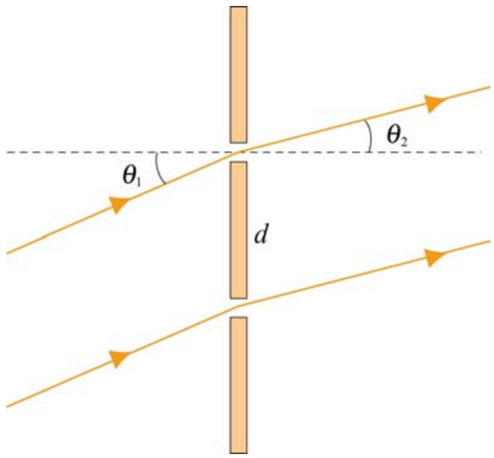
Interference
Challenge Problem Solutions

Problem 1:

Coherent light rays of wavelength λ are illuminated on a pair of slits separated by distance d at an angle θ_1, as shown in the figure below.



If an interference maximum is formed at an angle θ_2 far from the slits, find the relationship between θ_1, θ_2, d and λ.

Problem 1 Solution:

The path difference between the two rays is

$$\delta = d\sin\theta_1 - d\sin\theta_2 \qquad (0.1)$$

For a constructive interference, $\delta = m\lambda$, where $m = 0, \pm 1, \pm 2, \ldots$ is the order number. Thus, the condition is

$$d\left(\sin\theta_1 - \sin\theta_2\right) = m\lambda \qquad \textbf{(0.2)}$$

Problem 2:

In the Young's double-slit experiment, suppose the separation between the two slits is d=0.320 mm. If a beam of 500-nm light strikes the slits and produces an interference pattern. How many maxima will there be in the angular range $-30.0° < \theta < 30.0°$?

Problem 2 Solution:

On the viewing screen, light intensity is a maximum when the two waves interfere constructively. This occurs when

$$d \sin\theta = m\lambda, \quad m=0, \pm 1, \pm 2, ... \qquad (0.3)$$

where λ is the wavelength of the light. At the angle $\theta = 30.0°$, $d = 3.20 \times 10^{-4}$ m and $\lambda = 500 \times 10^{-9}$ m, we get

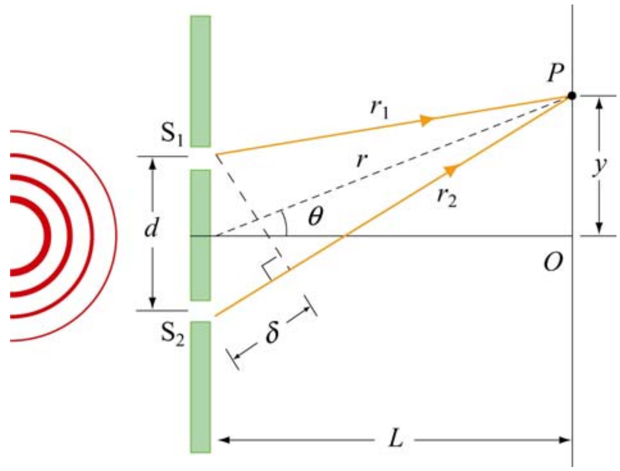
$$m = \frac{d \sin\theta}{\lambda} = 320 \qquad (0.4)$$

Thus, there are 320 maxima in the range $0 < \theta < 30.0°$. By symmetry, there are also 320 maxima in the range $-30.0° < \theta < 0$. Including the one for $m = 0$ straight ahead, the total number of maxima is

$$N = 320 + 320 + 1 = 641 \qquad (0.5)$$

Problem 3:

In the double-slit interference experiment shown in the figure, suppose $d = 0.100\,\text{mm}$ and $L = 1.20\,\text{m}$, and the incident light is monochromatic with a wavelength λ=600 nm.



(a) What is the phase difference between the two waves arriving at a point P on the screen when $\theta = 0.800°$?

(b) What is the phase difference between the two waves arriving at a point P on the screen when $y = 4.00\,\text{mm}$?

(c) If the phase difference between the two waves arriving at point P is ϕ=1/3 rad, what is the value of θ?

(d) If the path difference is $\delta = \lambda / 4$, what is the value of θ?

(e) In the double-slit interference experiment, suppose the slits are separated by $d = 1.00\,\text{cm}$ and the viewing screen is located at a distance $L = 1.20\,\text{m}$ from the slits. Let the incident light be monochromatic with a wavelength λ=500 nm. Calculate the spacing between the adjacent bright fringes on the viewing screen.

(f) What is the distance between the third-order fringe and the center line on the viewing screen?

Problem 3 Solutions:

(a)

$$\begin{aligned}
\phi &= 2\pi \frac{\delta}{\lambda} \\
&= 2\pi \frac{d \sin\theta}{\lambda} \\
&= 2(3.14) \frac{(1.00 \times 10^{-4}\ m) \sin 0.8°}{6.00 \times 10^{-7}\ m} \\
&= 14.6\, rad
\end{aligned}$$

(b)

$$\phi = 2\pi \frac{d\,y}{\lambda L} \quad (\because \sin\theta \approx \frac{y}{L})$$

$$= 2(3.14)\frac{(1.00\times10^{-4}\ m)(4.00\times10^{-3}\ m)}{(6.00\times10^{-7}\ m)(1.20\,m)}$$

$$= 3.49\,rad$$

(c)

$$\phi = \frac{1}{3}\ rad = 2\pi \frac{d\sin\theta}{\lambda} \quad \Rightarrow \quad \theta = \sin^{-1}\left(\frac{\lambda\phi}{2\pi d}\right) = 3.18\times10^{-4}\,rad = 0.0182^\circ$$

(d)

$$\delta = d\sin\theta \quad \Rightarrow \quad \theta = \sin^{-1}\left(\frac{\delta}{d}\right) = \sin^{-1}\left(\frac{\lambda}{4d}\right) = 1.50\times10^{-3}\ rad = 0.0860^\circ$$

(e)

Since $y_b = m\frac{\lambda L}{d}$, the spacing between adjacent bright fringes is

$$\Delta y_b = y_b(m+1) - y_b(m)$$

$$= (m+1)\frac{\lambda L}{d} - m\frac{\lambda L}{d}$$

$$= \frac{\lambda L}{d}$$

$$= \frac{(5.00\times10^{-7}\ m)(1.20\,m)}{(1.00\times10^{-2}\ m)}$$

$$= 6.00\times10^{-5}\ m$$

$$= 60.0\,\mu m$$

(f)

$$
\begin{aligned}
\Delta y_b &= y_b(3) - y_b(0) \\
&= (3)\frac{\lambda L}{d} - 0 \\
&= 3\frac{\lambda L}{d} \\
&= 3\frac{(5.00\times 10^{-7}\ m)(1.20\, m)}{(1.00\times 10^{-2}\ m)} \\
&= 1.80\times 10^{-4}\ m \\
&= 180\ \mu m
\end{aligned}
$$

Problem 4:

Let the intensity on the screen at a certain point in a double-slit interference pattern be 64.0% of the maximum value.

(a) What is the minimum phase difference (in radians) between sources that produces this result?

(b) Express this phase difference as a path difference for 486.1-nm light.

Problem 4 Solutions:

(a) The average intensity is given by

$$I = I_0 \cos^2\left(\frac{\phi}{2}\right) \qquad (0.6)$$

where I_0 is the maximum light intensity. Thus,

$$0.64 = \cos^2\left(\frac{\phi}{2}\right) \qquad (0.7)$$

which yields

$$\phi = 2\cos^{-1}\left(\frac{I}{I_0}\right)^{1/2} = 2\cos^{-1}(0.64)^{1/2} = 1.29 \text{ rad} \qquad (0.8)$$

(b) The phase difference ϕ is related to the path difference δ and the wavelength λ by

$$\delta = \frac{\lambda\phi}{2\pi} \qquad (0.9)$$

Therefore,

$$\delta = \frac{(486\,\text{nm})(1.29\,\text{rad})}{2\pi} = 99.8 \text{ nm} \qquad (0.10)$$

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