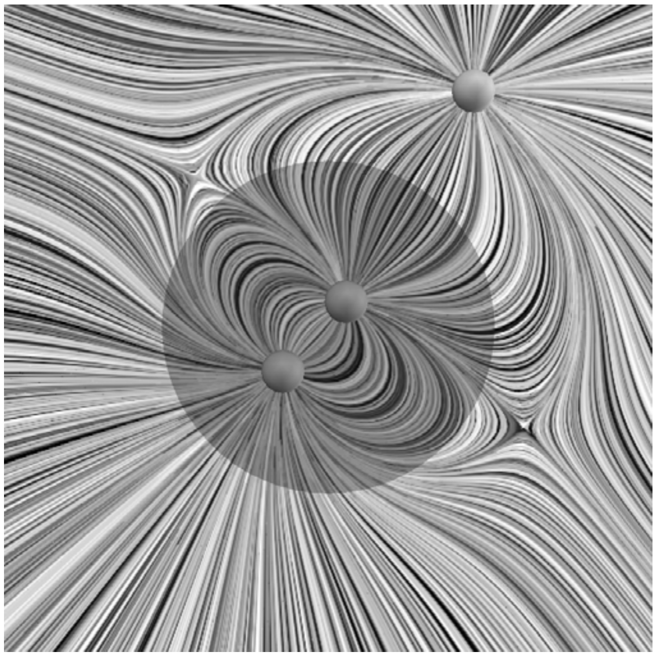
Gauss' Law

Challenge Problems

Problem 1:

The grass seeds figure below shows the electric field of three charges with charges +1, +1, and -1,   The Gaussian surface in the figure is a sphere containing two of the charges.



The total electric flux through the spherical Gaussian surface is

a) Positive

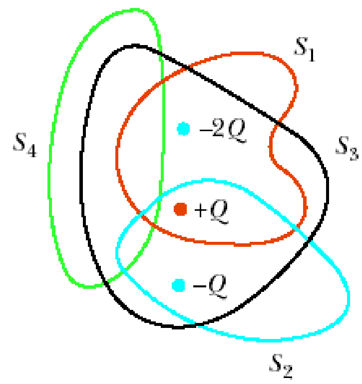
b) Negative

c) Zero

d) Impossible to determine without more information

Problem 2:

(a) Four closed surfaces, S_1 through S_4, together with the charges $-2Q$, Q, and $-Q$ are sketched in the figure at right. The colored lines are the intersections of the surfaces with the page. Find the electric flux through each surface.



(b) A pyramid has a square base of side a, and four faces which are equilateral triangles. A charge Q is placed at the center of the base of the pyramid. What is the net flux of electric field emerging from one of the triangular faces of the pyramid?

Problem 3:

Careful measurements reveal an electric field

$$\vec{\mathbf{E}}(r) = \begin{cases} \dfrac{a}{r^2}\left(1 - \dfrac{r^3}{R^3}\right)\hat{\mathbf{r}}\,; & r \le R \\ \vec{0}\,; & r \ge R \end{cases}$$

where a and R are constants. Which of the following best describes the charge distribution giving rise to this electric field?

a) A negative point charge at the origin with charge $q = 4\pi\varepsilon_0 a$ and a uniformly positive charged spherical shell of radius R with surface charge density $\sigma = -q / 4\pi R^2$.

b) A positive point charge at the origin with charge $q = 4\pi\varepsilon_0 a$ and a uniformly negative charged spherical shell of radius R with surface charge density $\sigma = -q / 4\pi R^2$.

c) A positive point charge at the origin with charge $q = 4\pi\varepsilon_0 a$ and a uniformly negative charged sphere of radius R with charge density $\rho = -q/(4\pi R^3/3)$.

d) A negative point charge at the origin with charge $-q = -4\pi\varepsilon_0 a$ and a uniformly positive charged sphere of radius R with charge density $\rho = q/(4\pi R^3/3)$.

e) Impossible to determine from the given information.

Problem 4:
A pyramid has a square base of side a, and four faces which are equilateral triangles. A charge Q is placed on the center of the base of the pyramid. What is the net flux of electric field emerging from one of the triangular faces of the pyramid?

1. 0
2. $\dfrac{Q}{8\varepsilon_0}$
3. $\dfrac{Qa^2}{2\varepsilon_0}$
4. $\dfrac{Q}{2\varepsilon_0}$
5. Undetermined: we must know whether Q is infinitesimally above or below the plane?

Problem 5:
A charge distribution generates a radial electric field

$$\vec{\mathbf{E}} = \frac{a}{r^2} e^{-r/b} \hat{\mathbf{r}}$$

where a and b are constants. The total charge giving rise to this electric field is

1. $4\pi\varepsilon_0 a$
2. 0
3. $4\pi\varepsilon_0 b$

Problem 6:

The bottom surface of a thundercloud of area A and the earth can be modeled as a pair of infinite parallel plate with equal and opposite surface uniform charge densities. Suppose the vertical electric field at the surface of the earth has a magnitude $\left|\vec{\mathbf{E}}_{atm}\right|$ and points towards the thundercloud.

a) Find an expression for the total charge density σ on the bottom surface of a thundercloud? Is this charge density positive or negative?

b) Suppose that the water in the thundercloud forms water droplets of radius r that carry all the charge of the thundercloud. The drops fall to the ground and make a height h of rainfall directly under the thundercloud. Find an expression for the charge on each droplet of water.

c) For the drops in part b), find an expression for the electric field $\left|\vec{\mathbf{E}}_{drop}\right|$ on the surface of the drop due only to the charge on the drop?

d) If a typical drop has radius $r = 5.0 \times 10^{-1}\,mm$ and the rainfall makes a height $h = 2.5 \times 10^{-3}\,m$, what is the ratio $f = \left|\vec{\mathbf{E}}_{drop}\right| / \left|\vec{\mathbf{E}}_{atm}\right|$?

Problem 7:
A sphere of radius R has a charge density $\rho = \rho_0 (r / R)$ where ρ_0 is a constant and r is the distance from the center of the sphere.

a) What is the total charge inside the sphere?

b) Find the electric field everywhere (both inside and outside the sphere).

Problem 8:

When two slabs of N-type and P-type semiconductors are put in contact, the relative affinities of the materials cause electrons to migrate out of the N-type material across the junction to the P-type material. This leaves behind a volume in the N-type material that is positively charged and creates a negatively charged volume in the P-type material.

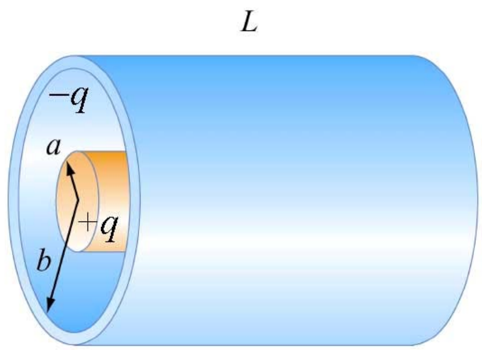
Let us model this as two infinite slabs of charge, both of thickness a with the junction lying on the plane $z = 0$. The N-type material lies in the range $0 < z < a$ and has uniform charge density $+\rho_0$. The adjacent P-type material lies in the range $-a < z < 0$ and has uniform charge density $-\rho_0$. Thus:

$$\rho(x, y, z) = \rho(z) = \begin{cases} +\rho_0 & 0 < z < a \\ -\rho_0 & -a < z < 0 \\ 0 & |z| > a \end{cases}$$

Find the electric field everywhere.

Problem 9

A very long conducting cylinder (length L and radius a) carrying a total charge $+q$ is surrounded by a thin conducting cylindrical shell (length L and radius b) with total charge $-q$, as shown in the figure.

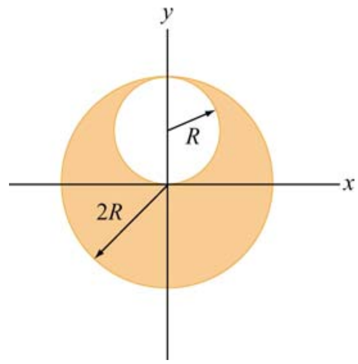


(a) Using Gauss's Law, find an expression for the direction and magnitude of the electric field $\vec{\mathbf{E}}$ for the region $r < a$.

(b) Similarly, find an expression for the direction and magnitude of the electric field $\vec{\mathbf{E}}$ for the region $a < r < b$.

Problem 10:

A sphere of radius $2R$ is made of a non-conducting material that has a uniform volume charge density ρ. (Assume that the material does not affect the electric field.) A spherical cavity of radius R is then carved out from the sphere, as shown in the figure below. Find the electric field within the cavity.



Problem 11:

(a) This problem demonstrates how one can use Gauss's law to draw important conclusions about the electric field associated with charged conductors. The following points will help you answer the questions posed in the problem

(i) In general charge resides *on* the surface of a conductor.

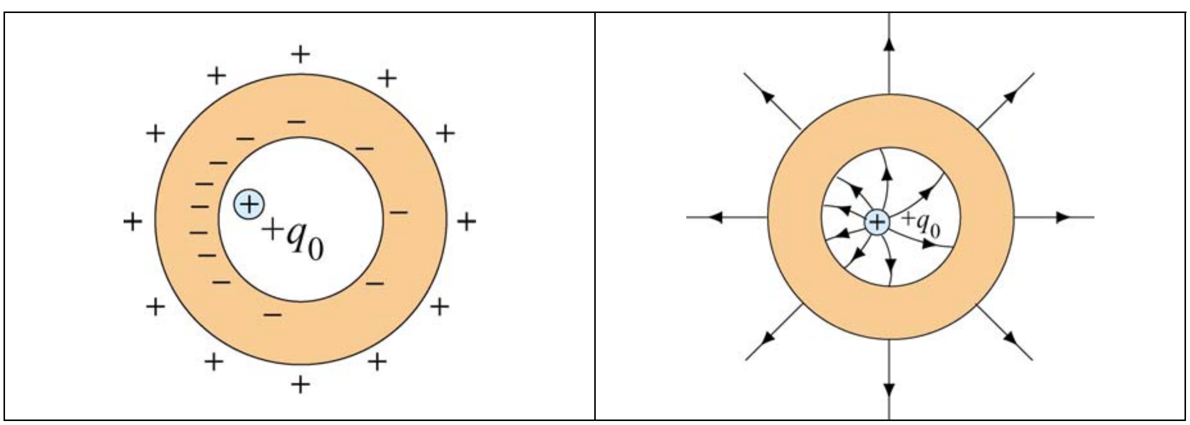
(ii) The electric field is zero inside the conductor. (This must be so in a static situation; otherwise electric currents would be flowing, contrary to the assumption.

(iii) Induced charge on the inner surface is exactly equal to $-q$. (A Gaussian surface, enclosing the $+q$ charge inside the cavity and the $-q$ charge on the inner surface, and staying entirely inside the conductor proves the above statement with the help of Gauss's Law.

(iv) Since the conductor has no net charge, the outer surface must carry $+q$ charge.

(v) The electric field outside any metallic surface is normal to the surface; its magnitude is σ/ε_0 by virtue of Gauss's law. (Recall, any metallic surface is an equipotential surface.)

Although we cannot derive the distribution of the charge $-q$ on the inner surface of the conductor without more sophisticated mathematics, we can nonetheless say that since the fields from all inner charges must add to give a zero field inside the metal, there must be more negative charge near the $+q$ (and less farther away). So the charge distribution must look like this:



In particular, notice that the charges on the outside of the shell are *uniformly distributed*!

(b) The electric field lines are shown in the sketch above. Notice that the field lines are closer together where the density of negative charges is greatest. Outside the sphere, the field looks like that from a point charge $+q_0$.

(c) No. The negative charge on the inside of the metal does, of course, rearrange itself in order to keep the field zero inside the conductor. The positive charge induced on the outside is totally uninfluenced because of the arguments presented in (a).

(d) When the "source charge" $+q_0$ touches the inner surface, a total neutralization inside the sphere (inner surface plus cavity) takes place; only the induced charge outside remains and is distributed uniformly on the surface.

(e) The behavior of the field just before contact is shown in the figure below: