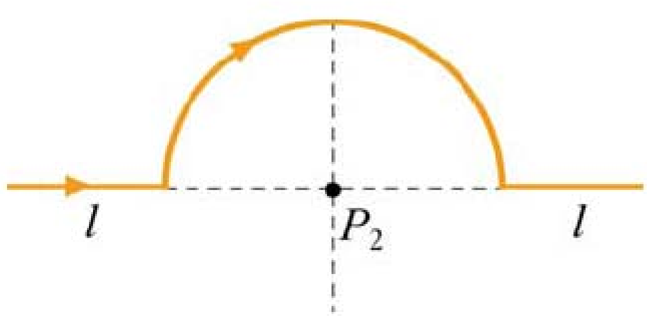
For $c \to d$, $\theta_1 = \frac{\pi}{4}$ and $\theta_2 = \frac{\pi}{2}$

$$B_{cd} = \frac{\mu_0 I}{4\pi l}\left(0 + \frac{1}{\sqrt{2}}\right) = \frac{\sqrt{2}\mu_0 I}{8\pi R}$$

Therefore,

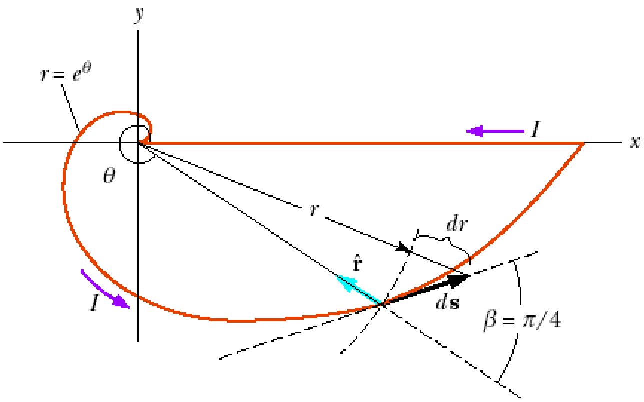
$$B_1 = B_{ab} + B_{bc} + B_{cd} = \frac{\sqrt{2}\mu_0 I}{2\pi l} \text{ (into page)}$$

The same segment of wire is then bent into a semi-circular shape shown in the figure below, and the magnetic field is measured at point P_2 when the current is again I. If the total length of wire is the same in each case, what is the ratio of B_1/B_2?



Problem 4:

A wire carrying a current I is bent into the shape of an exponential spiral, $r = e^{\theta}$, from $\theta = 0$ to $\theta = 2\pi$ as shown in the figure below.



To complete a loop, the ends of the spiral are connected by a straight wire along the x axis. Find the magnitude and direction of $\vec{\mathbf{B}}$ at the origin.

Hint: Use the Biot–Savart law. The angle β between a radial line and its tangent line at any point on the curve $r = f(\theta)$ is related to the function in the following way:

$$\tan\beta = \frac{r}{dr/d\theta}$$

Thus in this case $r = e^{\theta}$, $\tan\beta = 1$ and $\beta = \pi/4$. Therefore, the angle between $d\vec{\mathbf{s}}$ and $\hat{\mathbf{r}}$ is $\pi - \beta = 3\pi/4$. Also

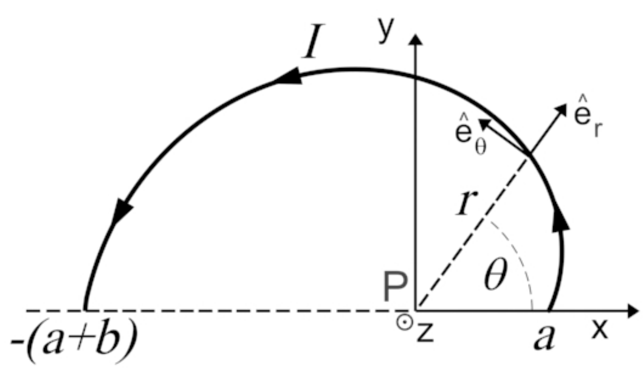
$$ds = \frac{dr}{\sin(\pi/4)} = \sqrt{2}\,dr$$

Problem 5:

A wire segment is bent into the shape of an Archimedes spiral (see sketch). The equation that describes the curve in the range $0 \le \theta \le \pi$ is

$$r(\theta) = a + \frac{b}{\pi}\theta, \quad \text{for } 0 \le \theta \le \pi$$

where θ is the angle from the x-axis in radians. The point P is located at the origin of our xy coordinate system. The vectors $\hat{\mathbf{e}}_r$ and $\hat{\mathbf{e}}_\theta$ are the unit vectors in the radial and azimuthal directions, respectively, as shown. The wire segment carries current I, flowing in the sense indicated.



What is the magnetic field at point P?

Problem 6:

Four infinitely long parallel wires carrying equal current I are arranged in such a way that when looking at the cross section, they are at the corners of a square, as shown in the figure below. Currents in A and D point out of the page, and into the page at B and C. What is the magnetic field at the center of the square?