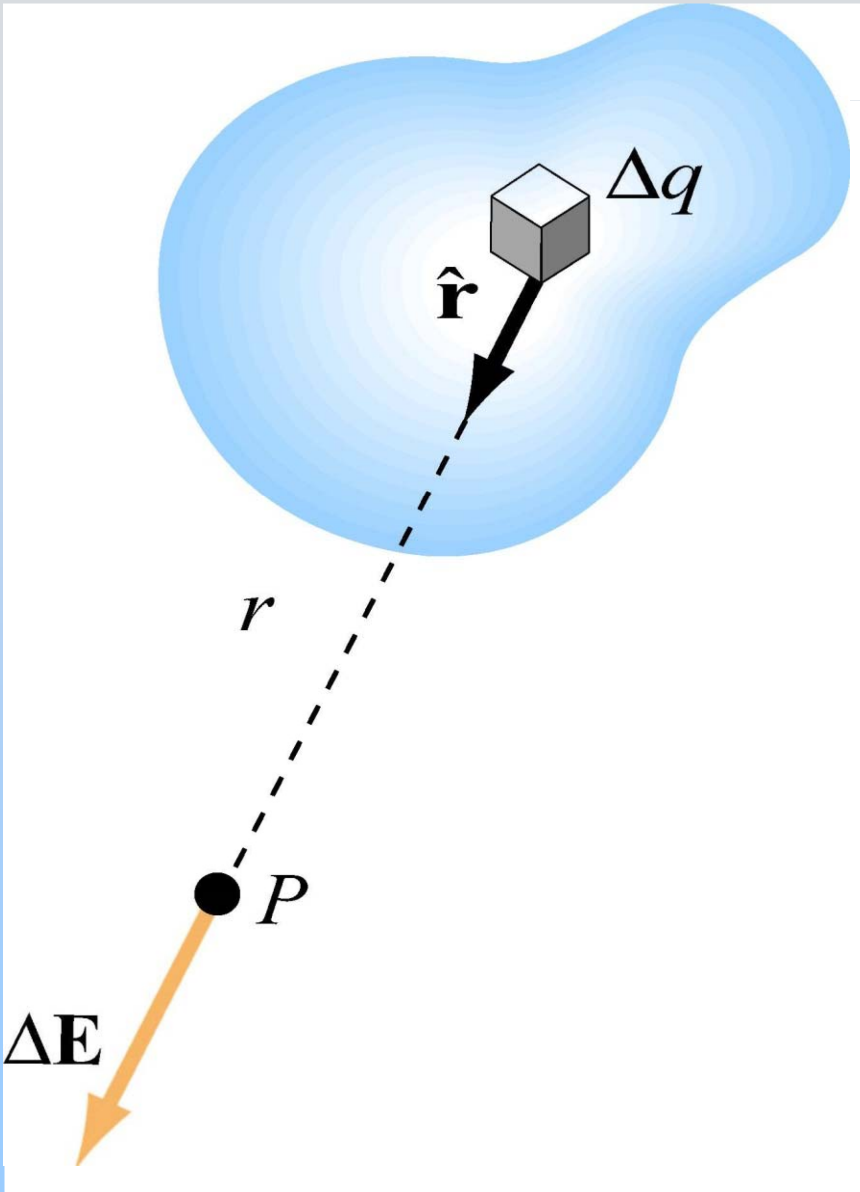
Module 04: Electric Fields and Continuous Charge Distributions

Continuous Charge Distributions



Break distribution into parts:

$$Q = \sum_i \Delta q_i \to \iiint_V dq$$

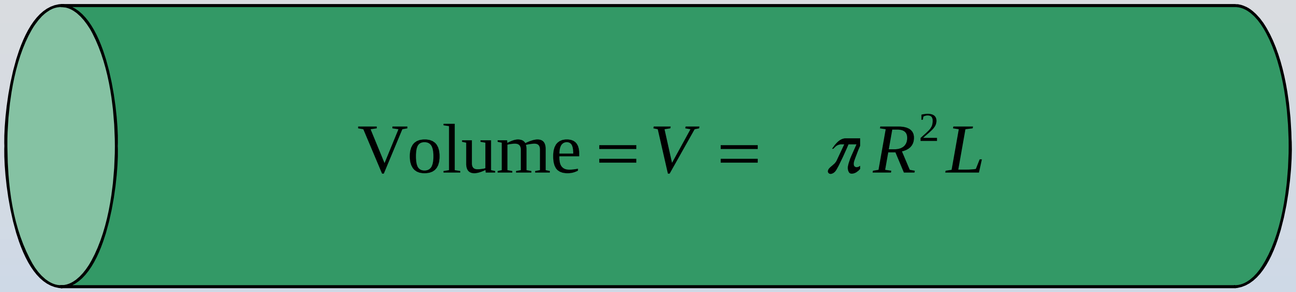
E field at P due to Δq

$$\Delta\vec{\mathbf{E}} = k_e \frac{\Delta q}{r^2}\hat{\mathbf{r}} \to d\vec{\mathbf{E}} = k_e \frac{dq}{r^2}\hat{\mathbf{r}}$$

Superposition:

$$\vec{\mathbf{E}} = \sum \Delta\vec{\mathbf{E}} \to \int d\vec{\mathbf{E}}$$

Continuous Sources: Charge Density



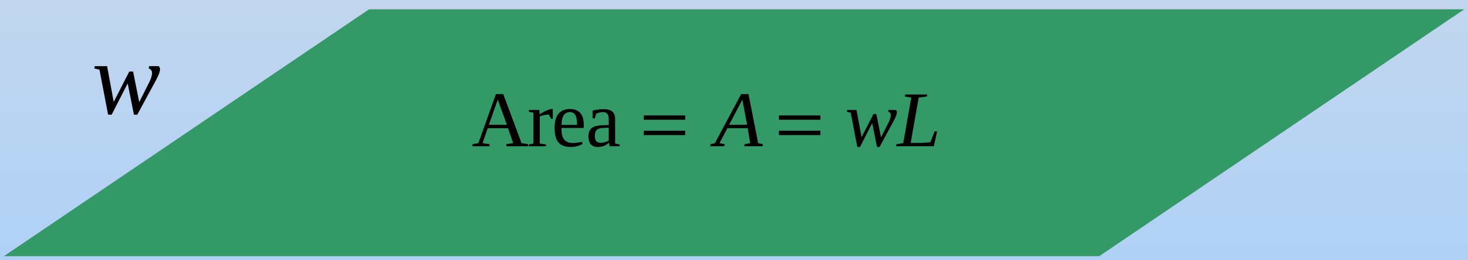
R

$$\text{Volume} = V = \pi R^2 L$$

L

$$dQ = \rho\, dV$$

$$\rho = \frac{Q}{V}$$



w

$$\text{Area} = A = wL$$

L

$$dQ = \sigma\, dA$$

$$\sigma = \frac{Q}{A}$$

$$\text{Length} = L$$

L

$$dQ = \lambda\, dL$$

$$\lambda = \frac{Q}{L}$$

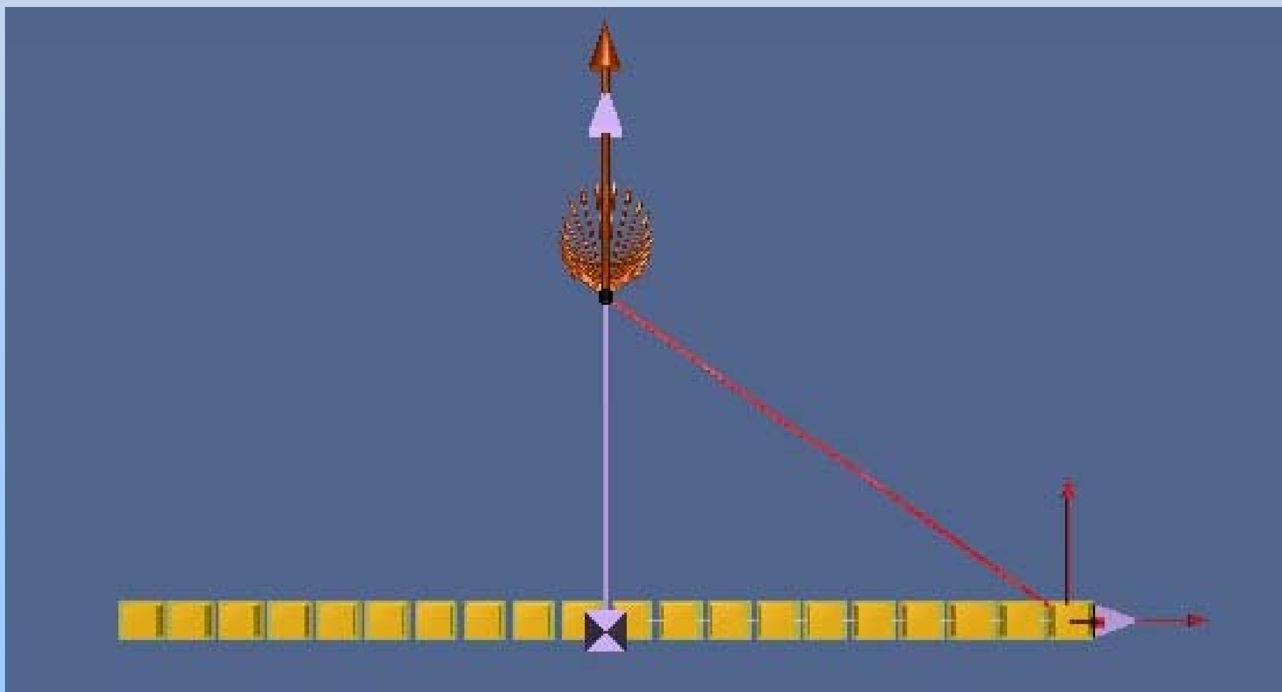
Examples of Continuous Sources: Line of charge

$$\text{Length} = L$$

$$L$$

$$dQ = \lambda \, dL$$

$$\lambda = \frac{Q}{L}$$



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Examples of Continuous Sources: Line of charge

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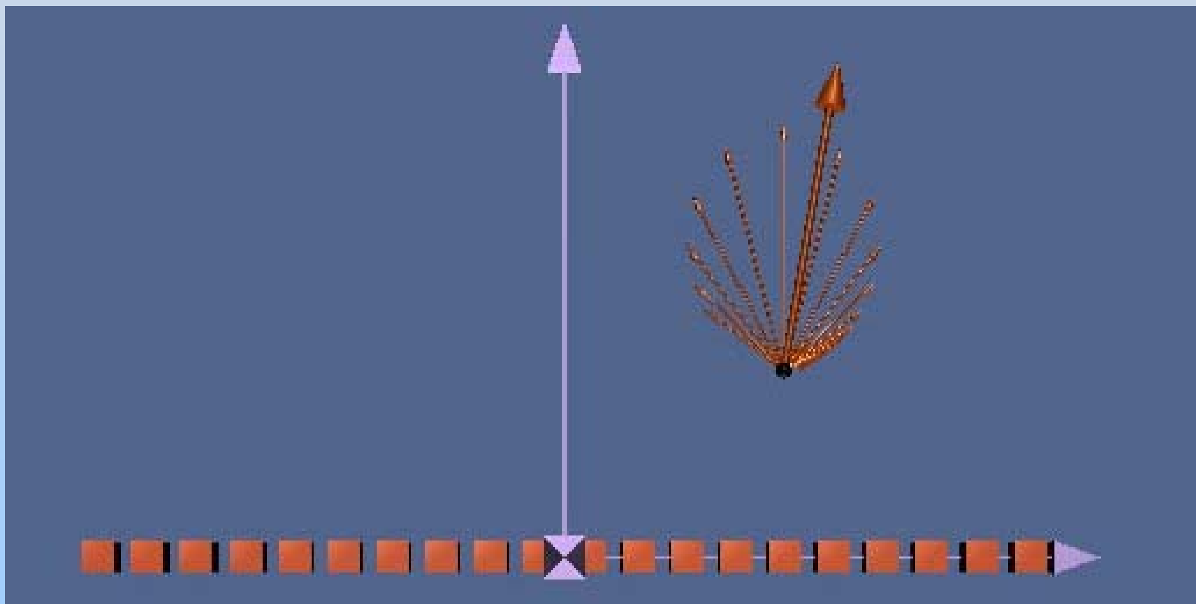

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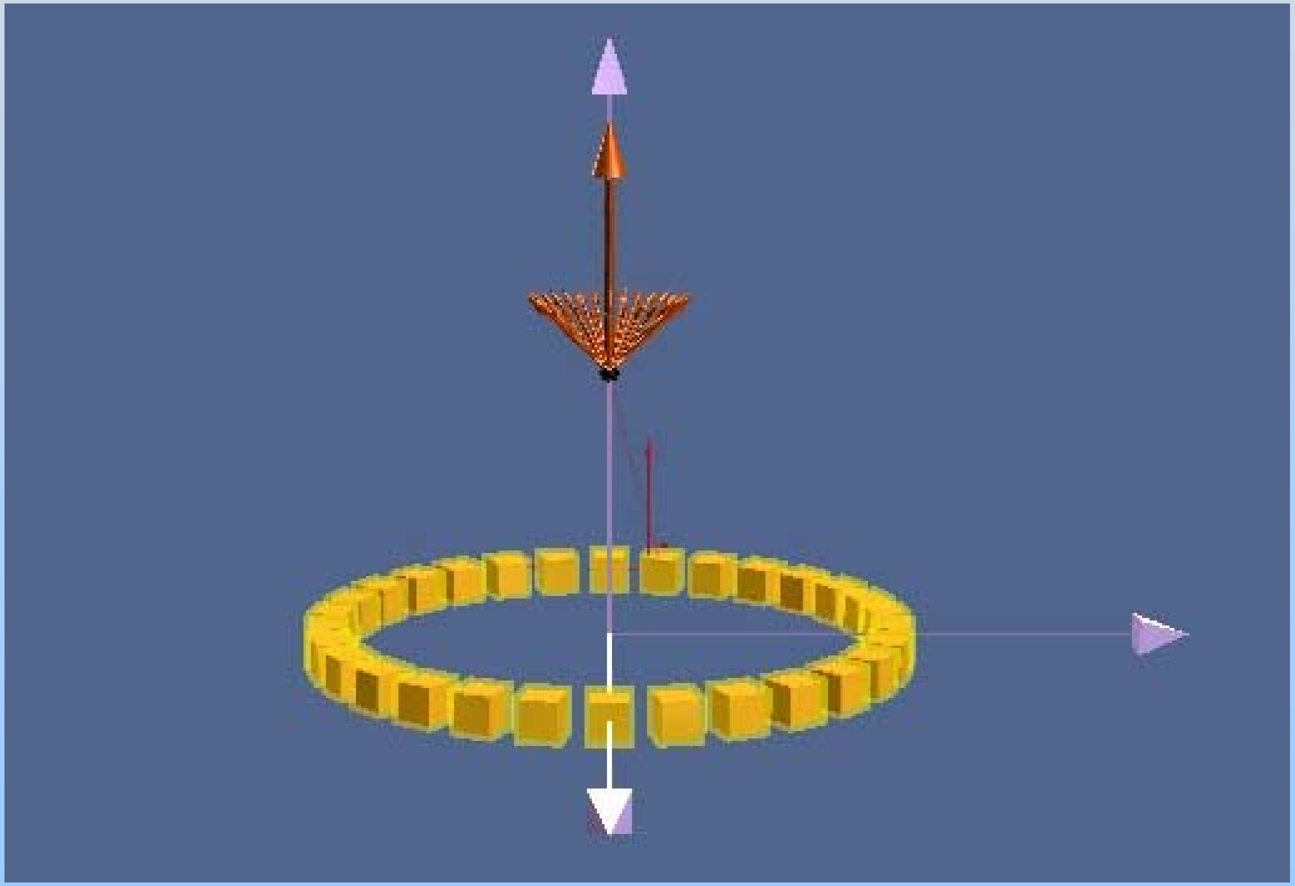
Examples of Continuous Sources: Ring of Charge

$$dQ = \lambda \, dL \qquad \lambda = \frac{Q}{2\pi R}$$



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Examples of Continuous Sources: Ring of Charge

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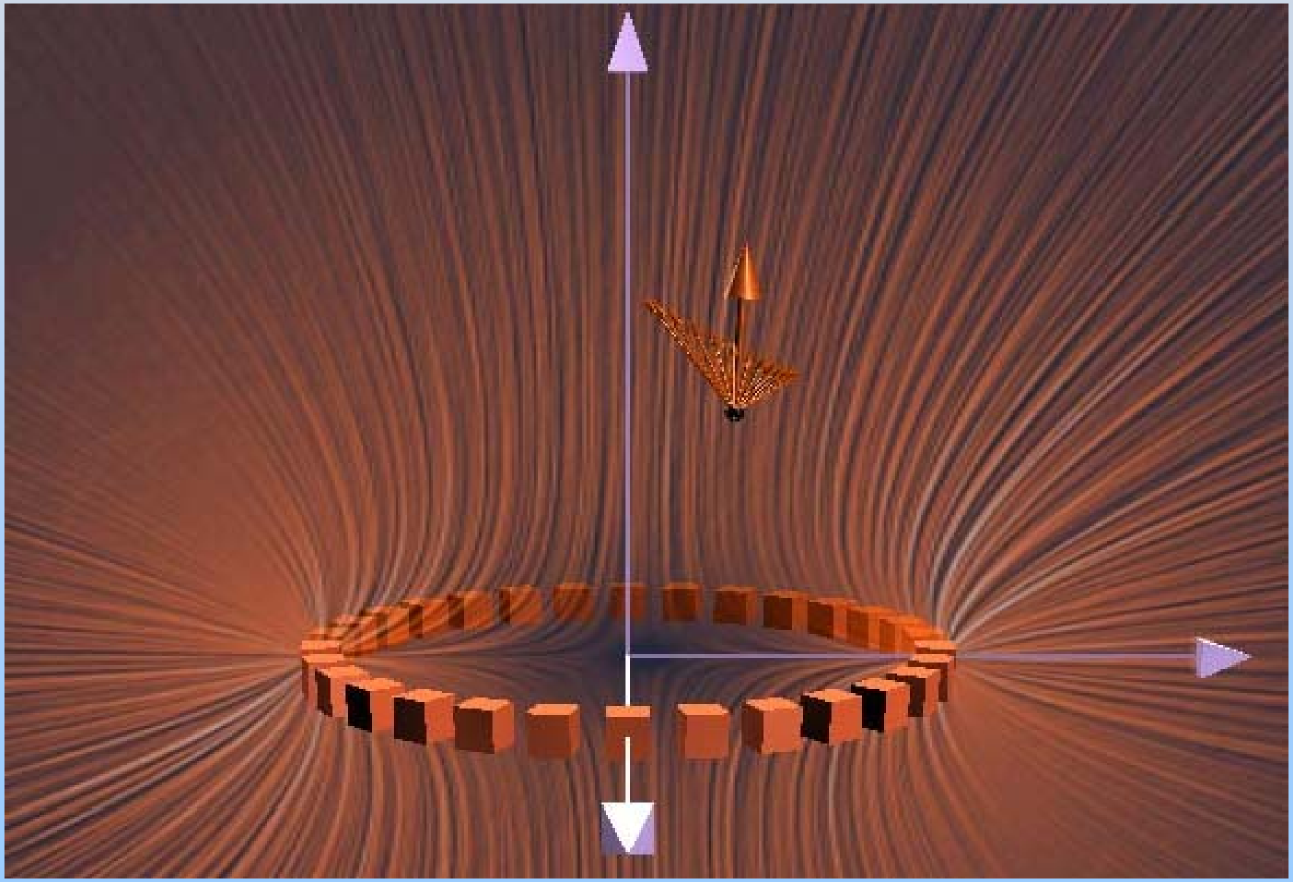

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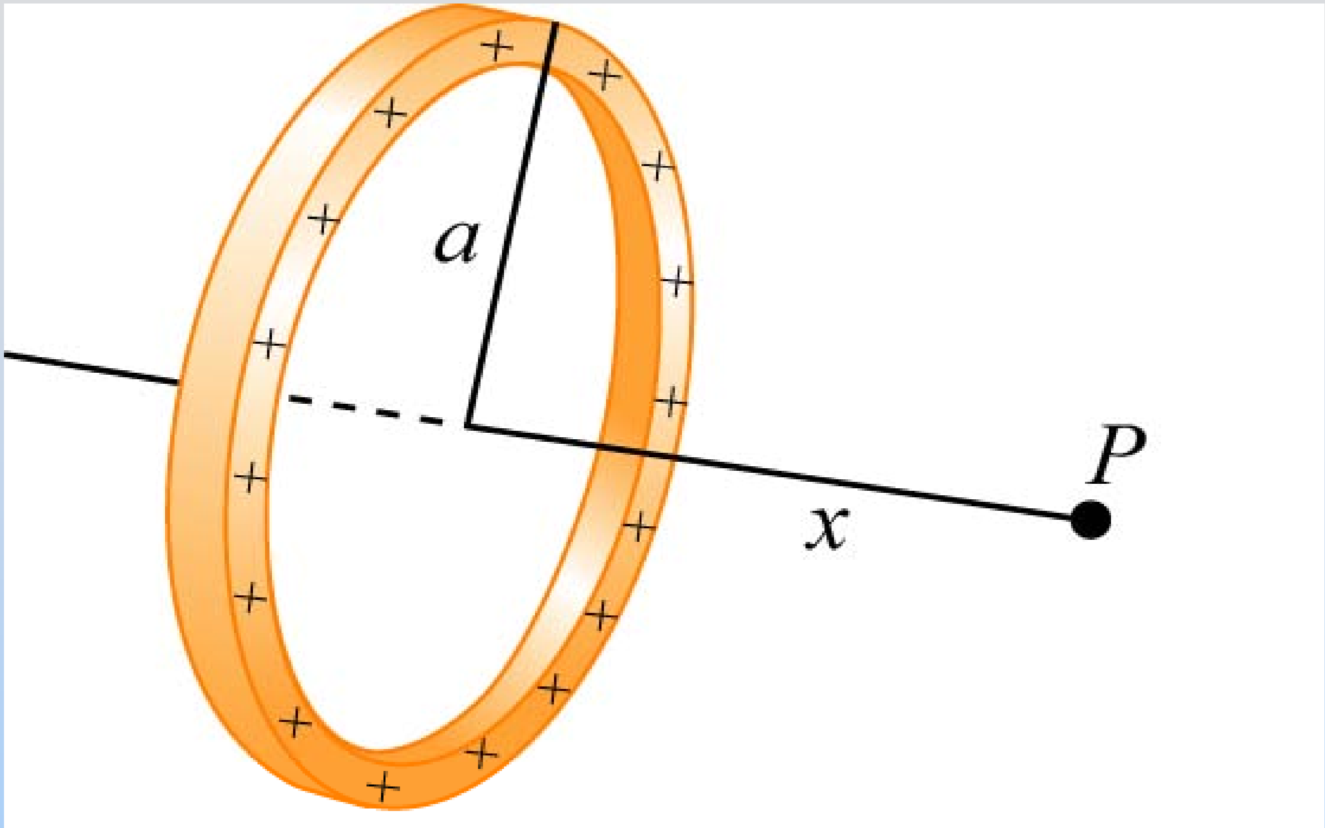
Example: Ring of Charge



P on axis of ring of charge, *x* from center

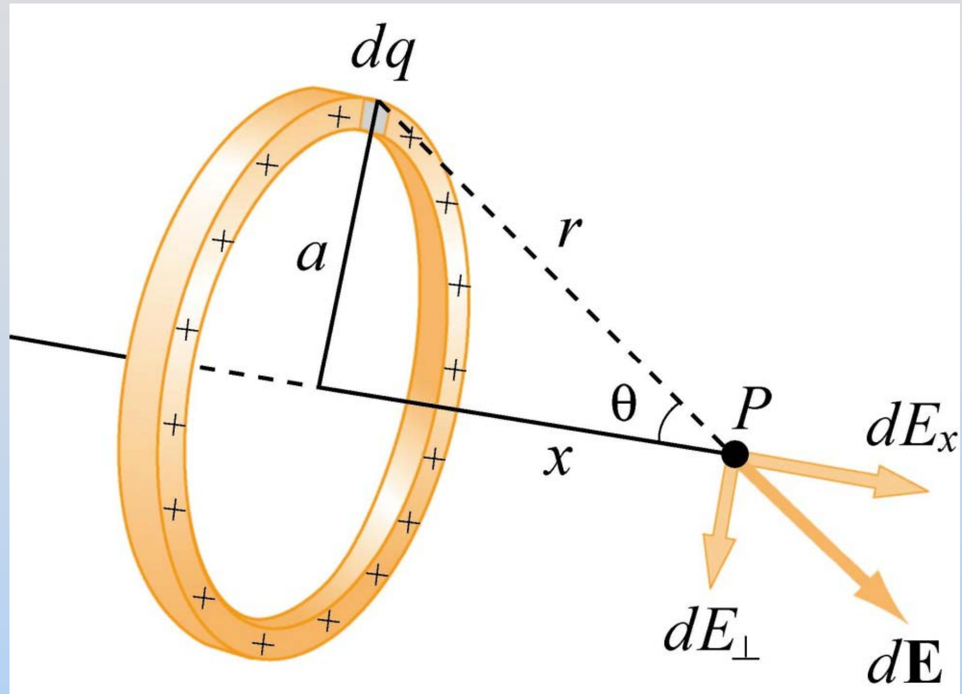
Radius *a*, charge density λ.

Find **E** at P

Ring of Charge

1) Think about it

$E_{\perp} = 0$ Symmetry!



2) Define Variables

$$dq - \lambda\, dl - \lambda\left(a\, d\varphi\right)$$

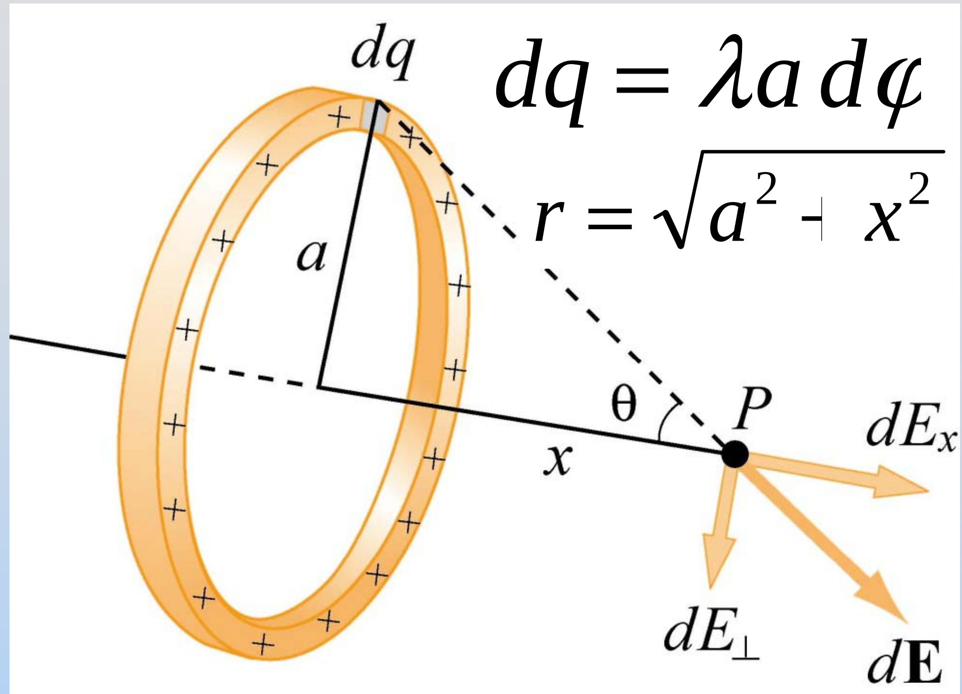
$$r = \sqrt{a^2 + x^2}$$

Ring of Charge

3) Write Equation

$$d\vec{\mathbf{E}} = k_e dq \frac{\hat{r}}{r^2} = k_e \, dq \frac{\vec{r}}{r^3}$$

$$dE_x = k_e dq \frac{x}{r^3}$$



$$dq = \lambda a \, d\varphi$$

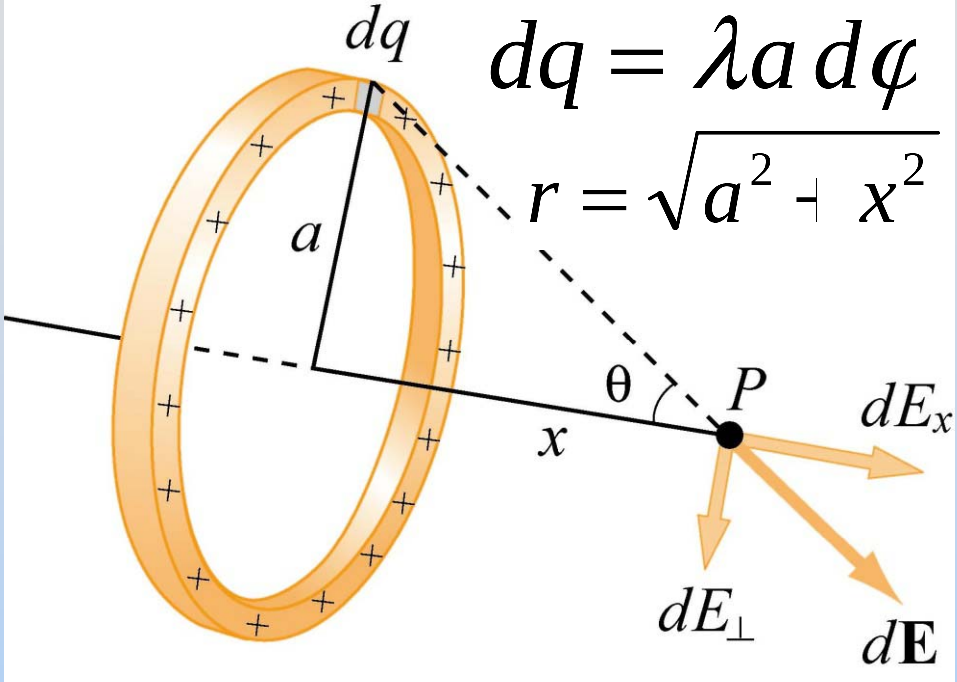
$$r = \sqrt{a^2 + x^2}$$

Ring of Charge

4) Integrate

$$E_x = \int dE_x = \int k_e \, dq \frac{x}{r^3}$$

$$= k_e \frac{x}{r^3} \int dq$$



$$dq = \lambda a \, d\varphi$$

$$r = \sqrt{a^2 + x^2}$$

Very special case: everything except *dq* is constant

$$\int dq = \int_0^{2\pi} \lambda a \, d\varphi = \lambda a \int_0^{2\pi} d\varphi = \lambda \cdot a 2\pi$$

$$= Q$$

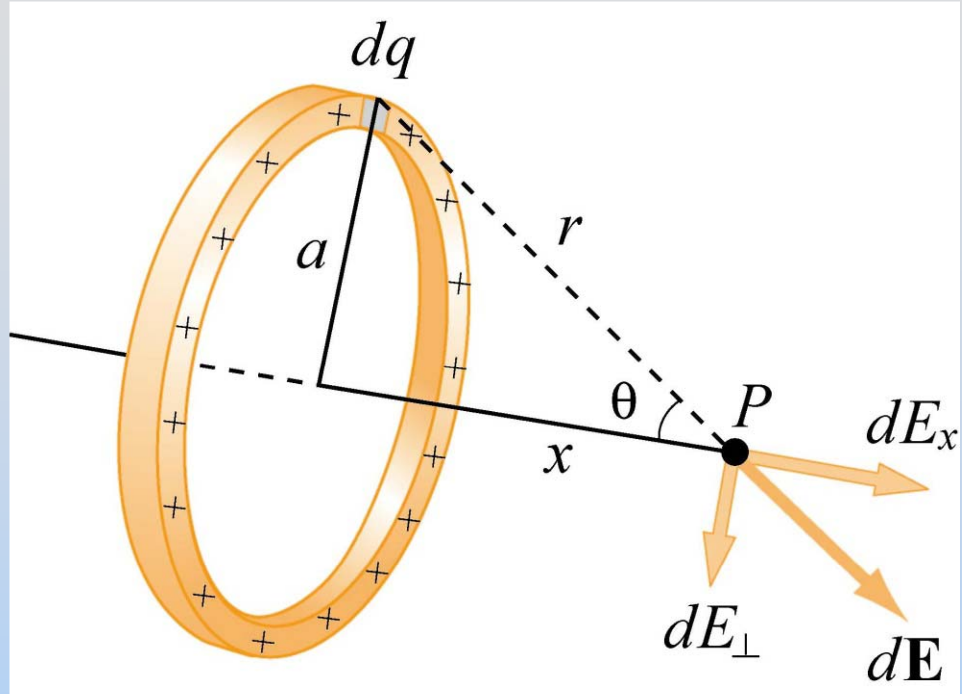
Ring of Charge

5) Clean Up

$$E_x = k_e Q \frac{x}{r^3}$$

$$E_x = k_e Q \frac{x}{\left(a^2 + x^2\right)^{3/2}}$$

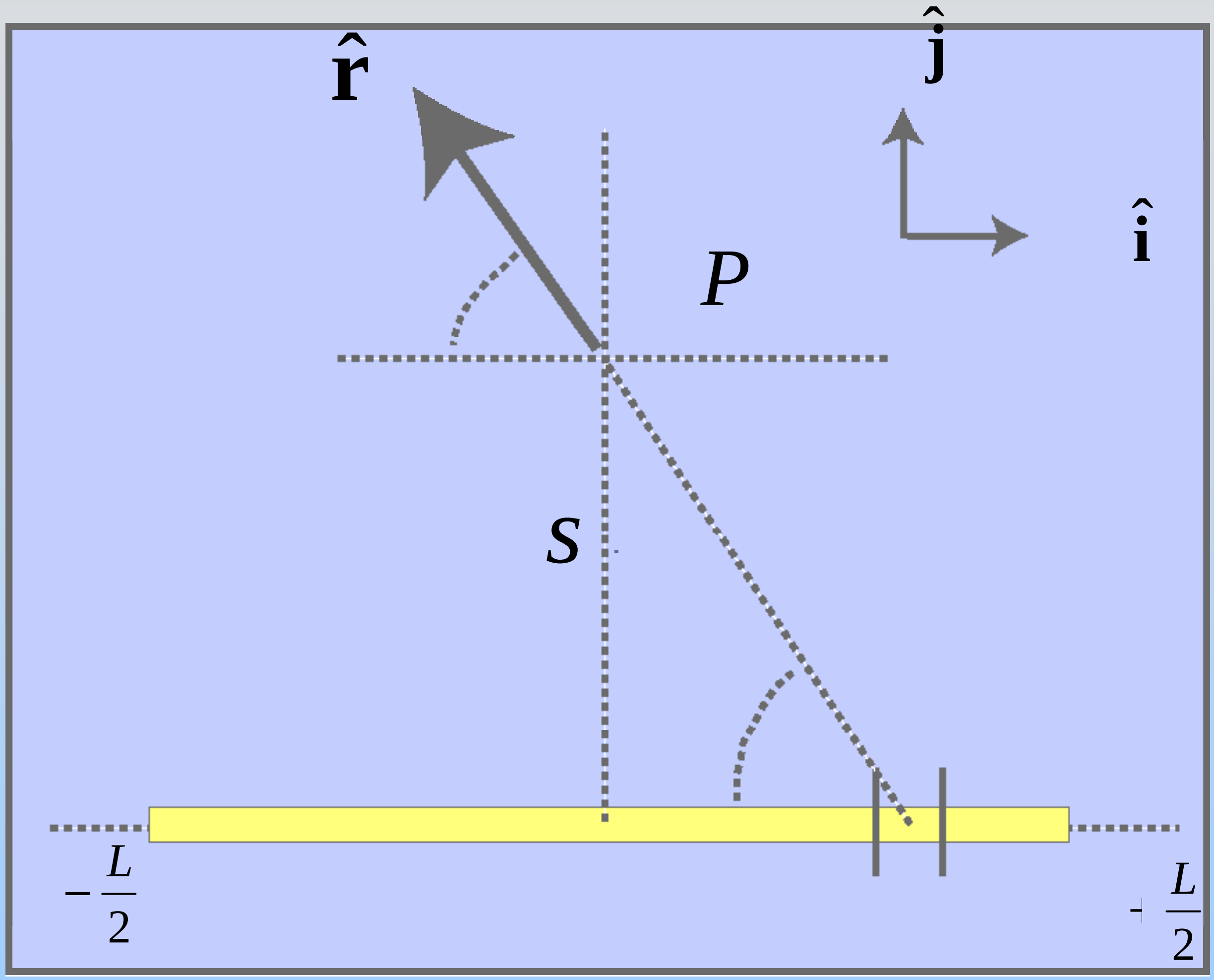
$$\vec{\mathbf{E}} = k_e Q \frac{x}{\left(a^2 + x^2\right)^{3/2}} \hat{\mathbf{i}}$$



6) Check Limit $a - \ 0$

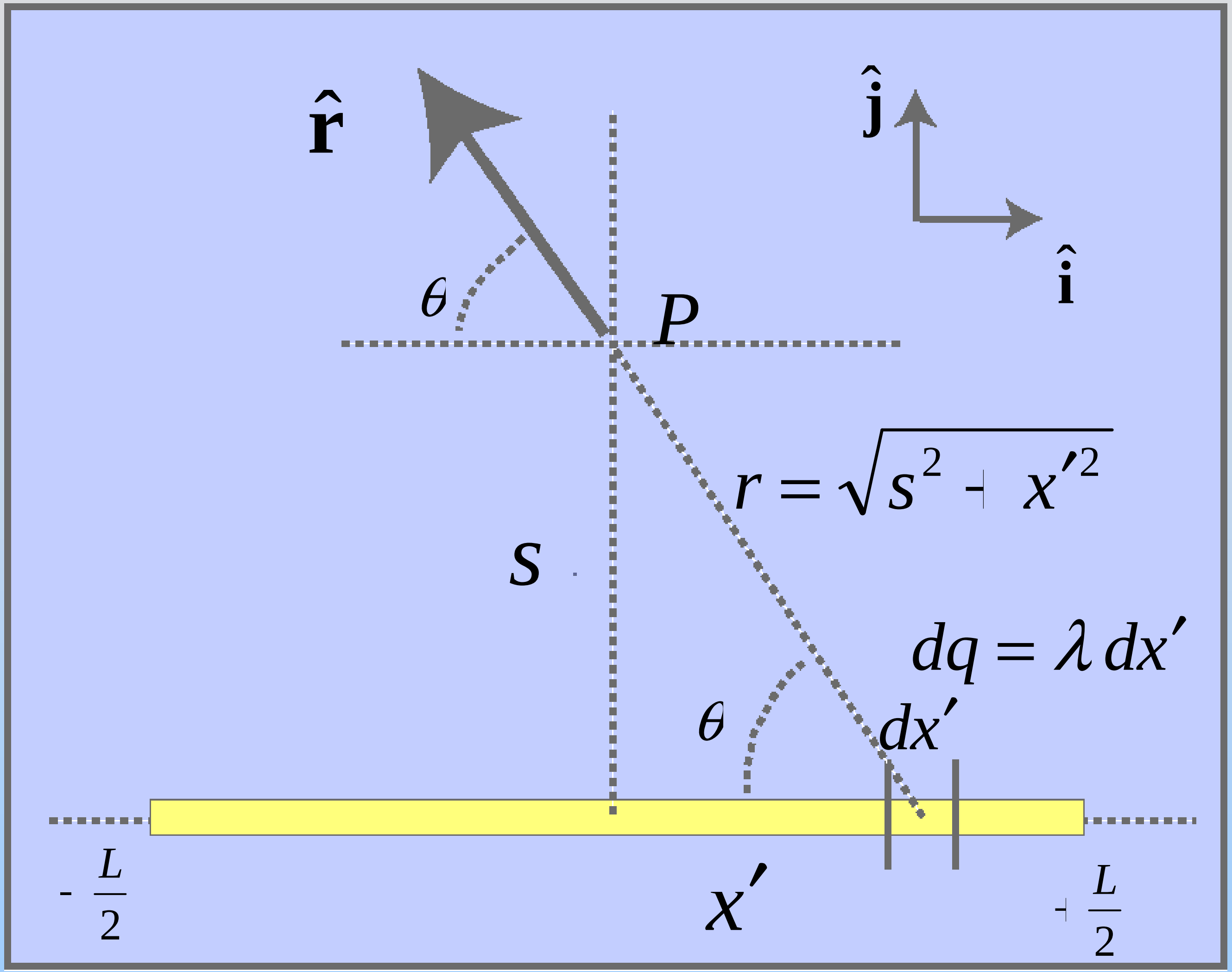
$$E_x \rightarrow k_e Q \frac{x}{\left(x^2\right)^{3/2}} = \frac{k_e Q}{x^2}$$

Chkpt. Problem: Line of Charge



Point *P* lies on perpendicular bisector of uniformly charged line of length *L*, a distance *s* away. The charge on the line is *Q*. What is ***E*** at *P*?

Hint: Line of Charge



Typically give the integration variable (*x'*) a "primed" variable name. ALSO: Difficult integral (trig. sub.)

E Field from Line of Charge

$$\vec{\mathbf{E}} = k_e \frac{Q}{s(s^2 + L^2/4)^{1/2}} \hat{\mathbf{j}}$$

Limits:

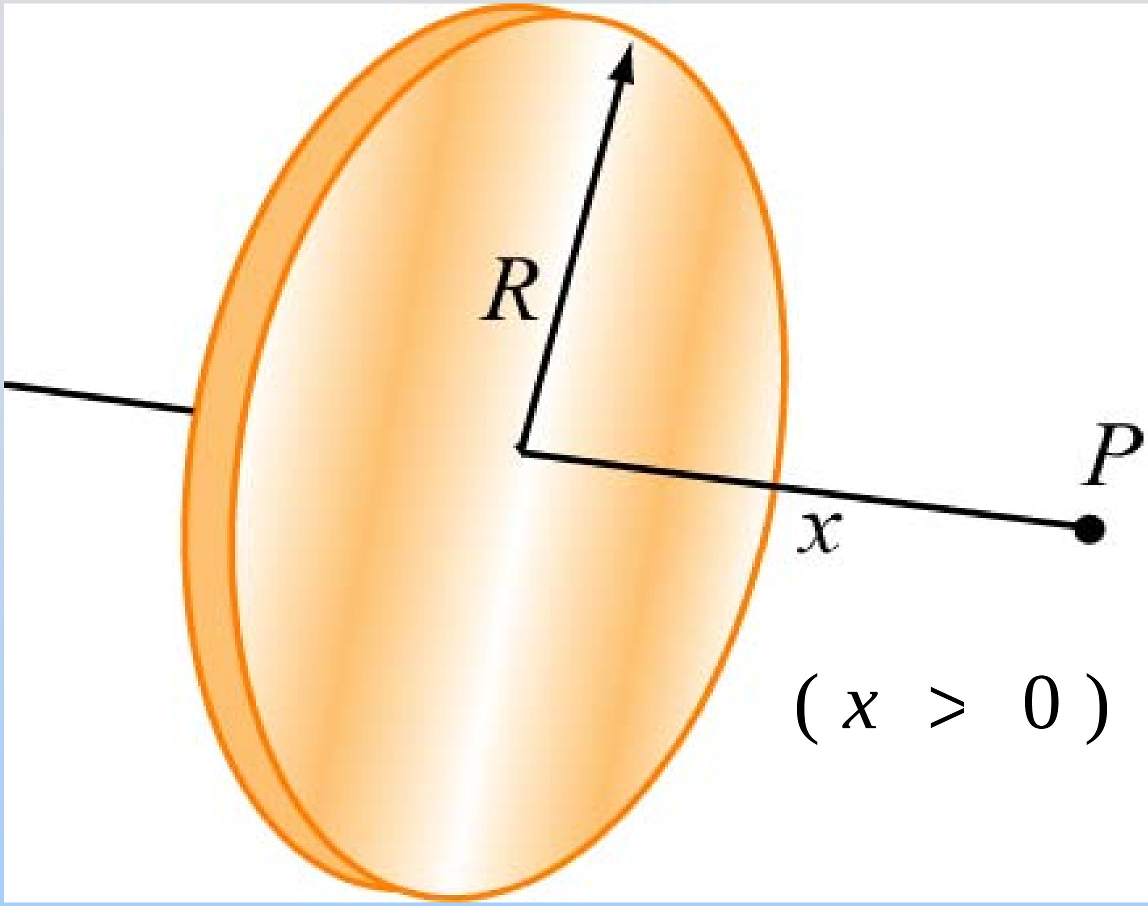
$$\lim_{s>>L} \vec{\mathbf{E}} \to k_e \frac{Q}{s^2} \hat{\mathbf{j}}$$

Point charge

$$\lim_{s<<L} \vec{\mathbf{E}} \to 2k_e \frac{Q}{Ls} \hat{\mathbf{j}} = 2k_e \frac{\lambda}{s} \hat{\mathbf{j}}$$

Infinite charged line

In-Class: Uniformly Charged Disk



P on axis of disk of charge, *x* from center

Radius R, charge density σ.

Find **E** at P

Disk: Two Important Limits

$$\vec{\mathbf{E}}_{disk} = \frac{\sigma}{2\varepsilon_o}\left[1 - \frac{x}{\left(x^2 + R^2\right)^{1/2}}\right]\hat{\mathbf{i}}$$

<u>Limits:</u>

$$\lim_{x>>R} \vec{\mathbf{E}}_{disk} \rightarrow \frac{1}{4\pi\varepsilon_o}\frac{Q}{x^2}\hat{\mathbf{i}}$$ Point charge

$$\lim_{x<<R} \vec{\mathbf{E}}_{disk} \rightarrow \frac{\sigma}{2\varepsilon_o}\hat{\mathbf{i}}$$ Infinite charged plane

Scaling: E for Plane is Constant

1) Dipole: E falls off like $1/r^3$
2) Point charge: E falls off like $1/r^2$
3) Line of charge: E falls off like $1/r$
4) Plane of charge: E constant

8.02SC Physics II: Electricity and Magnetism
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