

Electric Potential and Gauss's Law, Configuration Energy Challenge Problem Solutions

Problem 1:

Consider a very long rod, radius R and charged to a uniform linear charge density λ .

- a) Calculate the electric field everywhere outside of this rod (i.e. find $\vec{E}(\vec{r})$).
- b) Calculate the electric potential everywhere outside, where the potential is defined to be zero at a radius $R_0 > R$ (i.e. $V(R_0) \equiv 0$)

Problem 1 Solution:

(a) This is easily calculated using Gauss's Law and a cylindrical Gaussian surface of radius r and length l . By symmetry, the electric field is completely radial (this is a "very long" rod), so all of the flux goes out the sides of the cylinder:

$$\oiint \vec{E} \cdot d\vec{A} = 2\pi r l E = \frac{Q_{enc}}{\epsilon_0} = \frac{\lambda l}{\epsilon_0} \Rightarrow \boxed{\vec{E} = \frac{\lambda}{2\pi r \epsilon_0} \hat{r}}$$

(b) To get the potential we simply integrate the electric field from R to wherever we want to know it (in this case r):

$$V(r) = V(r) - \underbrace{V(R_0)}_0 = - \int_{R_0}^r \vec{E}(\vec{r}') \cdot d\vec{r}' = - \int_{R_0}^r \frac{\lambda}{2\pi r' \epsilon_0} dr' = - \frac{\lambda}{2\pi \epsilon_0} \ln(r') \Big|_{R_0}^r = \boxed{\frac{\lambda}{2\pi \epsilon_0} \ln\left(\frac{R_0}{r}\right)}$$

Problem 2:

Estimate the largest voltage at which it's reasonable to hold high voltage power lines. Then check out [this video](#), care of a Boulder City, Nevada power company. Air ionizes when electric fields are on the order of $3 \times 10^6 \text{ V} \cdot \text{m}^{-1}$.

Problem 2 Solution: In order to answer this question we have to think about what happens if we go to very high voltages. What breaks down? The problem with high voltages is that they lead to high fields. And high fields mean breakdown.

You derived the voltage and field in problem 3

:

$$E(r) = \lambda/2\pi\epsilon_0 r; V(r) = (\lambda/2\pi\epsilon_0) \ln(R_0/r) \Rightarrow V(r) = E(r)r \ln(R_0/r)$$

The strongest field, and hence breakdown, appears at $r = R \sim 1 \text{ cm}$, the radius of a power line (that makes the diameter just under 1 inch – it might be 3 or 4 times that big but probably not ten times). The voltage is defined relative to some ground, either another cable (probably $R_0 \sim 1 \text{ m}$ away) or at the most the real ground ($R_0 \sim 10 \text{ m}$ away). So,

$$V_{\max} = E_{\max} R \ln(R_0/R) = (3 \times 10^6 \text{ V} \cdot \text{m}^{-1})(1 \text{ cm}) \ln(10 \text{ m}/1 \text{ cm}) \cong \boxed{2 \times 10^5 \text{ V}}$$

As it turns out, a typical power-line voltage is about 250 kV, about as large as we estimate here. Some high voltage lines can even go up to 600 kV though (or double that for AC voltages). They must use larger diameter cables.

By the way, you can tell that breakdown is a real concern. In humid weather (during rainstorms for example) you will sometimes hear crackling coming from the power lines. This is corona discharge, a high voltage, low current breakdown, similar to the crackling you hear from the Van de Graff generator in class. The movie is of an arc discharge, a very high current phenomenon that can be very dangerous.

Problem 3:

Consider a uniformly charged sphere of radius R and charge Q . Find the electric potential difference between any point lying on a sphere of radius r and the point at the origin, i.e. $V(r) - V(0)$. Choose the zero reference point for the potential at $r = 0$, i.e. $V(0) = 0$. How does your expression for $V(r)$ change if you chose the zero reference point for the potential at $r = \infty$, i.e. $V(\infty) = 0$.

Problem 3 Solution: In order to solve this problem we must first calculate the electric field as a function of r for the regions $0 < r < R$ and $r > R$. Then we integrate the electric field to find the electric potential difference between any point lying on a sphere of radius r and the point at the origin. Because we are computing the integral along a path we must be careful to use the correct functional form for the electric field in each region that our path crosses.

There are two distinct regions of space defined by the charged sphere: region I: $r < R$, and region II: $r > R$. So we shall apply Gauss's Law in each region to find the electric field in that region.

For region I: $r < R$, we choose a sphere of radius r as our Gaussian surface. Then, the electric flux through this closed surface is

$$\oiint \mathbf{E}_I \cdot d\mathbf{A} = E_I \cdot 4\pi r^2.$$

$$r < R$$



The sphere has a uniform charge density $\rho = Q/(4/3)\pi R^3$. Because the charge distribution is uniform, the charge enclosed in our Gaussian surface is given by

$$\frac{Q_{enc}}{\epsilon_0} = \frac{\rho(4/3)\pi r^3}{\epsilon_0} = \frac{Q}{\epsilon_0} \frac{r^3}{R^3}.$$

Now we apply Gauss's Law:

$$\oiint \mathbf{E}_I \cdot d\mathbf{A} = \frac{Q_{enc}}{\epsilon_0}.$$

to arrive at:

$$E_I \cdot 4\pi r^2 = \frac{Q}{\epsilon_0} \frac{r^3}{R^3}.$$

which we can solve for the electric field inside the sphere

$$\vec{E}_I = E_I \hat{r} = \frac{Qr}{4\pi\epsilon_0 R^3} \hat{r}, \quad 0 < r < R.$$

For region II: $r > R$: we choose the same spherical Gaussian surface of radius $r > R$, and the electric flux has the same form

$$\oiint \vec{E}_{II} \cdot d\vec{A} = E_{II} \cdot 4\pi r^2.$$

$r > R$



All the charge is now enclosed, $Q_{enc} = Q$, then Gauss's Law becomes

$$E_{II} \cdot 4\pi r^2 = \frac{Q}{\epsilon_0}.$$

We can solve this equation for the electric field

$$\vec{\mathbf{E}}_{II} = E_{II} \hat{\mathbf{r}} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{\mathbf{r}}, \quad r > R.$$

In this region of space, the electric field falls off as $1/r^2$ as we expect since outside the charge distribution, the sphere acts as if all the charge were concentrated at the origin.

Our complete expression for the electric field as a function of r is then

$$\vec{\mathbf{E}}(r) = \begin{cases} \vec{\mathbf{E}}_I = E_I \hat{\mathbf{r}} = \frac{Qr}{4\pi\epsilon_0 R^3} \hat{\mathbf{r}}, & 0 < r < R \\ \vec{\mathbf{E}}_{II} = E_{II} \hat{\mathbf{r}} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{\mathbf{r}}, & r > R \end{cases}$$

We can now find the electric potential difference between any point lying on a sphere of radius r and the origin, i.e. $V(r) - V(0)$.

We begin by considering values of r such that $0 < r < R$. We shall calculate the potential difference by calculating the line integral

$$V(r) - V(0) = - \int_{r'=0}^{r'=r} \vec{\mathbf{E}}_I \cdot d\vec{\mathbf{r}}'; \quad 0 < r < R$$

We use as integration variable r' and integrate from $r' = 0$ to $r' = r$:

$$V(r) - V(0) = - \int_{r'=0}^{r'=r} \frac{Qr'}{4\pi\epsilon_0 R^3} \hat{\mathbf{r}} \cdot dr' \hat{\mathbf{r}} = - \int_{r'=0}^{r'=r} \frac{Qr'}{4\pi\epsilon_0 R^3} dr' = - \frac{Qr^2}{8\pi\epsilon_0 R^3}; \quad 0 < r < R$$

For $r > R$: we are taking a path from the origin through regions I and regions II and so we need to use both functional forms for the electric field in the appropriate regions. The potential difference between any point lying on a sphere of radius $r > R$ and the origin is given by the line integral expression

$$V(r) - V(0) = - \int_{r'=0}^{r'=R} \vec{E}_I \cdot d\vec{r}' - \int_{r'=R}^{r'=r} \vec{E}_{II} \cdot d\vec{r}'; \quad r > R.$$

Using our results for the electric field we get that

$$V(r) - V(0) = - \int_{r'=0}^{r'=R} \frac{Qr'}{4\pi\epsilon_0 R^3} \hat{r} \cdot dr' \hat{r} - \int_{r'=R}^{r'=r} \frac{Q}{4\pi\epsilon_0 r'^2} \hat{r} \cdot dr' \hat{r}; \quad r > R$$

This becomes

$$V(r) - V(0) = - \int_{r'=0}^{r'=R} \frac{Qr'}{4\pi\epsilon_0 R^3} dr' - \int_{r'=R}^{r'=r} \frac{Q}{4\pi\epsilon_0 r'^2} dr'; \quad r > R$$

Integrating yields

$$V(r) - V(0) = - \left. \frac{Qr'^2}{8\pi\epsilon_0 R^3} \right|_{r'=0}^{r'=R} + \left. \frac{Q}{4\pi\epsilon_0 r'} \right|_{r'=R}^{r'=r}; \quad r > R$$

Substituting in the endpoints yields

$$V(r) - V(0) = V(r) - V(0) = - \frac{Q}{8\pi\epsilon_0 R} + \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{r} - \frac{1}{R} \right); \quad r > R$$

A little algebra then yields

$$V(r) - V(0) = \frac{Q}{4\pi\epsilon_0 r} - \frac{3Q}{8\pi\epsilon_0 R}; \quad r > R$$

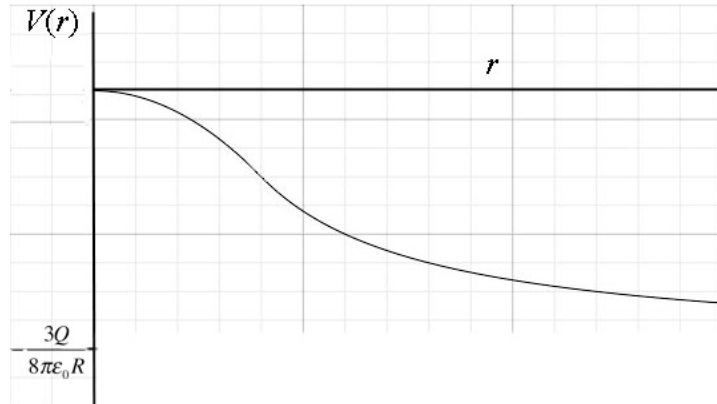
Thus the electric potential difference between any point lying on a sphere of radius r and the origin (where $V(0) = 0$) is given by

$$V(r) - V(0) = \begin{cases} -\frac{Qr^2}{8\pi\epsilon_0 R^3}; & 0 < r < R \\ \frac{Q}{4\pi\epsilon_0 r} - \frac{3Q}{8\pi\epsilon_0 R}; & r > R \end{cases}$$

When we set $V(0) = 0$, we have an expression for the electric potential function

$$V(r) = \begin{cases} -\frac{Qr^2}{8\pi\epsilon_0 R^3}; & 0 < r < R \\ \frac{Q}{4\pi\epsilon_0 r} - \frac{3Q}{8\pi\epsilon_0 R}; & r > R \end{cases}$$

We plot $V(r)$ vs. r in the figure below. Note that the graph of the electric potential function is continuous at $r = R$.



When we set $r = \infty$, the potential difference between the sphere at infinity and the origin is

$$V(\infty) - V(0) = -\frac{3Q}{8\pi\epsilon_0 R}.$$

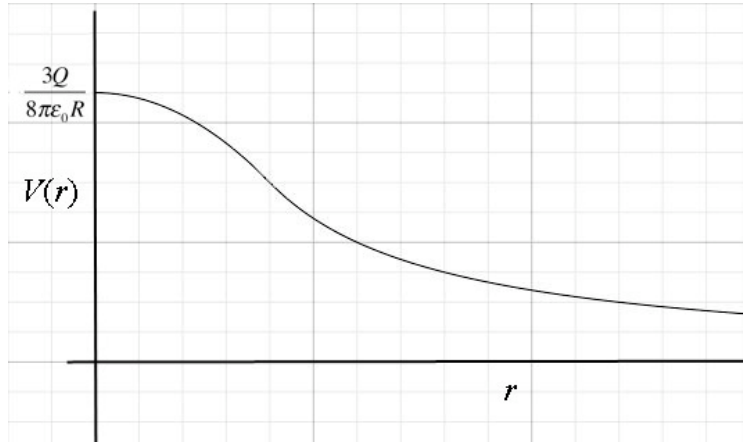
If we had chosen the zero reference point for the electric potential at $r = \infty$, with $V(\infty) = 0$. Then with that choice, we have that $V(0) = \frac{3Q}{8\pi\epsilon_0 R}$. Therefore using our results above the new form for the potential function is

$$V(r) = \begin{cases} V(0) - \frac{Qr^2}{8\pi\epsilon_0 R^3}; & 0 < r < R \\ V(0) + \frac{Q}{4\pi\epsilon_0 r} - \frac{3Q}{8\pi\epsilon_0 R}; & r > R \end{cases}$$

This amounts to just adding the constant $\frac{3Q}{8\pi\epsilon_0 R}$ to the above results for the potential function $V(r)$ giving

$$V(r) = \begin{cases} \frac{3Q}{8\pi\epsilon_0 R} - \frac{Qr^2}{8\pi\epsilon_0 R^3}; & 0 < r < R \\ \frac{Q}{4\pi\epsilon_0 r}; & r > R \end{cases} .$$

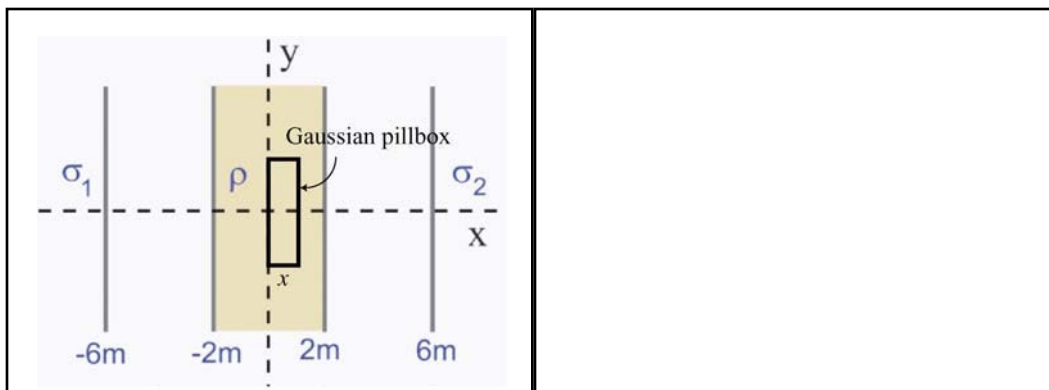
In the above expression we can easily check that $V(\infty) = 0$. Equivalently we shift our previous graph up by $3Q/8\pi\epsilon_0 R$ as shown in the graph below.



Problem 4:

An infinite slab of charge carrying a charge per unit volume ρ has a charged sheet carrying charge per unit area σ_1 to its left and a charged sheet carrying charge per unit area σ_2 to its right (see top part of sketch). The lower plot in the sketch shows the electric potential $V(x)$ in volts due to this slab of charge and the two charged sheets as a function of horizontal distance x from the center of the slab. The slab is 4 meters wide in the x -direction, and its boundaries are located at $x = -2$ m and $x = +2$ m, as indicated. The slab is infinite in the y direction and in the z direction (out of the page). The charge sheets are located at $x = -6$ m and $x = +6$ m, as indicated.

- (a) The potential $V(x)$ is a linear function of x in the region $-6 \text{ m} < x < -2 \text{ m}$. What is the electric field in this region?
- (b) The potential $V(x)$ is a linear function of x in the region $2 \text{ m} < x < 6 \text{ m}$. What is the electric field in this region?
- (c) In the region $-2 \text{ m} < x < 2 \text{ m}$, the potential $V(x)$ is a quadratic function of x given by the equation $V(x) = \frac{5}{16}x^2 \frac{\text{V}}{\text{m}^2} - \frac{25}{4} \text{V}$. What is the electric field in this region?
- (d) Use Gauss's Law and your answers above to find an expression for the charge density ρ of the slab. Indicate the Gaussian surface you use on a figure.



- (e) Use Gauss's Law and your answers above to find the two surface charge densities of the left and right charged sheets. Indicate the Gaussian surface you use on a figure.

Problem 4 Solution:

(a)

$$\vec{E} = -\frac{\partial V}{\partial x} \hat{i} = -\frac{\Delta V}{\Delta x} \hat{i} = \frac{-5 \text{ V}}{4 \text{ m}} = 1.25 \frac{\text{V}}{\text{m}} \hat{i}$$

(b)

$$\vec{\mathbf{E}} = -\frac{\partial V}{\partial x} \hat{\mathbf{i}} = -\frac{\Delta V}{\Delta x} \hat{\mathbf{i}} = -\frac{5 \text{ V}}{4 \text{ m}} = -1.25 \frac{\text{V}}{\text{m}} \hat{\mathbf{i}}$$

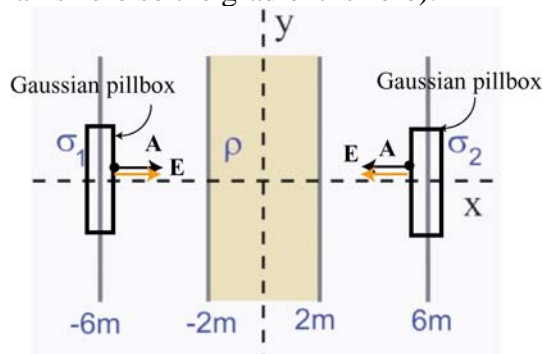
(c) In the region inside the slab, the electric field is

$$\vec{\mathbf{E}} = -\frac{\partial V}{\partial x} \hat{\mathbf{i}} = \left[-\frac{5 \text{ V}}{8 \text{ m}^2} \right] x \hat{\mathbf{i}}$$

(d)

$$\oiint_s \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = EA = \left[-\frac{5 \text{ V}}{8 \text{ m}^2} \right] xA = \frac{q_{in}}{\epsilon_0} = \frac{\rho xA}{\epsilon_0}$$
$$\Rightarrow \rho = \left[-\frac{5 \text{ V}}{8 \text{ m}^2} \right] \epsilon_0$$

(e)**Solution:** The electric field vanishes in the regions $x > 6 \text{ m}$ and $x < -6 \text{ m}$ (the electric potential is zero and remains zero so the gradient is zero).



Using Gauss's law with the Gaussian pillboxes indicated in the figure, we have

$$\oiint_s \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = EA = \left[\frac{5 \text{ V}}{4 \text{ m}} \right] A = \frac{q_{in}}{\epsilon_0} = \frac{\sigma_1 A}{\epsilon_0}$$
$$\Rightarrow \sigma_1 = \left[\frac{5 \text{ V}}{4 \text{ m}} \right] \epsilon_0$$

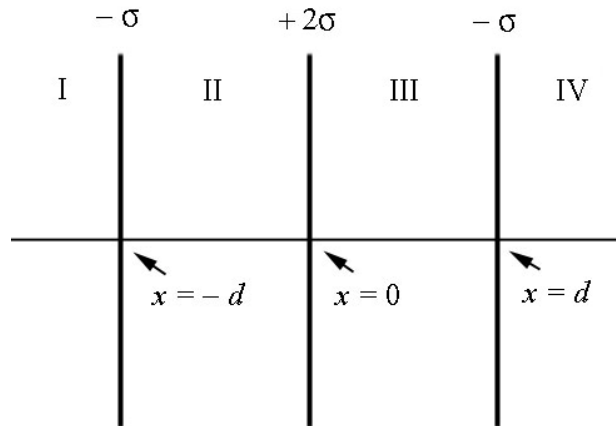
In a similar manner, $\sigma_2 = \frac{5 \text{ V}}{4 \text{ m}} \epsilon_0$.

A common mistake is to think that the sign must flip because the electric field sign flips. Note that because the area vector of the Gaussian pillbox also flips direction this is NOT true. It is very important to draw pictures and show the vector directions. If the vectors

(\vec{E} and $d\vec{A}$) are in the same direction then the dot product (and the enclosed charge) is positive.

Problem 5:

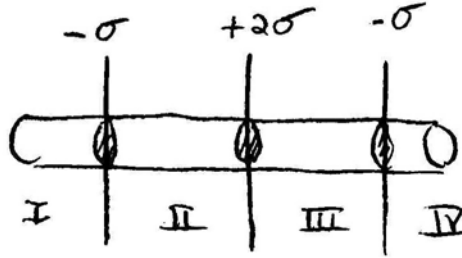
Three infinite sheets of charge are located at $x = -d$, $x = 0$, and $x = d$, as shown in the sketch. The sheet at $x = 0$ has a charge per unit area of 2σ , and the other two sheets have charge per unit area of $-\sigma$.



- a) What is the electric field in each of the four regions I-IV labeled in the sketch? Clearly present your reasoning, relevant figures, and any accompanying calculations. Plot the x component of the electric field, E_x , on the graph on the bottom of the next page. Clearly indicate on the vertical axis the values of E_x for the different regions.
- b) Find the electric potential in each of the four regions I-IV labeled above, with the choice that the potential is zero at $x = +\infty$ i.e. $V(+\infty) = 0$. Show your calculations. Plot the electric potential as a function of x on the graph on the bottom of the next page. Indicate units on the vertical axis.
- c) How much work must you do to bring a point-like object with charge $+Q$ in from infinity to the origin $x = 0$?

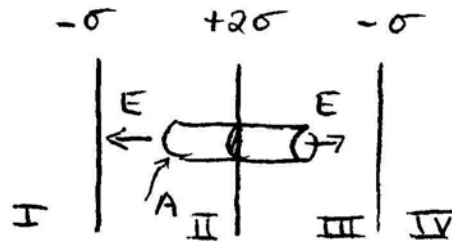
Problem 5 Solutions:

(a) We begin by choosing a Gaussian cylinder with end caps in regions I and IV as shown in the figure below. The total charge enclosed is zero and hence the electric flux on the endcaps must be zero. Thus the electric field must be zero in regions I and IV.



This turns out to be correct but the conclusion depends on an additional argument based on symmetry. If the electric field is non-zero on the endcaps it must point either in the $+x$ -direction in both regions I and IV or in the $-x$ -direction in both regions I and IV. Neither is possible due to the symmetry of the charge distribution. For example, if the electric field pointed in the $+x$ -direction in both regions I and IV. Then if we looked at the charge distribution from the other side of the plane of the paper, the field should point in the $-x$ -direction. However the charge distribution is identical when looking from the other side of the paper. Therefore the field must point in the $+x$ -direction according to our original assertion. Therefore by symmetry the only possibility is for the fields in regions I and IV to point toward $x=0$ or away from $x=0$. In the first case the flux would be non-zero on our Gaussian surface but it must be zero because the charge enclosed is zero. Hence the electric field in regions I and IV is zero. (A similar argument holds if we assume that the field points in the $-x$ -direction in both regions I and IV.)

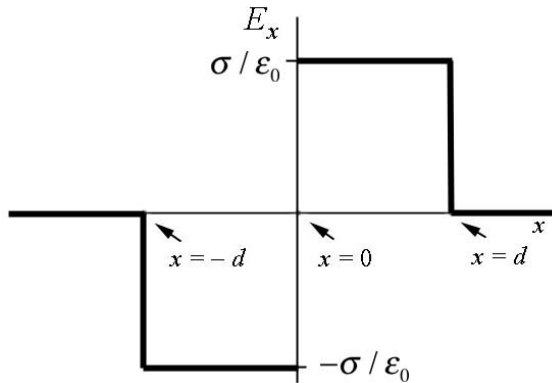
For regions II and III, we choose a Gaussian cylinder with end caps in regions II and III as shown in the figure below.



The electric flux on the endcaps is $\oiint \vec{E} \cdot d\vec{A} = 2EA$. The charge enclosed divided by ϵ_0 is $Q_{enc} / \epsilon_0 = 2\sigma A / \epsilon_0$. Therefore by Gauss's Law, $2EA = 2\sigma A / \epsilon_0$ which implies that the magnitude of the electric field is $E = \sigma / \epsilon_0$. Thus the electric field is given by

$$\vec{E} = \begin{cases} \vec{0} & ; \quad x < -d \\ -\frac{\sigma}{\epsilon_0} \hat{i} & ; \quad -d < x < 0 \\ \frac{\sigma}{\epsilon_0} \hat{i} & ; \quad 0 < x < +d \\ \vec{0} & ; \quad d < x \end{cases}$$

The graph of the x component of the electric field, E_x vs x is shown on the graph below.



(b) The electric potential difference between infinity and a point P located at x , is given by

$$V(x) - V(\infty) = -\int_{\infty}^P \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}.$$

We shall evaluate this integral for points in each region. We start with P anywhere in region IV, $d < x$. Because the electric field in region IV is zero, the integral is zero,

$$V(x) - V(\infty) = -\int_{\infty}^x \vec{\mathbf{E}}_{IV} \cdot d\vec{\mathbf{s}} = 0.$$

If P is anywhere in region III, $0 < x < +d$ then

$$\begin{aligned} V(x) - V(\infty) &= -\int_{\infty}^d \vec{\mathbf{E}}_{IV} \cdot d\vec{\mathbf{s}} - \int_d^x \vec{\mathbf{E}}_{III} \cdot d\vec{\mathbf{s}} \\ &= 0 - \int_d^x E_x dx = -\int_d^x \frac{\sigma}{\epsilon_0} dx = -\frac{\sigma}{\epsilon_0} (x - d) = \frac{\sigma}{\epsilon_0} d - \frac{\sigma}{\epsilon_0} x \end{aligned}$$

If P is anywhere in region II, $-d < x < 0$ then

$$\begin{aligned} V(x) - V(\infty) &= -\int_{\infty}^d \vec{\mathbf{E}}_{IV} \cdot d\vec{\mathbf{s}} - \int_d^0 \vec{\mathbf{E}}_{III} \cdot d\vec{\mathbf{s}} - \int_0^x \vec{\mathbf{E}}_{II} \cdot d\vec{\mathbf{s}} \\ &= 0 - \int_d^0 \frac{\sigma}{\epsilon_0} dx - \int_0^x -\frac{\sigma}{\epsilon_0} dx = \frac{\sigma}{\epsilon_0} d + \frac{\sigma}{\epsilon_0} x \end{aligned}$$

If P is anywhere in region I, $x < -d$ then

$$\begin{aligned}
 V(x) - V(\infty) &= -\int_{\infty}^d \vec{\mathbf{E}}_{IV} \cdot d\vec{\mathbf{s}} - \int_d^0 \vec{\mathbf{E}}_{III} \cdot d\vec{\mathbf{s}} - \int_0^{-d} \vec{\mathbf{E}}_{II} \cdot d\vec{\mathbf{s}} - \int_{-d}^x \vec{\mathbf{E}}_I \cdot d\vec{\mathbf{s}} \\
 &= 0 - \int_d^0 \frac{\sigma}{\epsilon_0} dx - \int_0^{-d} -\frac{\sigma}{\epsilon_0} dx - 0 = \frac{\sigma}{\epsilon_0} d - \frac{\sigma}{\epsilon_0} d = 0
 \end{aligned}$$

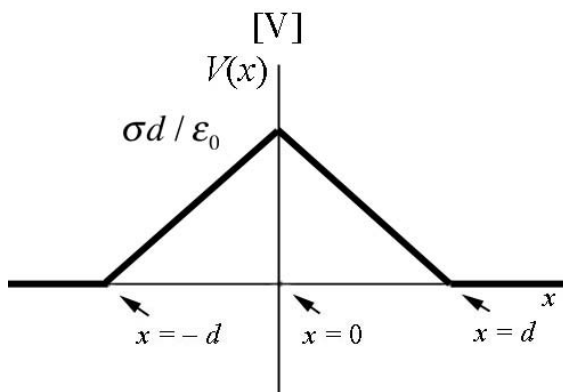
Because the electric field is continuous we can write our result as

$$V(x) - V(\infty) = \begin{cases} 0 & ; & x \leq -d \\ \frac{\sigma}{\epsilon_0} d + \frac{\sigma}{\epsilon_0} x & ; & -d \leq x \leq 0 \\ \frac{\sigma}{\epsilon_0} d - \frac{\sigma}{\epsilon_0} x & ; & 0 \leq x \leq +d \\ 0 & ; & d \leq x \end{cases}$$

Note this can be written as

$$V(x) - V(\infty) = \begin{cases} 0 & ; & x \leq -d \\ \frac{\sigma}{\epsilon_0} d - \frac{\sigma}{\epsilon_0} |x| & ; & -d \leq x \leq d \\ 0 & ; & d \leq x \end{cases}$$

This result looks good because the area under the graph of the x component of the electric field, E_x vs x for the region $-d < x < d$ is zero. The plot of the electric potential as a function of x on the graph is shown below with units of [V] on the vertical axis.



(c) The work you must do is equal to the change in potential energy (assuming the point-like object begins and ends at rest). Therefore

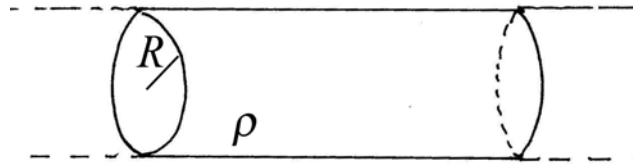
$$W = U(0) - U(\infty) = +Q(V(0) - V(\infty)) = +\frac{Q\sigma}{\epsilon_0}d.$$

Problem 6:

You may find the following integrals helpful in this answering this question.

$$\int_{r_a}^{r_b} r^n dr = \frac{1}{n+1} (r_b^{n+1} - r_a^{n+1}); n \neq -1, \quad \int_{r_a}^{r_b} \frac{dr}{r} = \ln(r_b / r_a).$$

Consider a charged infinite cylinder of radius R .



The charge density is non-uniform and given by

$$\rho(r) = br; r < R,$$

where r is the distance from the central axis and b is a constant.

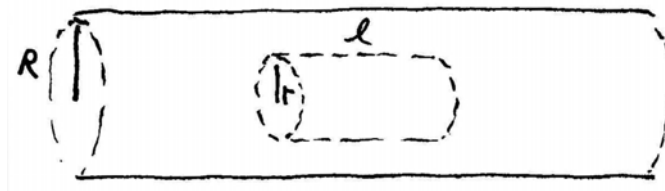
a) Find an expression for the direction and magnitude of the electric field everywhere i.e. inside and outside the cylinder. Clearly present your reasoning, relevant figures, and any accompanying calculations.

b) A point-like object with charge $+q$ and mass m is released from rest at the point a distance $2R$ from the central axis of the cylinder. Find the speed of the object when it reaches a distance $3R$ from the central axis of the cylinder.

Problem 6 Solutions:

(a) Because the charge distribution defines two distinct regions of space, region I defined by $r < R$ and region II defined by $r > R$, we must apply Gauss's Law twice to find the electric field everywhere.

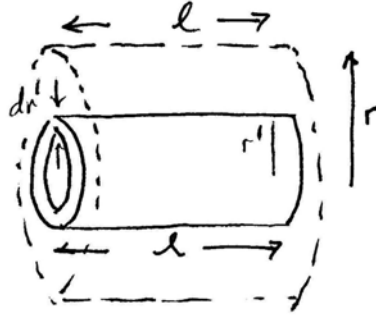
In region I, where $r < R$, we choose a Gaussian cylinder of radius r and length l .



Because the electric field points away from the central axis, the electric flux on our Gaussian surface is

$$\oiint \vec{E}_l \cdot d\vec{A} = E_l 2\pi r l .$$

Because the charge density is non-uniform, we must integrate the charge density. We choose as our integration volume a cylindrical shell of radius r' , length l and thickness dr' . The integration volume is then $dV' = 2\pi r' l dr'$.



Therefore the charge divided by ϵ_0 enclosed within our Gaussian surface is

$$Q_{enc} / \epsilon_0 = \frac{1}{\epsilon_0} \int_0^r \rho 2\pi r' l dr' = \frac{1}{\epsilon_0} \int_0^r b r' 2\pi r' l dr' = \frac{2\pi l b}{\epsilon_0} \int_0^r r'^2 dr' = \frac{2\pi l b r^3}{3\epsilon_0} .$$

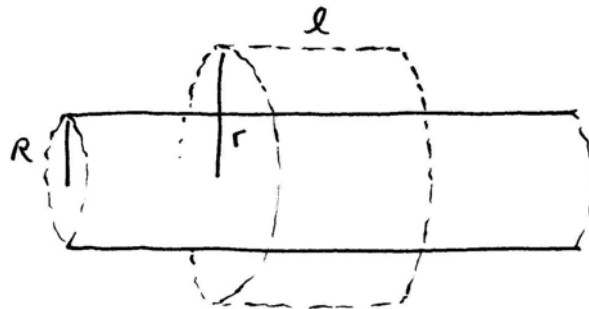
Therefore Gauss's Law becomes

$$E_l 2\pi r l = 2\pi l b r^3 / 3 .$$

We can now solve for the direction and magnitude of the electric field when $r < R$,

$$\vec{E}_l = \frac{b r^2}{3\epsilon_0} \hat{r} .$$

In region II where $r > R$, we choose a Gaussian cylinder of radius r and length l .



Because the electric field points away from the central axis, the electric flux on our Gaussian surface is

$$\oiint \vec{\mathbf{E}}_{||} \cdot d\vec{\mathbf{A}} = E_{||} 2\pi r l.$$

We again must integrate the charge density but this time taking our endpoints as $r = 0$ and $r = R$. Therefore the charge divided by ϵ_0 enclosed within our Gaussian surface is

$$Q_{enc} / \epsilon_0 = \frac{1}{\epsilon_0} \int_0^r \rho 2\pi r' l dr' = \frac{1}{\epsilon_0} \int_0^R br' 2\pi r' l dr' = \frac{2\pi lb}{\epsilon_0} \int_0^R r'^2 dr' = \frac{2\pi lb R^3}{3\epsilon_0}.$$

Therefore Gauss's Law becomes

$$E_{||} 2\pi r l = 2\pi lb R^3 / 3.$$

We can now solve for the direction and magnitude of the electric field when $r > R$,

$$\vec{\mathbf{E}}_{||} = \frac{bR^3}{3\epsilon_0} \frac{1}{r} \hat{\mathbf{r}}.$$

Collected our results we have that

$$\vec{\mathbf{E}} = \begin{cases} \frac{br^2}{3\epsilon_0} \hat{\mathbf{r}}; & r < R \\ \frac{bR^3}{3\epsilon_0} \frac{1}{r} \hat{\mathbf{r}}; & r > R \end{cases}$$

(b) The change in kinetic energy when the object moves from a distance $2R$ from the central axis of the cylinder to a distance $3R$ is given by

$$K(3R) - K(2R) = -(U(3R) - U(2R)) = -q(V(3R) - V(2R)).$$

Because the particle was released at rest, $K(2R) = 0$, and $K(3R) = (1/2)mv_f^2$, the final speed of the object is

$$v_f = \sqrt{-\frac{2q}{m}(V(3R) - V(2R))}.$$

The electric potential difference between two points in region II is given by

$$\begin{aligned} V(3R) - V(2R) &= -\int_{2R}^{3R} \vec{\mathbf{E}}_{II} \cdot d\vec{\mathbf{s}} = -\int_{2R}^{3R} \frac{bR^3}{3\epsilon_0} \frac{1}{r} \hat{\mathbf{r}} \cdot d\vec{\mathbf{s}} \\ &= -\int_{2R}^{3R} \frac{bR^3}{3\epsilon_0} \frac{1}{r} dr = -\frac{bR^3}{3\epsilon_0} \ln \frac{3R}{2R} = -\frac{bR^3}{3\epsilon_0} \ln(3/2) \end{aligned}$$

Therefore the speed of the object when it reaches a distance $3R$ from the central axis of the cylinder is

$$v_f = \sqrt{\frac{2qbR^3}{3m\epsilon_0} \ln(3/2)} .$$

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