Introduction to Electric Fields
Challenge Problem Solutions

Problem 1:

We have defined a vector field as a family of vectors, with a vector at every point in space. A scalar field can be likewise defined as a family of scalars, namely at every point in space the field has a value but no direction. Name as many examples of scalar and vector fields as possible.

Vector fields

Make a plot of the following vector fields:

(a) $\vec{\mathbf{v}} = 3\hat{\mathbf{i}} - 5\hat{\mathbf{j}}$

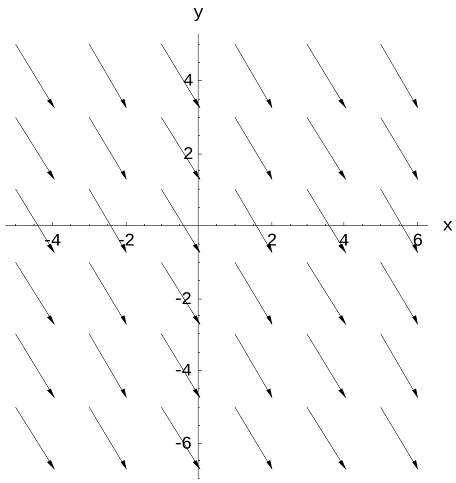
(b) $\vec{\mathbf{v}} = \vec{\mathbf{r}}$

(c) $\vec{\mathbf{v}} = \dfrac{\hat{\mathbf{r}}}{r^2}$

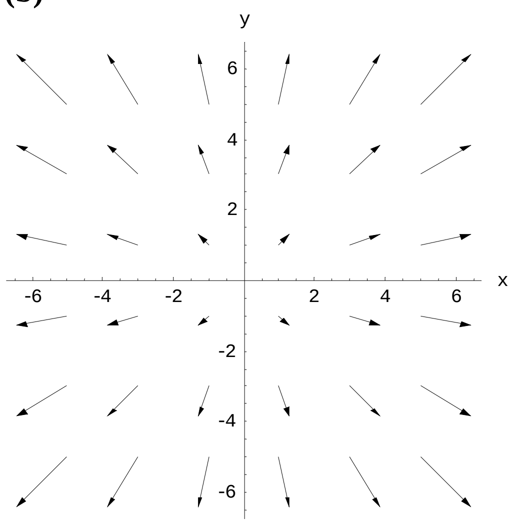
(d) $\vec{\mathbf{v}} = \dfrac{3xy}{r^5}\hat{\mathbf{i}} + \dfrac{2y^2 - x^2}{r^5}\hat{\mathbf{j}}$

Problem 1 Solutions:

(a) This is an example of a constant vector field in two dimensions. The plot is depicted below:



(b)

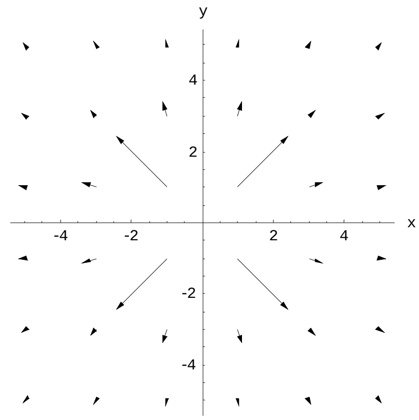


(c)

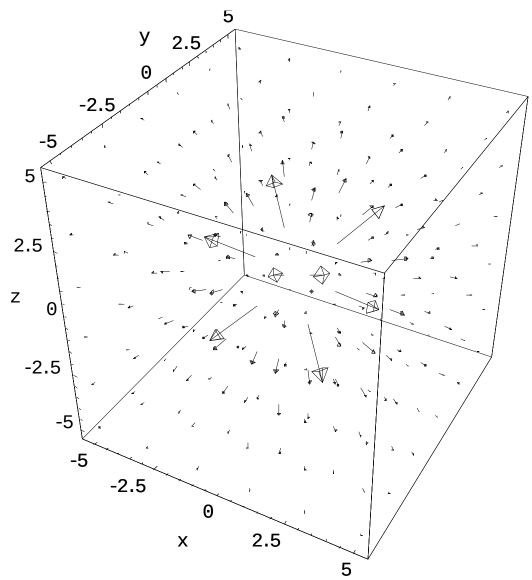
In two dimensions, using the Cartesian coordinates where $\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$, $\vec{\mathbf{v}}$ can be written as

$$\vec{\mathbf{v}} = \frac{\hat{\mathbf{r}}}{r^2} = \frac{\vec{\mathbf{r}}}{r^3} = \frac{x\hat{\mathbf{i}} + y\hat{\mathbf{j}}}{(x^2+y^2)^{3/2}}$$

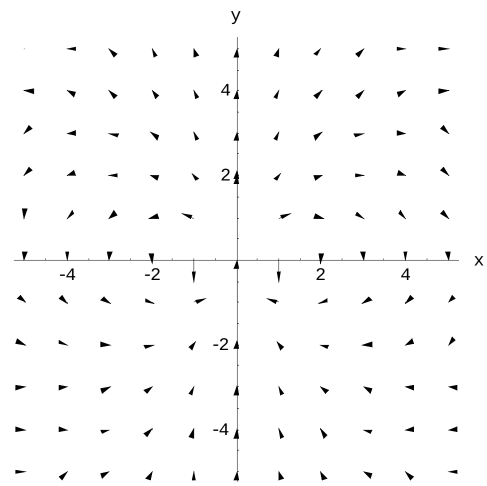
The plot is shown below:



Both the gravitational field of the Earth $\vec{\mathbf{g}}$ and the electric field $\vec{\mathbf{E}}$ due to a point charge have the same characteristic behavior as $\vec{\mathbf{v}}$. Note that in three dimensions, we would have $\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$, and the plot would look like



(d)



The plot is characteristic of the electric field due to a point electric dipole located at the origin.

Problem 2:

Scalar fields

Make a plot of the following scalar functions in two dimensions:

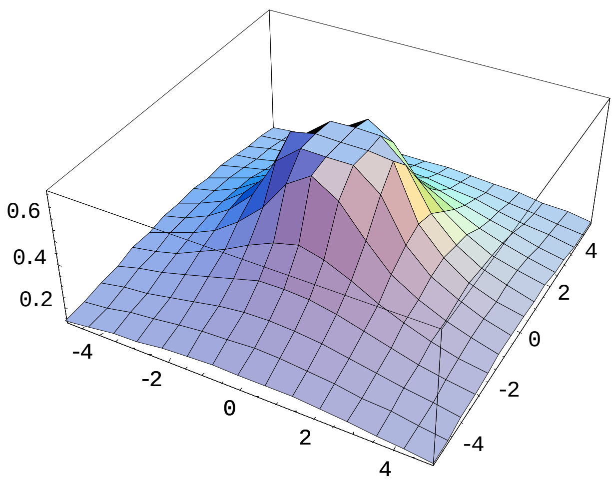
(a) $f(r) = \dfrac{1}{r}$

(b) $f(x, y) = \dfrac{1}{\sqrt{x^2 + (y-1)^2}} - \dfrac{1}{\sqrt{x^2 + (y+1)^2}}$

Problem 2 Solutions:

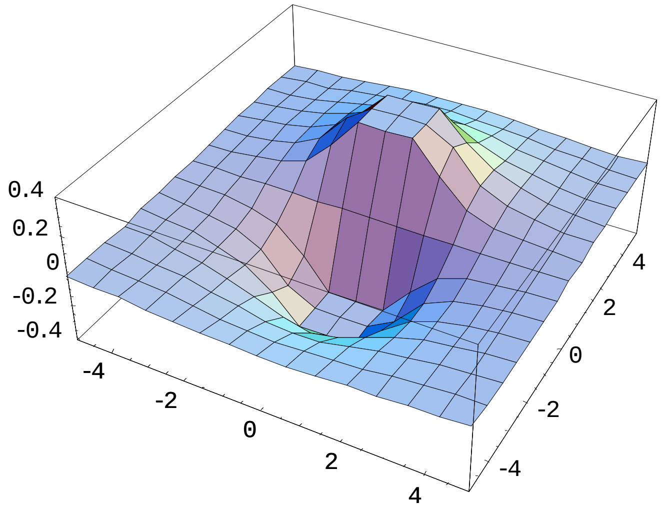
(a)

In two dimensions, $r = \sqrt{x^2 + y^2}$



The plot represents the electric potential due to a point charge located at the origin.

(b)



This plot represents the potential due to a dipole with the positive charge located at $y = 1$ and the negative charge at $y = -1$.