## Introduction to Electric Fields

## Challenge Problems

## Problem 1:

Vector fields
Make a plot of the following vector fields:
(a) $\overrightarrow{\mathbf{v}}=3 \hat{\mathbf{i}}-5 \hat{\mathbf{j}}$
(b) $\overrightarrow{\mathbf{v}}=\overrightarrow{\mathbf{r}}$
(c) $\overrightarrow{\mathbf{v}}=\frac{\hat{\mathbf{r}}}{r^{2}}$
(d) $\overrightarrow{\mathbf{v}}=\frac{3 x y}{r^{5}} \hat{\mathbf{i}}+\frac{2 y^{2}-x^{2}}{r^{5}} \hat{\mathbf{j}}$

## Problem 2:

## Scalar fields

Make a plot of the following scalar functions in two dimensions:
(a) $f(r)=\frac{1}{r}$
(b) $f(x, y)=\frac{1}{\sqrt{x^{2}+(y-1)^{2}}}-\frac{1}{\sqrt{x^{2}+(y+1)^{2}}}$

## Problem 3:



Above we show the grass seeds representation of the field of four point charges, located at the positions indicated by the numbers. Which statement is true about the signs of these charges:
a) All four charges have the same sign.
b) Charges 1 and 2 have the same sign, and that sign is opposite the sign of 3 and 4 .
c) Charges 1 and 3 have the same sign, and that sign is opposite the sign of 2 and 4 .
d) Charges 1 and 4 have the same sign, and that sign is opposite the sign of 2 and 3 .
e) None of the above.

## Problem 4:

## Equations of Field Lines

The "grass seeds" representation ${ }^{1}$ of the vector function $\overrightarrow{\mathbf{F}}(x, y)=y \hat{\mathbf{i}}+x^{2} \hat{\mathbf{j}}$ is shown below. We also show one of the field lines for this vector field ${ }^{2}$.


Figure 1: A grass seeds representation of the vector field given above and one of the field lines of the same field. See visualizations link for the grass seeds applet. Click on the figure above to get a movie of the family of field lines for this vector field.

Find by explicit construction the equation of the field lines for this vector field. That is, integrate the equation

$$
\frac{d y}{d x}=\frac{F_{y}(x, y)}{F_{x}(x, y)}
$$

to find the functions $y(x)$ that satisfy this differential equation. To do this, set up the differential equation and then isolate all the variables depending on $y$ on the left side of the equation with the differential $d y$, and all the variables depending on $x$ on the right side of the equation, with the differential $d x$, and integrate (for example, the integral of ( $y d y$ ) is $1 / 2 y^{2}$, and so on).

We only show one of the field lines in the above figure. If you click on this figure you will download a movie that shows that field line as the constant of integration varies, thereby sweeping out the family of possible field lines.

## Problem 5:

## Ratio of Electric and Gravitational Forces (10 points)

What is the ratio of the magnitudes of the electrostatic force and the gravitational force between two protons if the protons are separated by a distance $r=1.0 \times 10^{-15} \mathrm{~m}$ ? In SI units the magnitude of the charge of the proton is $e=1.6 \times 10^{-19} \mathrm{C}$ and the mass of the proton is $m_{p}=1.67 \times 10^{-27} \mathrm{~kg}$.

## Problem 6:

## Bohr Theory

In the Bohr theory of the hydrogen atom, an electron moves in a circular orbit about a proton, where the radius of the orbit is $0.529 \times 10^{-10} \mathrm{~m}$.
(a) Find the electric force between the two.
(b) If this force causes the centripetal acceleration of the electron, what is the speed of the electron?

## Problem 7:

Two objects with charges $-q$ and $+3 q$ are placed on a line as shown in the figure below.


Besides an infinite distance away from the charges, where else can the electric field possibly be zero?

1. Between the two charges.
2. To the right of the charge on the right.
3. To the left of the charge on the left.
4. The electric field is only zero an infinite distance away from the charges.

## Explain your reasoning

## Problem 8:

Two objects with charges $-4 Q$ and $-Q$ lie on the $y$-axis. The object with the charge $-4 Q$ is above the object with charge $-Q$. Below are four possible "grass seed" representations of the electric field of the two charges. Which of these representations is most nearly right for the two charges in this problem?


## Explain your reasoning.

MIT OpenCourseWare
http://ocw.mit.edu

### 8.02SC Physics II: Electricity and Magnetism

Fall 2010

For information about citing these materials or our Terms of Use, visit: http://ocw.mit.edu/terms.

