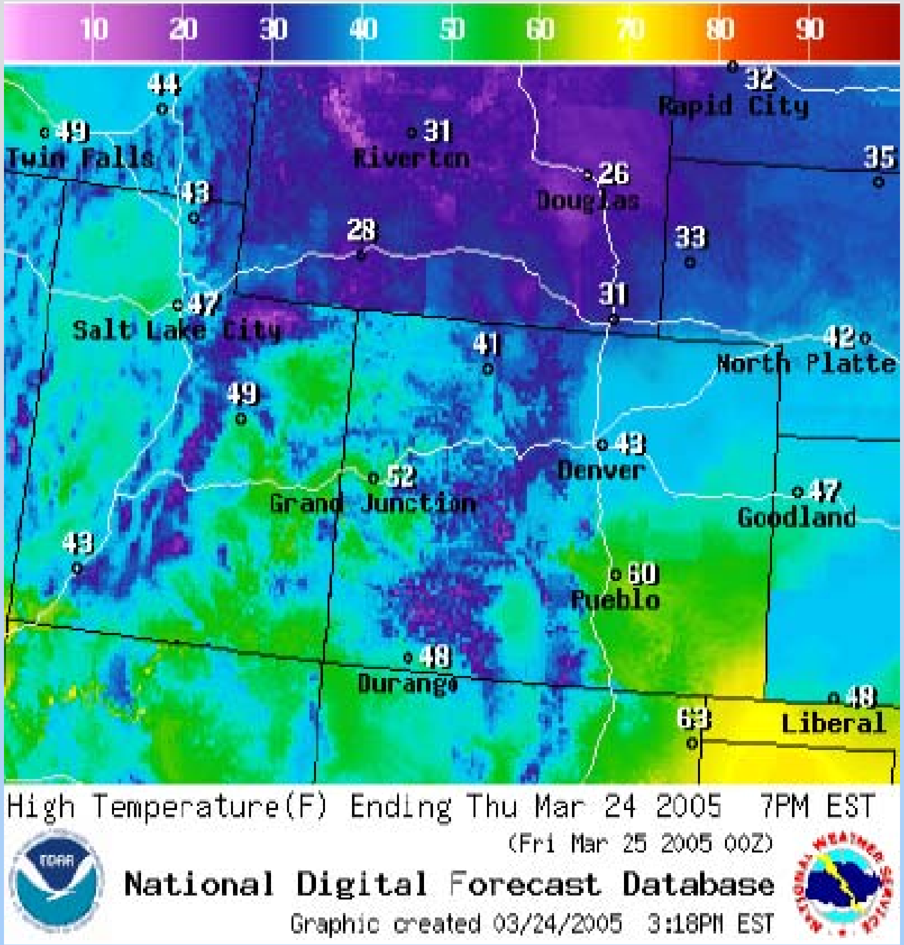
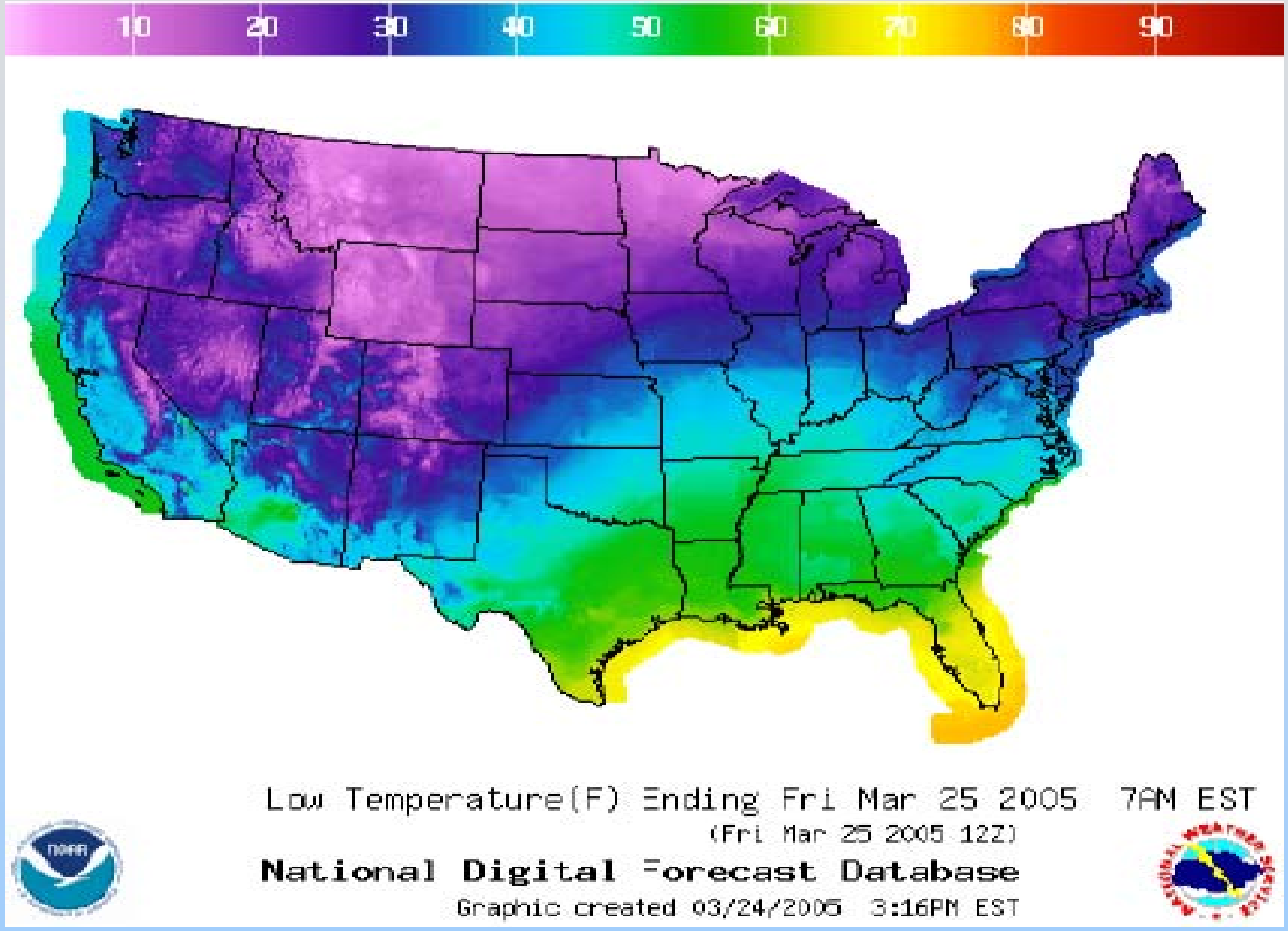
Module 01: Introduction to Electric Fields

Scalar Fields



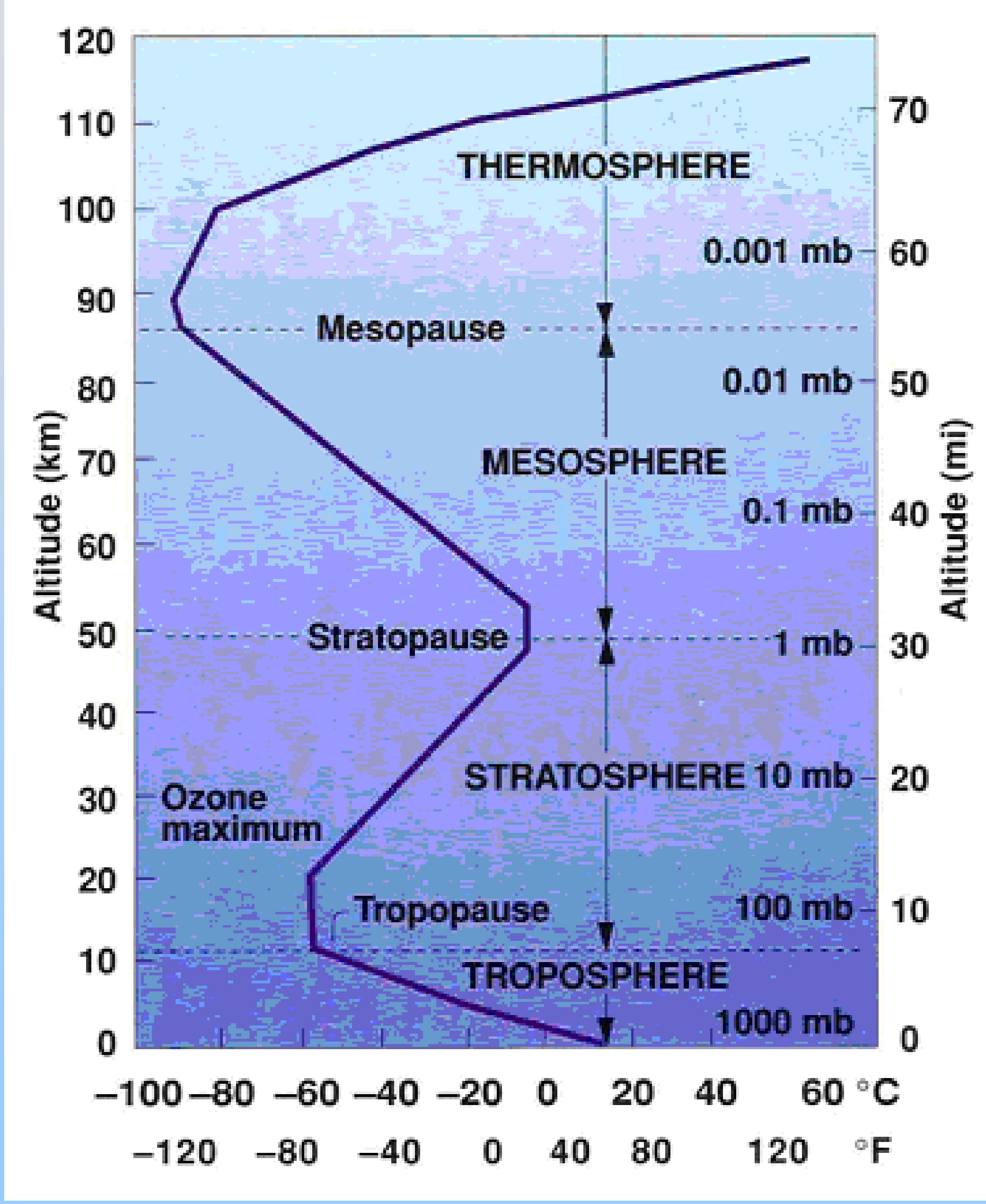
e.g. Temperature: Every location has associated value (number with units)

Scalar Fields - Contours



- Colors represent surface temperature
- Contour lines show constant temperatures

Fields are 3D

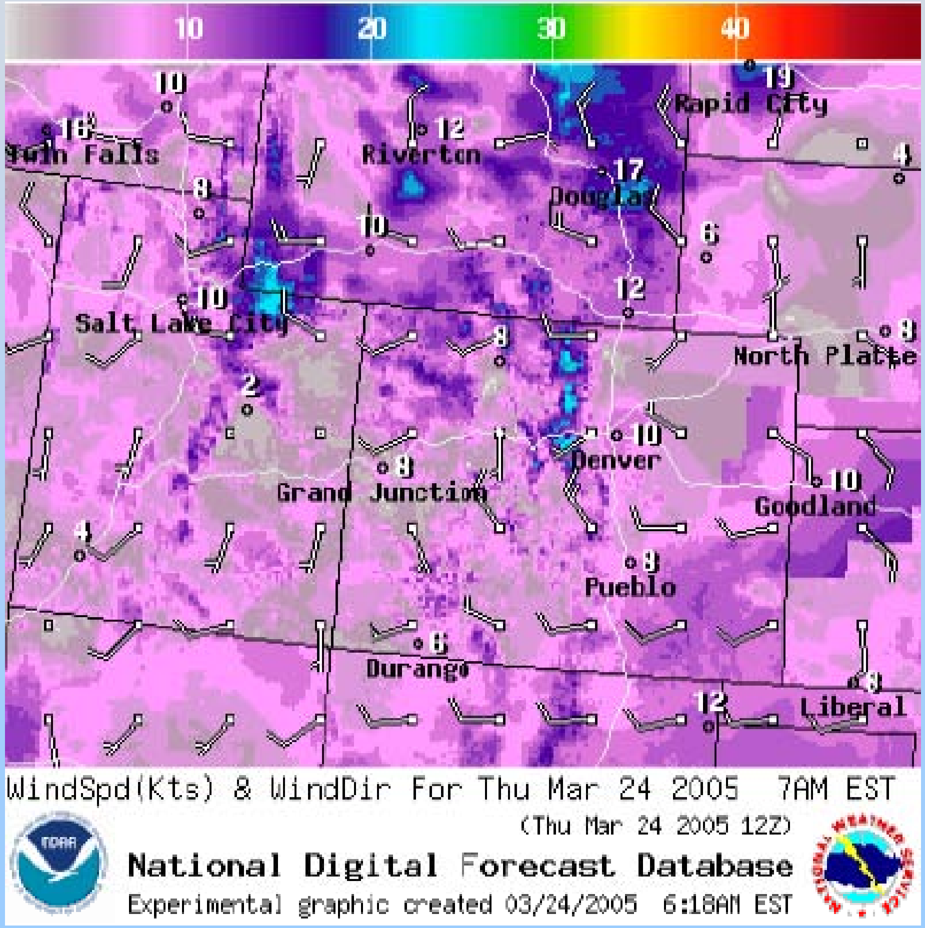


- T − T(x,y,z)
- Hard to visualize

→ Work in 2D

Vector Fields

Vector (magnitude, direction) at every point in space



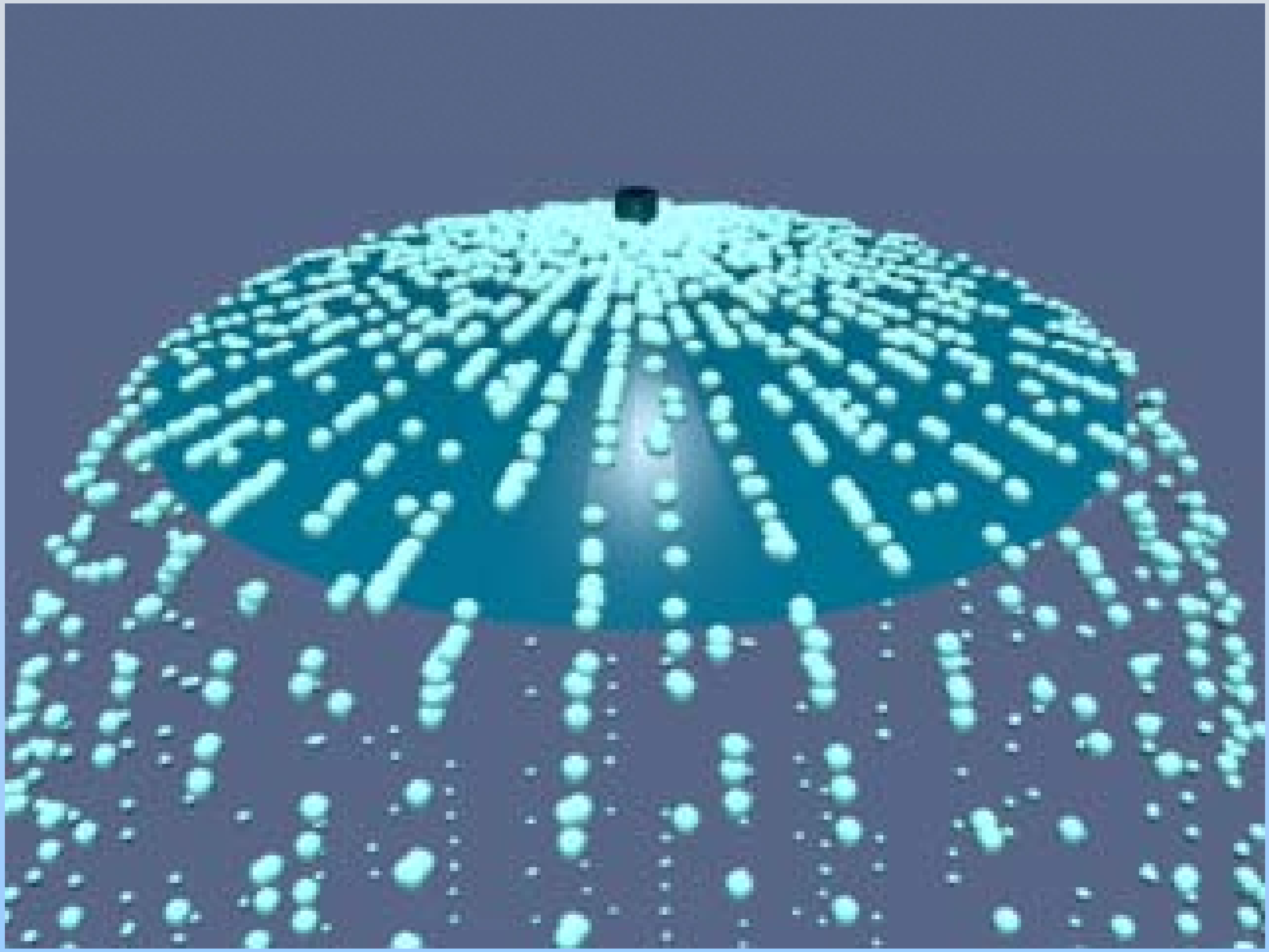
Example: Velocity vector field - jet stream

Vector Field Examples

Begin with Fluid Flow

Vector Field Examples

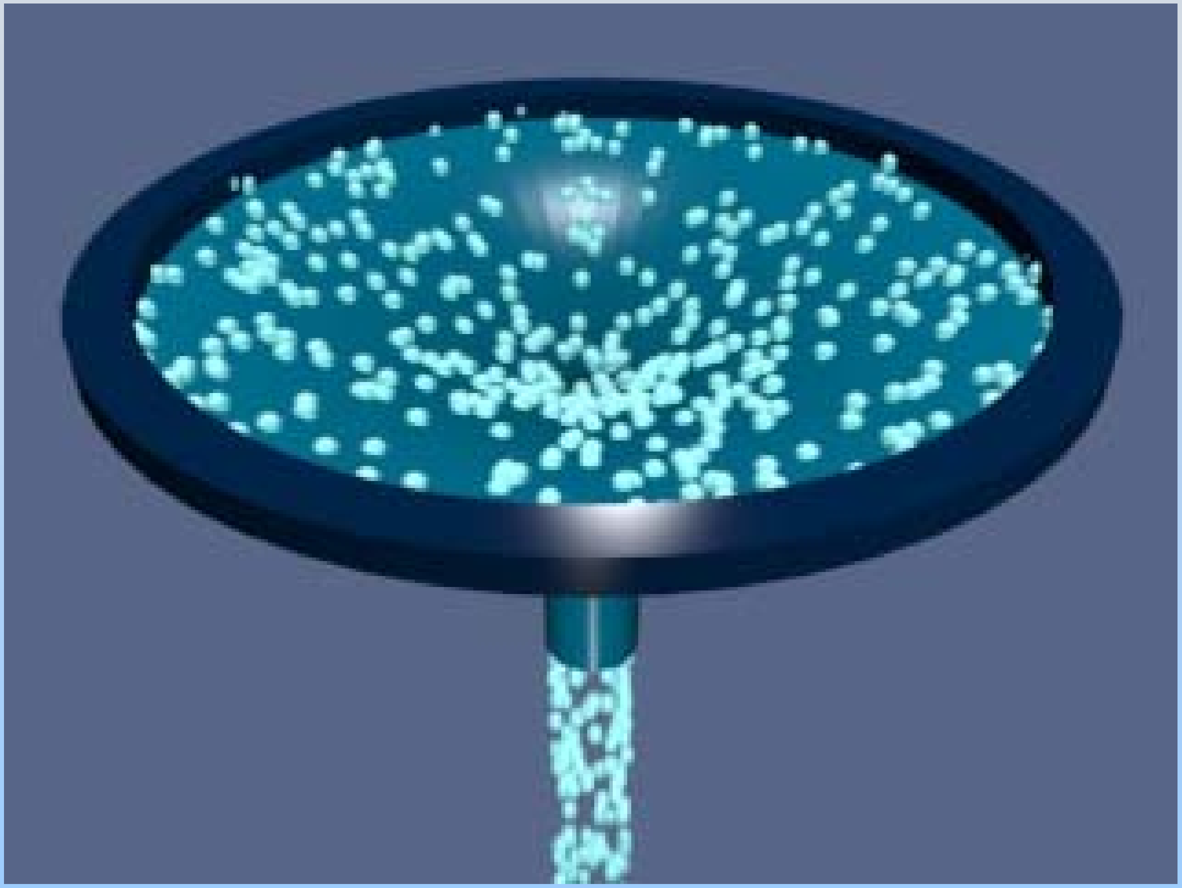
Flows With Sources



Link to movie

Vector Field Examples

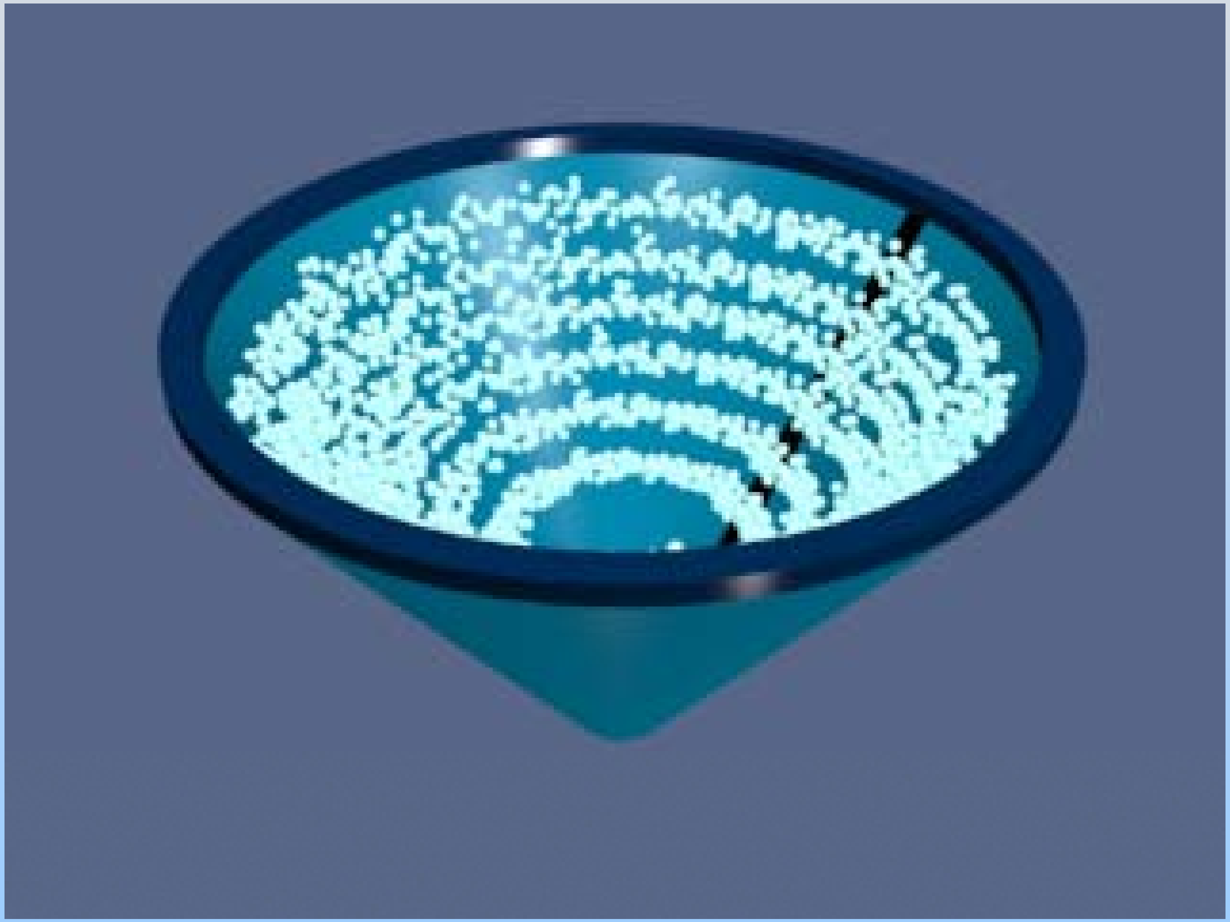
Flows With Sinks



Link to movie

Vector Field Examples

Circulating Flows



Link to movie

Visualizing Vector Fields: Three Methods

Vector Field Diagram

Arrows (different colors or length) in direction of field on uniform grid.

Field Lines

Lines tangent to field at every point along line

Grass Seeds

Textures with streaks parallel to field direction

All methods illustrated in

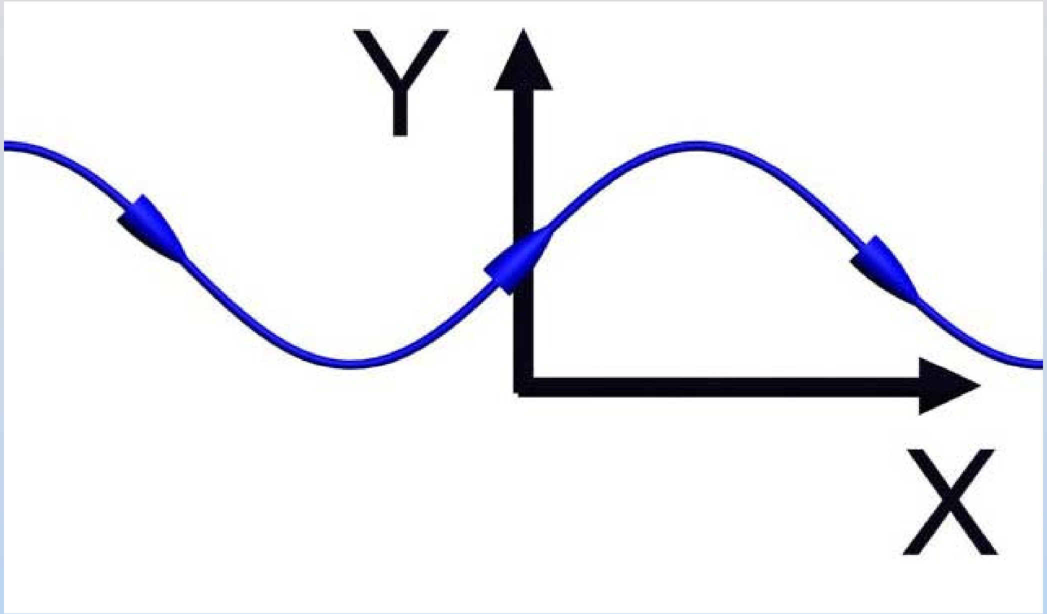
Vector Field Diagram Java Applet (link)

Vector Fields – Field Lines

- Direction of field line at any point is tangent to field at that point
- Field lines never cross each other

Concept Question Question: Vector Field

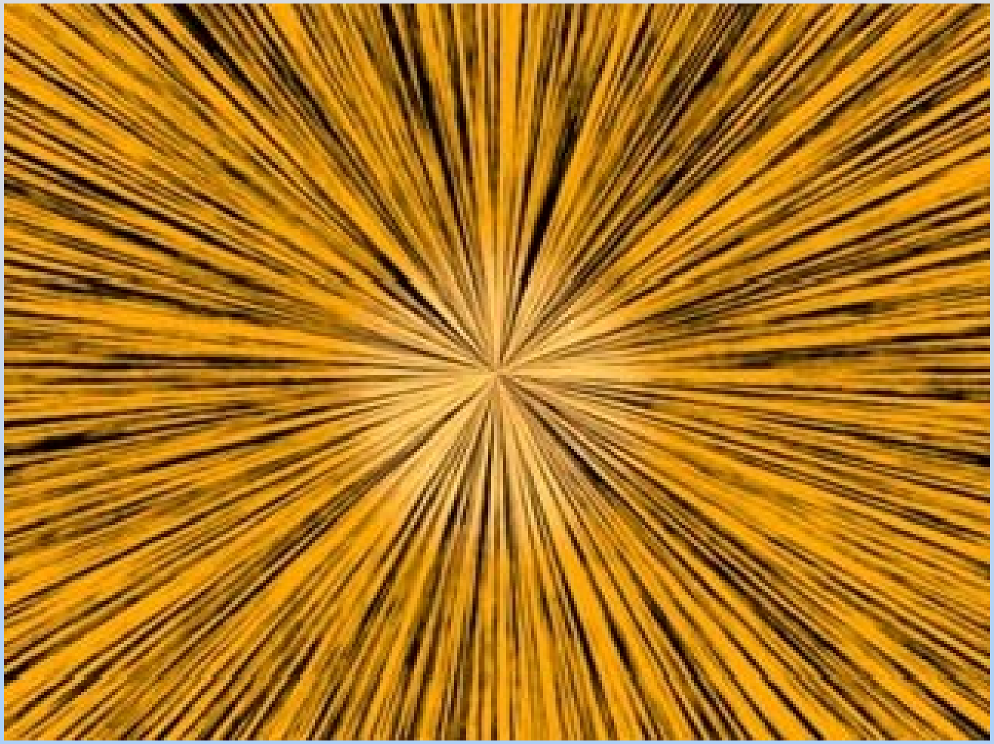
Concept Question: Vector Field



The field line at left corresponds to the vector field:

1. $\vec{\mathbf{F}}(x,y)=\sin(x)\,\hat{\mathbf{i}}+\hat{\mathbf{j}}$
2. $\vec{\mathbf{F}}(x,y)=\hat{\mathbf{i}}+\sin(x)\,\hat{\mathbf{j}}$
3. $\vec{\mathbf{F}}(x,y)=\cos(x)\,\hat{\mathbf{i}}+\hat{\mathbf{j}}$
4. $\vec{\mathbf{F}}(x,y)=\hat{\mathbf{i}}+\cos(x)\,\hat{\mathbf{j}}$
5. I don't know

Vector Fields – “Grass Seeds”



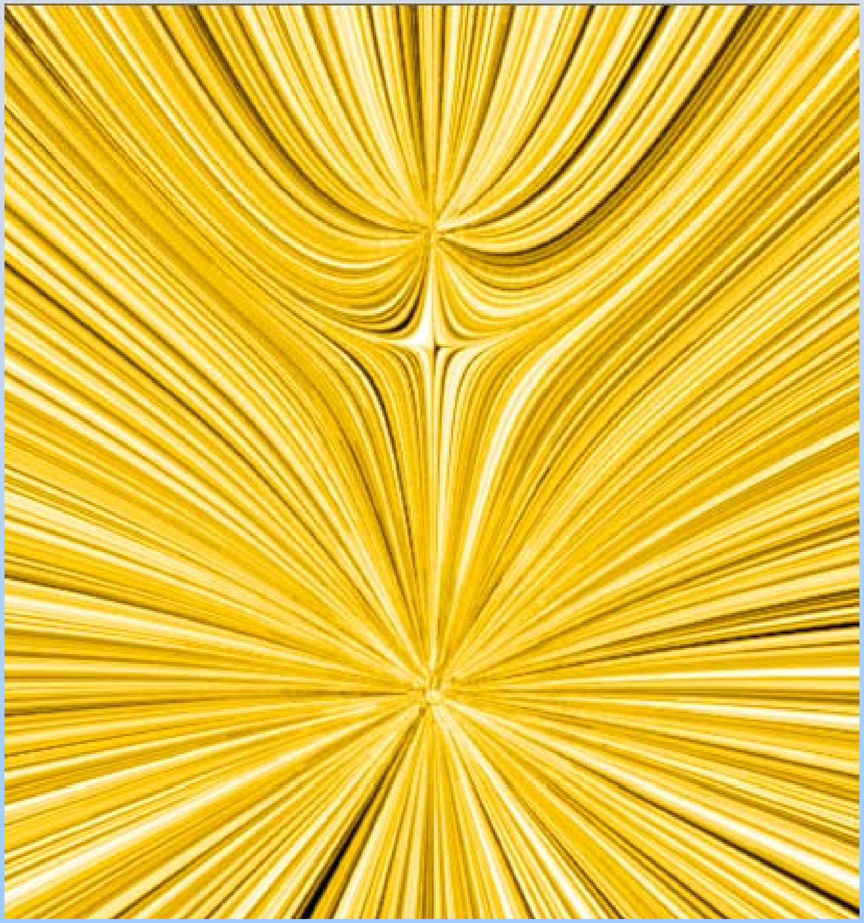
Source/Sink
(link)

Circulating
(link)

“Grass seeds” still does not give us absolute direction, only modulo 180 degrees

Concept Question Questions: “Grass Seed” Visualizations

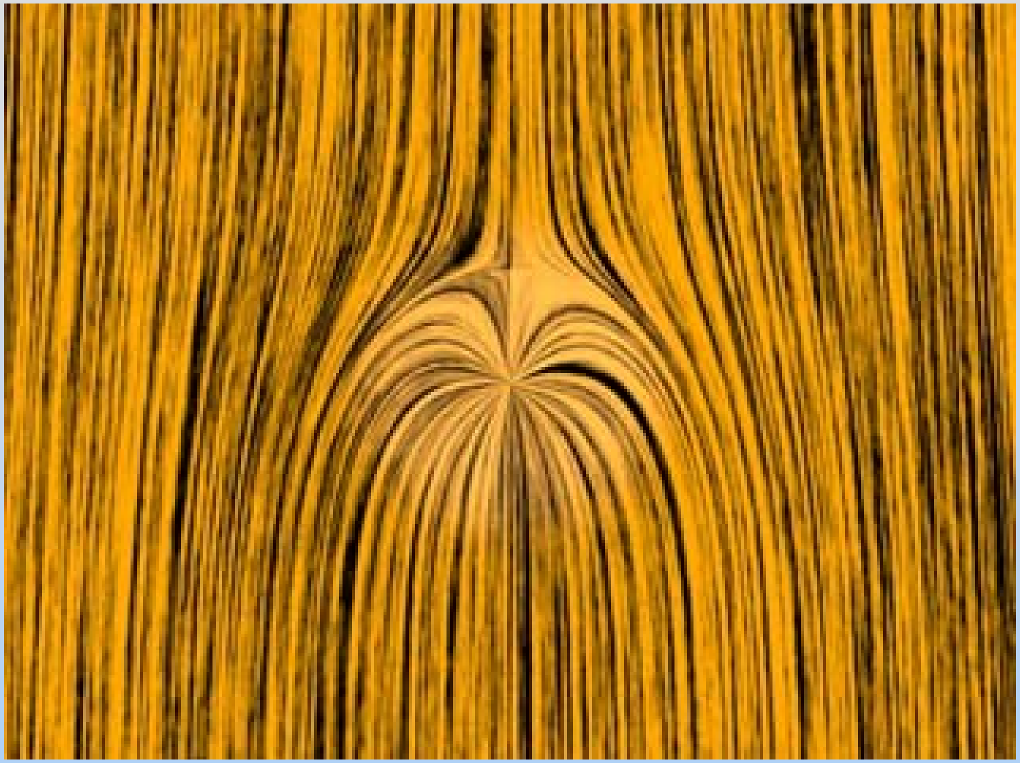
Concept Question: Grass Seeds



The vector field at left is created by:

1. Two sources (equal strength)
2. Two sources (top stronger)
3. Two sources (bottom stronger)
4. Source & Sink (equal strength)
5. Source & Sink (top stronger)
6. Source & Sink (bottom stronger)
7. I don't know

Concept Question: Grass Seeds



Here there is an initial downward flow.

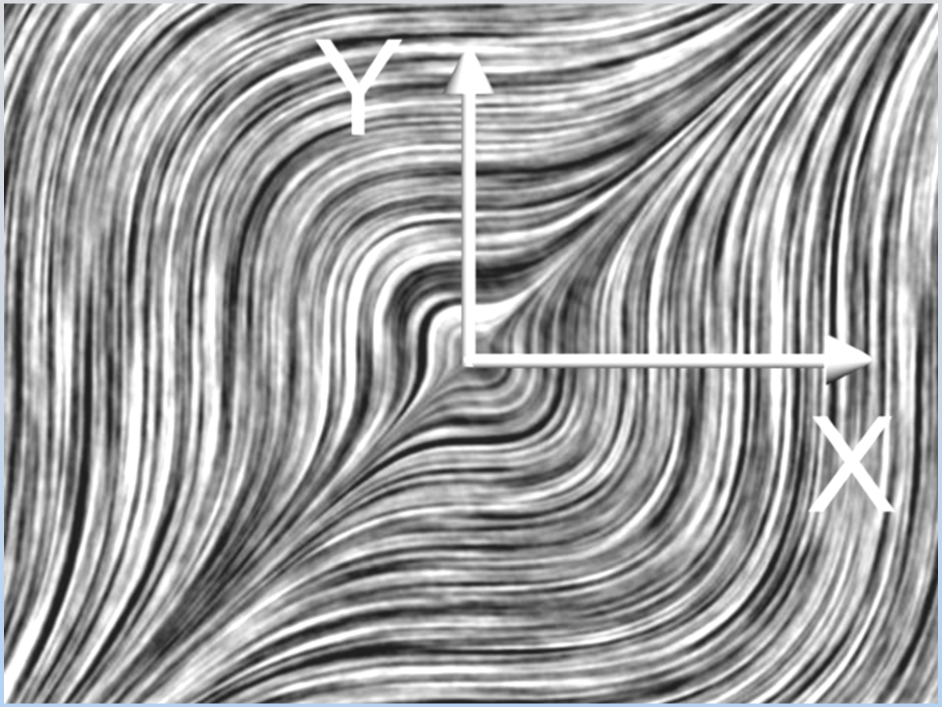
1. The point is a source
2. The point is a sink
3. I don't know

Concept Question: Circulation

These two circulations are in:

1. The same direction (e.g. both clockwise)
2. Opposite directions (e.g. one cw, one ccw)
3. I don't know

Concept Question: Vector Field



The grass seeds field plot at left is a representation of the vector field:

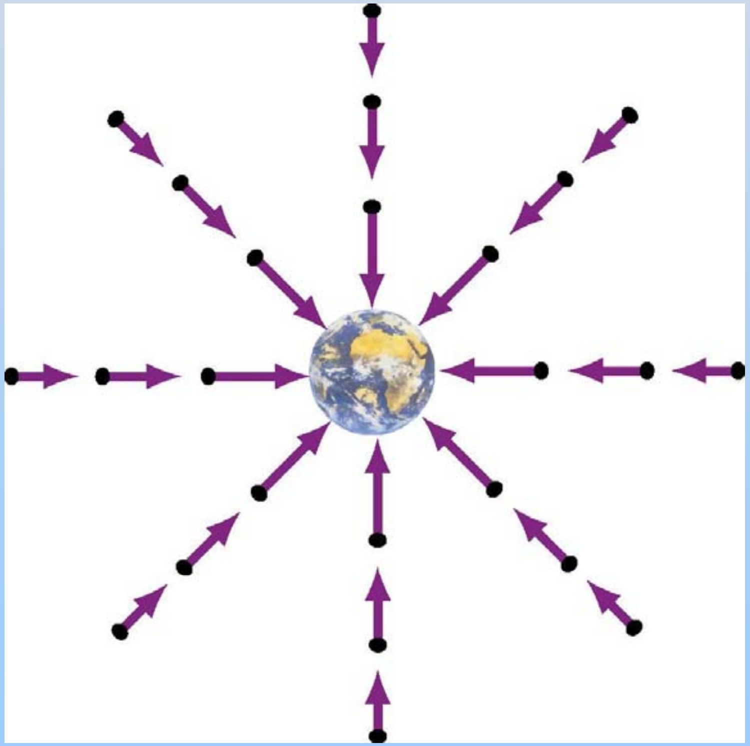
1. $\vec{\mathbf{F}}(x, y) = x^2\hat{\mathbf{i}} + y^2\hat{\mathbf{j}}$
2. $\vec{\mathbf{F}}(x, y) = y^2\hat{\mathbf{i}} + x^2\hat{\mathbf{j}}$
3. $\vec{\mathbf{F}}(x, y) = \sin(x)\,\hat{\mathbf{i}} + \cos(y)\,\hat{\mathbf{j}}$
4. $\vec{\mathbf{F}}(x, y) = \cos(x)\,\hat{\mathbf{i}} + \sin(y)\,\hat{\mathbf{j}}$
5. I don’t know

Another Vector Field: Gravitational Field

Example Of Vector Field: Gravitation

Gravitational Force:

$$\vec{\mathbf{F}}_g = -G\frac{Mm}{r^2}\hat{\mathbf{r}}$$



Gravitational Field:

$$\vec{\mathbf{g}} = \frac{\vec{\mathbf{F}}_g}{m} = -\frac{GMm/r^2}{m}\hat{\mathbf{r}} = -G\frac{M}{r^2}\hat{\mathbf{r}}$$

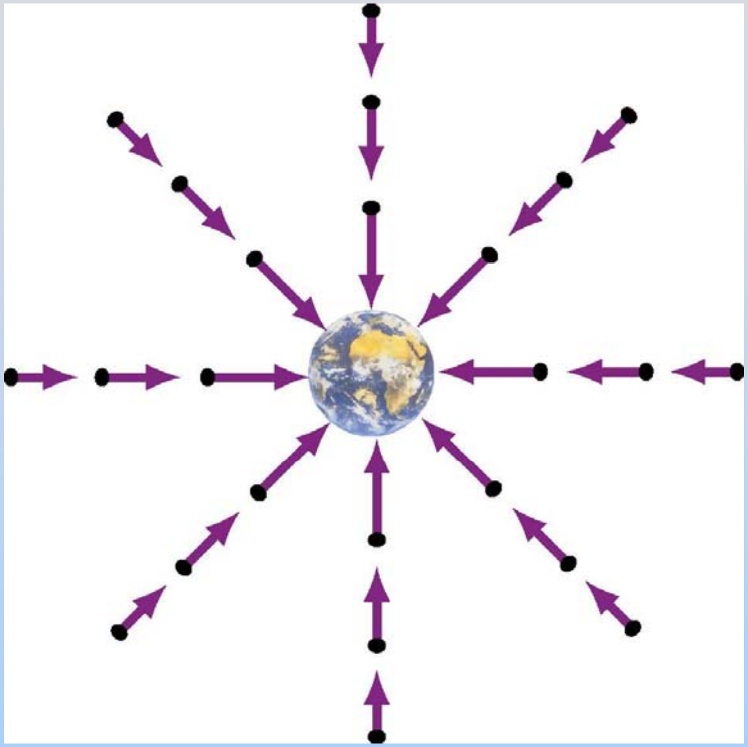
M : Mass of Earth

Example Of Vector Field: Gravitation

Gravitational Field:

$$\vec{\mathbf{g}} = -G\frac{M}{r^2}\hat{\mathbf{r}} \qquad \vec{\mathbf{F}}_g = m\vec{\mathbf{g}}$$

Created by M *Felt* by m



$\hat{\mathbf{r}}$: unit vector from M to m

$\vec{\mathbf{r}}$: vector from M to m

M : Mass of Earth

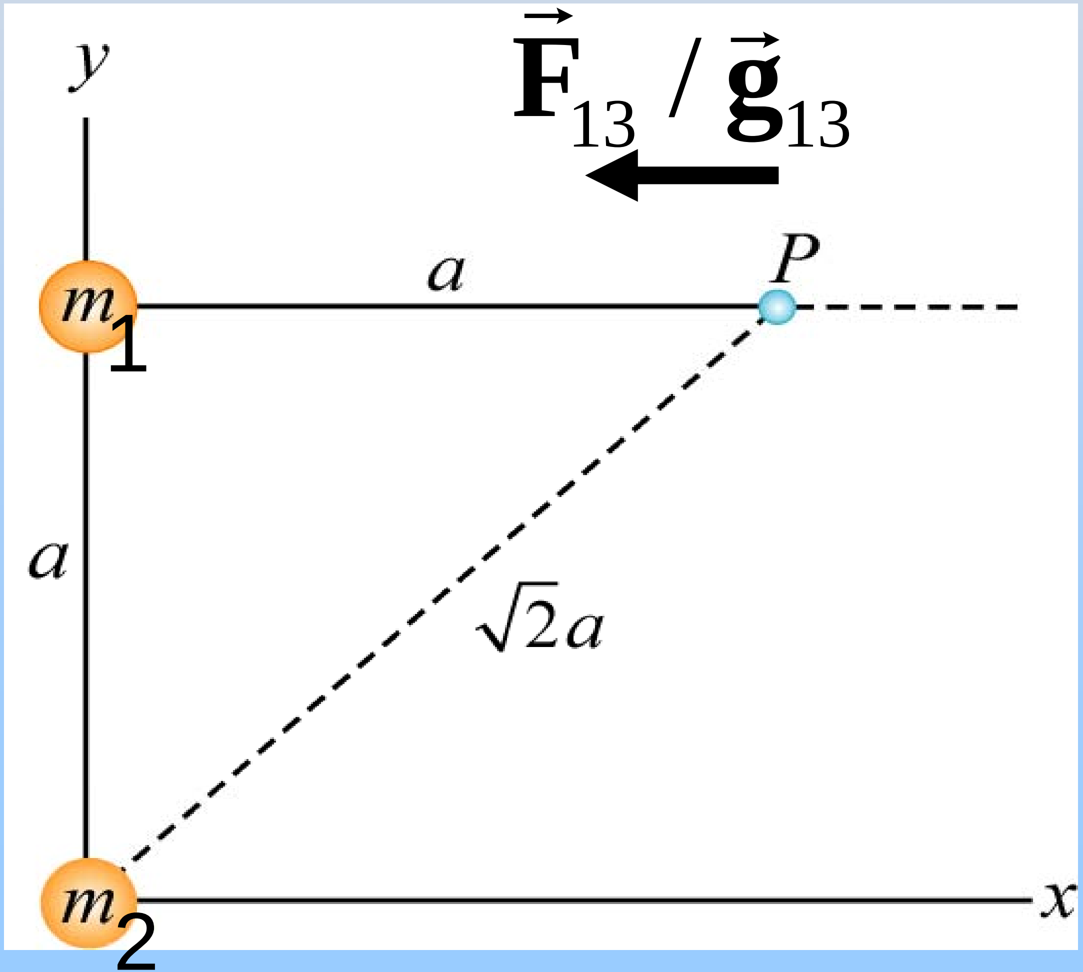
$$\hat{\mathbf{r}} = \frac{\vec{\mathbf{r}}}{r} \quad \Rightarrow \vec{\mathbf{g}} = -G\frac{M}{r^3}\vec{\mathbf{r}}$$

This form is easier to use

The Superposition Principle

Net force/field is vector sum of forces/fields

Example:

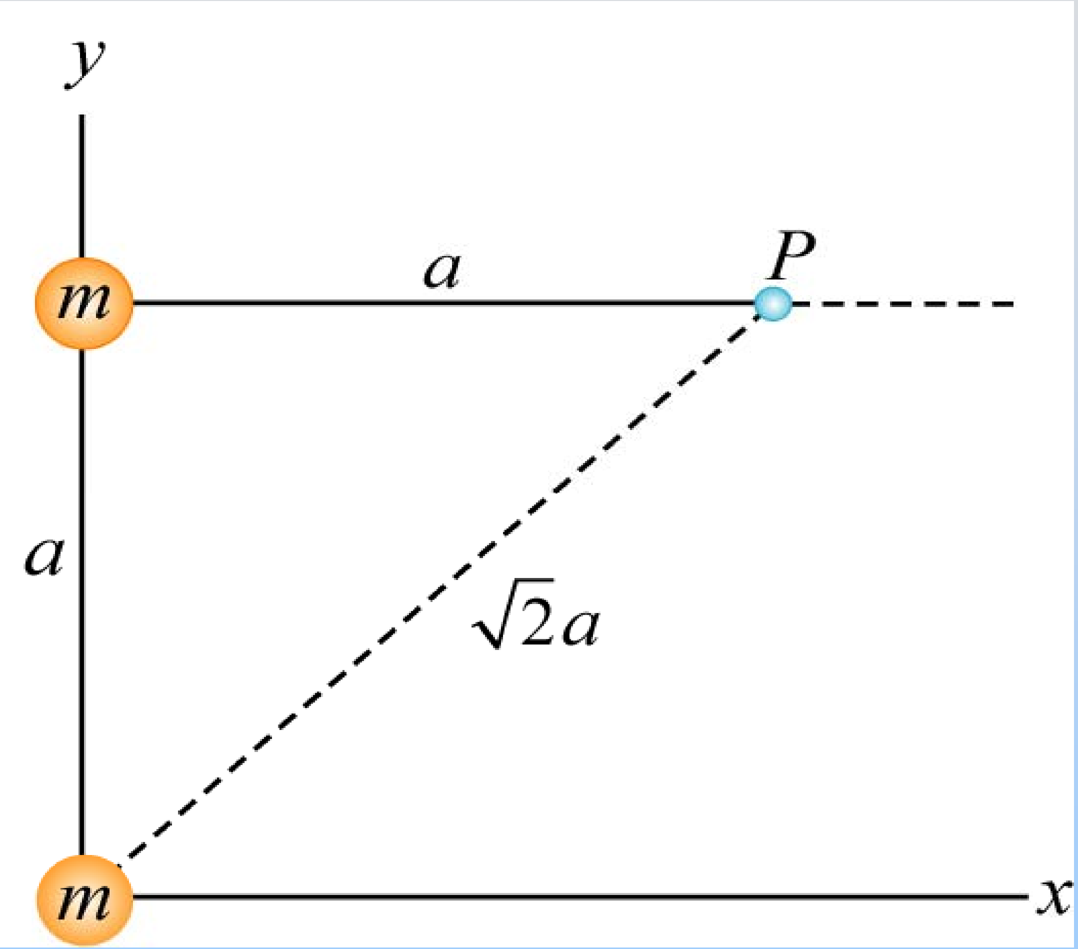


$$\vec{\mathbf{F}}_3 = \vec{\mathbf{F}}_{13} + \vec{\mathbf{F}}_{23}$$

In general:

$$\vec{\mathbf{F}}_j = \sum_{i=1}^{N} \vec{\mathbf{F}}_{ij}$$

Checkpoint Problem



Find the gravitational field $\vec{\mathbf{g}}$ at point P

Bonus: Where would you put another mass m to make the field $\vec{\mathbf{g}}$ become 0 at P?

Use $\vec{\mathbf{g}} = -G\dfrac{M}{r^3}\vec{\mathbf{r}}$

From Gravitational to Electric Fields

Electric Charge (~Mass)

Two types of electric charge: positive and negative

Unit of charge is the ***coulomb*** [C]

Charge of electron (negative) or proton (positive) is

$$\pm e, \quad e = 1.602 \times 10^{-19} C$$

Charge is quantized

$$Q = \pm Ne$$

Charge is conserved

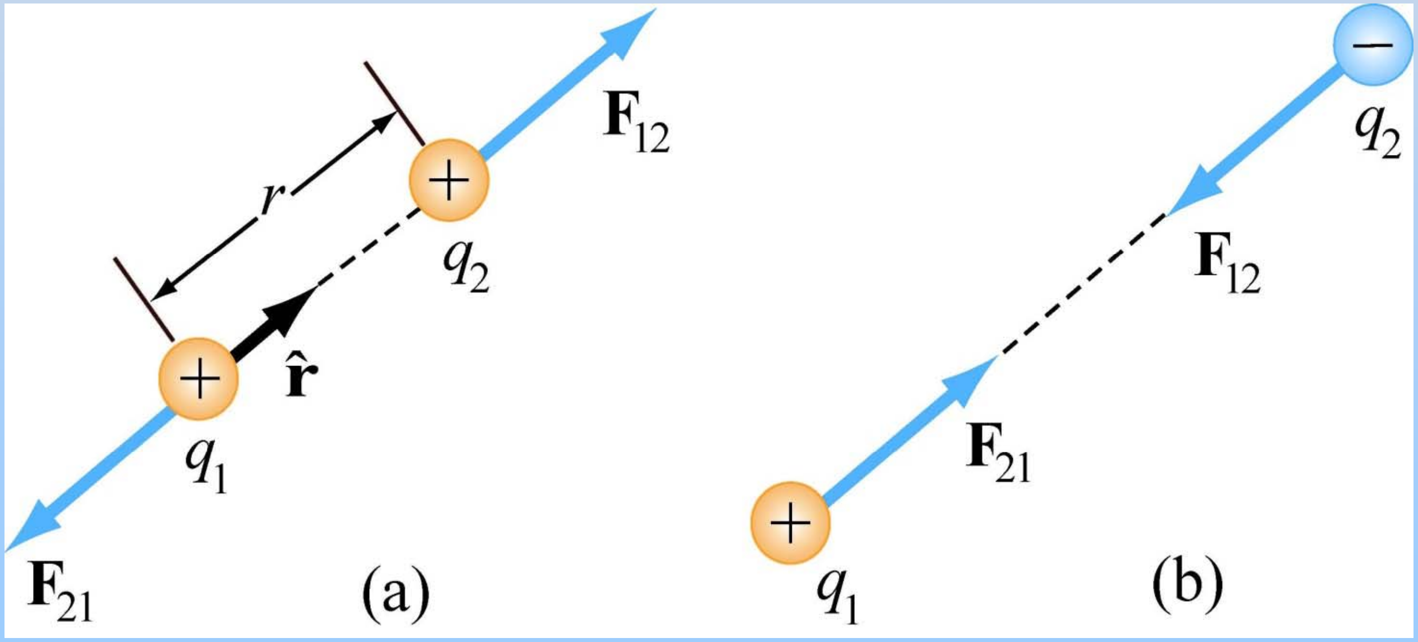
$$n \rightarrow p + e^{-} + \bar{\nu} \qquad e^{+} + e^{-} \rightarrow \gamma + \gamma$$

Electric Force (~Gravity)

The electric force between charges q_1 and q_2 is

(a) repulsive if charges have same signs

(b) attractive if charges have opposite signs

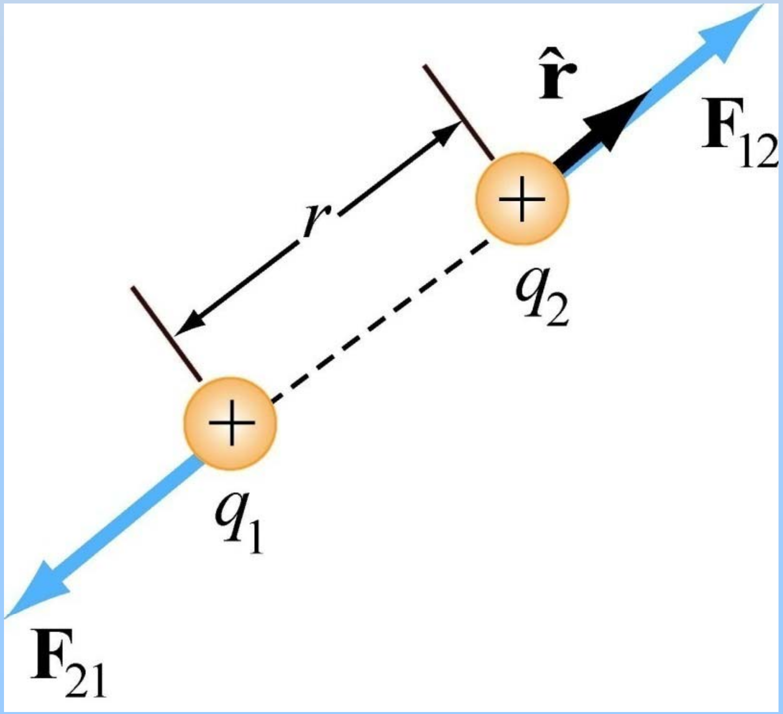


Like charges repel and opposites attract !!

Coulomb's Law

Coulomb's Law: Force on q_2 due to interaction between q_1 and q_2

$$\vec{\mathbf{F}}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{\mathbf{r}}$$



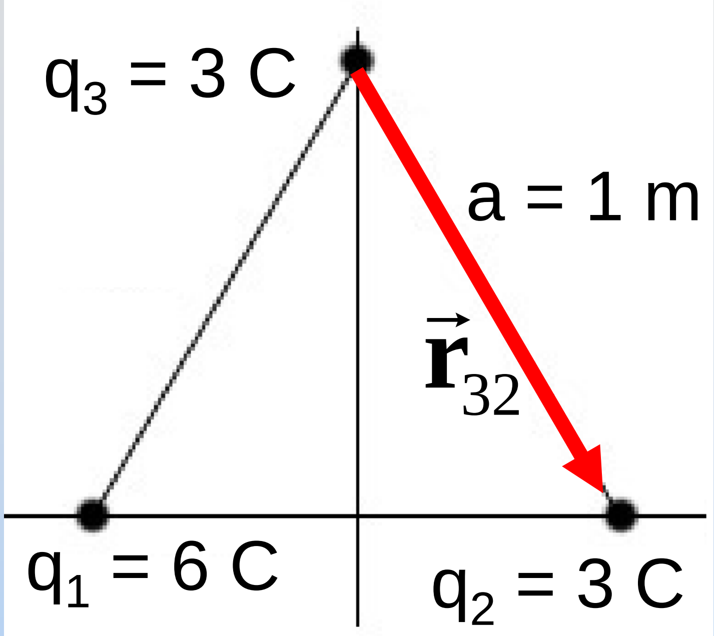
$$k_e = \frac{1}{4\pi\varepsilon_0} = 8.9875 \times 10^9 \text{ N m}^2/\text{C}^2$$

$\hat{\mathbf{r}}$: unit vector from q_1 to q_2

$\vec{\mathbf{r}}$: vector from q_1 to q_2

$$\hat{\mathbf{r}} = \frac{\vec{\mathbf{r}}}{r} \qquad \Rightarrow \vec{\mathbf{F}}_{12} = k_e \frac{q_1 q_2}{r^3} \vec{\mathbf{r}}$$

Coulomb's Law: Example



$$\vec{\mathbf{F}}_{32} = ?$$

$$\vec{\mathbf{r}}_{32} = \left(\tfrac{1}{2}\,\hat{\mathbf{i}} - \tfrac{\sqrt{3}}{2}\,\hat{\mathbf{j}}\right)\ \mathrm{m}$$

$$r - 1\mathrm{m}$$

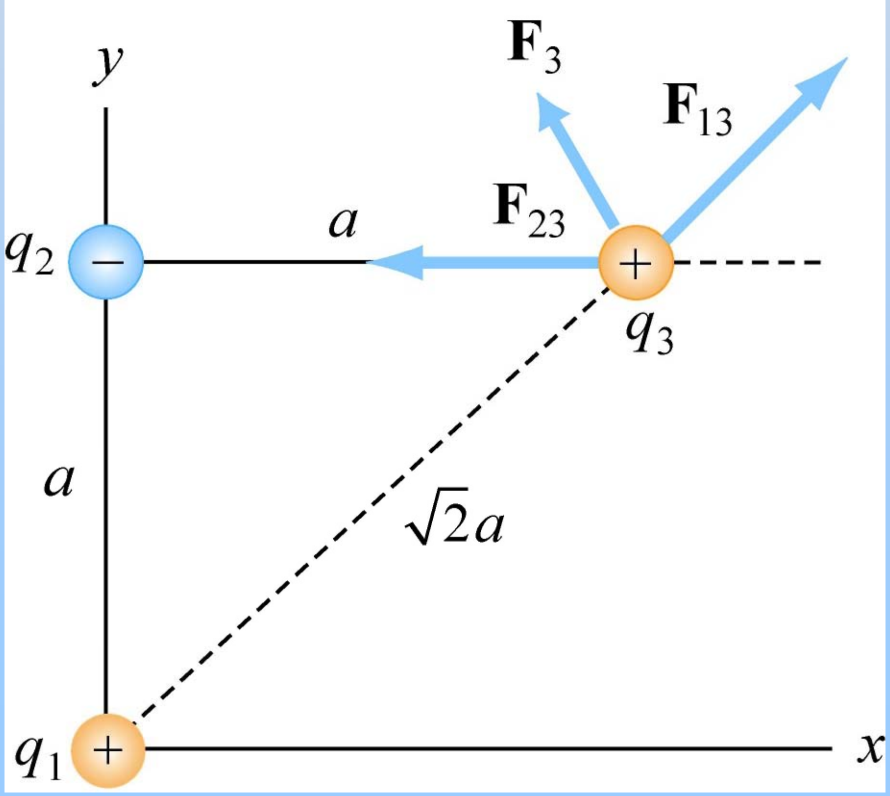
$$\vec{\mathbf{F}}_{32} = k_e q_3 q_2 \frac{\vec{\mathbf{r}}}{r^3} = \left(9\times10^9\,\mathrm{N\,m^2/C^2}\right)(3\mathrm{C})(3\mathrm{C})\frac{\frac{1}{2}\left(\hat{\mathbf{i}} - \sqrt{3}\hat{\mathbf{j}}\right)\mathrm{m}}{(1\mathrm{m})^3}$$

$$= \frac{81\times10^9}{2}\left(\hat{\mathbf{i}} - \sqrt{3}\hat{\mathbf{j}}\right)\ \mathrm{N}$$

The Superposition Principle

Many Charges Present:
Net force on any charge is vector sum of forces from other individual charges

Example:



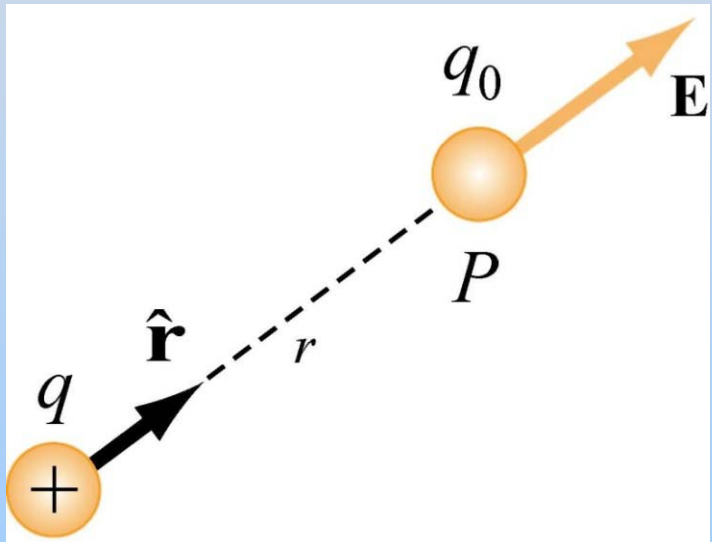
$$\vec{\mathbf{F}}_3 = \vec{\mathbf{F}}_{13} + \vec{\mathbf{F}}_{23}$$

In general:

$$\vec{\mathbf{F}}_j = \sum_{i=1}^{N} \vec{\mathbf{F}}_{ij}$$

Electric Field (~g)

The electric field at a point *P* due to a charge q is the force acting on a test charge q_0 at that point *P*, divided by the charge q_0 :



$$\vec{\mathbf{E}}_q(P) \equiv \frac{\vec{\mathbf{F}}_{qq_0}}{q_0}$$

For a point charge q: $$\vec{\mathbf{E}}_q(P) = k_e \frac{q}{r^2}\hat{\mathbf{r}}$$

Units: Newtons/Coulomb, same as Volts/meter

Superposition Principle

The electric field due to a collection of *N* point charges is the vector sum of the individual electric fields due to each charge

$$\vec{\mathbf{E}}_{total} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 + \ldots\ldots = \sum_{i=1}^{N} \vec{\mathbf{E}}_i$$

Summary Thus Far

SOURCE: Mass M_s Charge q_s (±)

CREATE: $\vec{\mathbf{g}} = -G\frac{M_s}{r^2}\hat{\mathbf{r}}$ $\vec{\mathbf{E}} = k_e\frac{q_s}{r^2}\hat{\mathbf{r}}$

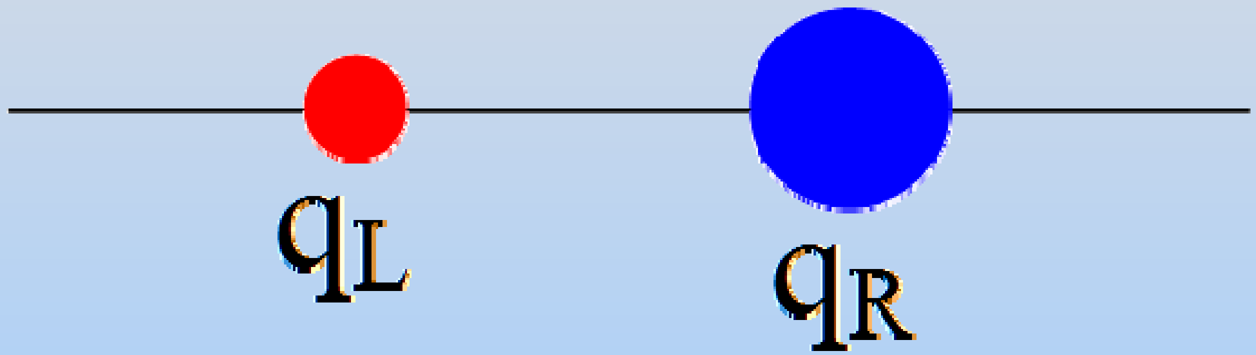
FEEL: $\vec{\mathbf{F}}_g = m\vec{\mathbf{g}}$ $\vec{\mathbf{F}}_E = q\vec{\mathbf{E}}$

This is easiest way to picture field

Concept Question Question: Electric Field

Concept Question: Electric Field

Two opposite charges are placed on a line as shown below. The charge on the right is three times larger than the charge on the left. Other than at infinity, where is the electric field zero?



1. Between the two charges
2. To the right of the charge on the right
3. To the left of the charge on the left
4. The electric field is nowhere zero
5. Not enough info – need to know which is positive
6. I don’t know