Driven RLC Circuits
Challenge Problems

Problem 1:

Using the same circuit as in problem 6, only this time leaving the function generator on and driving below resonance, which in the following pairs leads (if either):

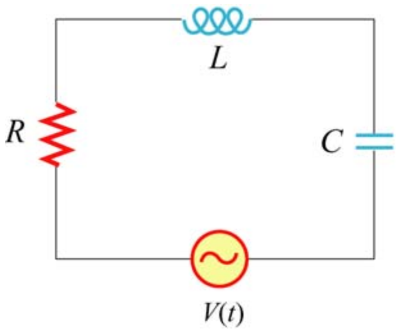
(a) Voltage across the capacitor or voltage across the resistor

(b) Voltage across the function generator or voltage measured across the inductor

(c) Current or voltage across the resistor

(d) Current or voltage across the function generator

Problem 2:



Consider the circuit at left, consisting of an AC function generator ($V(t) = V_0 \sin(\omega t)$, with V_0 = 5 V), an inductor L = 8.5 mH, resistor R = 5 Ω, capacitor C = 100 μF and switch S.

The circuit has been running in equilibrium for a long time. We are now going to shut off the function generator (instantaneously replace it with a wire).

a) Assuming that our driving frequency ω is not necessarily on resonance, what is the frequency with which the system will ring down (in other words, that current will oscillate at after turning off the function generator)? Feel free to use an approximation if you wish, just make sure you know you are.

(b) What (numerical) frequency f should we drive at to maximize the peak magnetic energy in the inductor?

(c) In this case, if we time the shut off to occur when the magnetic energy in the inductor peaks, after how long will the electric energy in the capacitor peak?

(d) Approximately how much energy will the resistor have dissipated during that time?

Problem 3:

A series *RLC* circuit with $R = 10.0\ \Omega$, $L = 400\ \text{mH}$ and $C = 2.0\ \mu\text{F}$ is connected to an AC voltage source $V(t) = V_0 \sin \omega t$ which has a maximum amplitude $V_0 = 100\ \text{V}$.

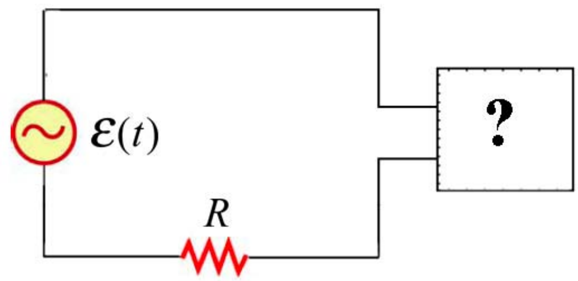
(a) What is the resonant frequency ω_0?

(b) Find the rms current at resonance.

(c) Let the driving frequency be $\omega = 4000\ \text{rad/s}$. Assume the current response is given by $I(t) = I_0 \sin(\omega t - \phi)$. Calculate the amplitude of the current and the phase shift between the current and the driving voltage.

Problem 4:

The circuit shown below contains an AC generator which provides a source of sinusoidally varying emf $\mathcal{E} = \mathcal{E}_0 \sin \omega t$, a resistor with resistance $R = 6\,\Omega$, and a "black box", which contains ***either*** an inductor ***or*** a capacitor, ***or both***. The amplitude of the driving emf is $\mathcal{E}_0 = 6\text{ volt}$. We measure the current in the circuit at an angular frequency $\omega = 2\text{ rad}\cdot\text{s}^{-1}$ and find that it is exactly in phase with the driving emf. We measure the current in the circuit at an angular frequency $\omega = 1\text{ rad}\cdot\text{s}^{-1}$ and find that it is out of phase from the driving emf by exactly $\pi/4$ radians.



a) What does the black box contain – an inductor or a capacitor, or both? Explain your reasoning. Does current lead or lag at $\omega = 1\text{ rad}\cdot\text{s}^{-1}$?

b) What is the numerical value of the capacitance *or* of the inductance, *or of both*, as the case may be? Indicate units. Your answer(s) will involve simple fractions only, you will not need a calculator to find the value(s).

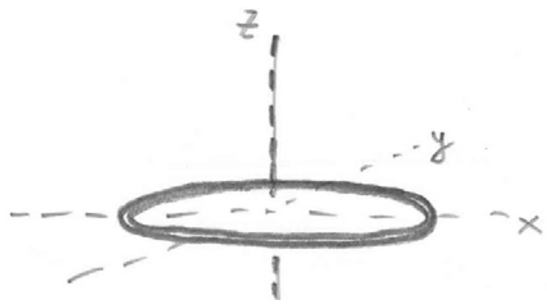
c) What is ratio of the amplitudes of the current $\dfrac{I_0(\omega = 2\text{ rad}\cdot\text{s}^{-1})}{I_0(\omega = 1\text{ rad}\cdot\text{s}^{-1})}$?

Problem 5:

Loop Antenna. An electromagnetic wave propagating in air has a magnetic field given by

$$B_x = 0 \qquad B_y = 0 \qquad B_z = B_0 \cos(\omega t - kx) .$$

It encounters a circular loop antenna of radius a centered at the origin $(x, y, z) = (0,0,0)$ and lying in the x-y plane. The radius of the antenna $a << \lambda$ where λ is the wavelength of the wave. So you can assume that at any time t the magnetic field inside the loop is approximately equal to its value at the center of the loop.



a) What is the magnetic flux, $\Phi_{mag}(t) \equiv \iint_{disk} \vec{\mathbf{B}} \cdot d\vec{\mathbf{a}}$, through the plane of the loop of the antenna?

The loop has a self-inductance L and a resistance R. Faraday's law for the circuit is

$$IR = -\frac{d\Phi_{mag}}{dt} - L\frac{dI}{dt} .$$

b) Assume a solution for the current of the form $I(t) = I_0 \sin(\omega t - \phi)$ where ω is the angular frequency of the electromagnetic wave, I_0 is the amplitude of the current, and ϕ is a phase shift between the changing magnetic flux and the current.. Find expressions for the constants ϕ and I_0.

c) What is the magnetic field created at the center of the loop by this current $I(t)$?

Problem 6:

Driven RLC circuit

Suppose you want a series RLC circuit to tune to your favorite FM radio station that broadcasts at a frequency of $89.7\,MHz$. You would like to avoid the obnoxious station that broadcasts at $89.5\,MHz$. In order to achieve this, for a given input voltage signal from your antenna, you want the width of your resonance to be narrow enough at $89.7\,MHz$ such that the current flowing in your circuit will be 10^{-2} times less at $89.5\,MHz$ than at $89.7\,MHz$. You cannot avoid having a resistance of $R = 0.1\Omega$, and practical considerations also dictate that you use the minimum L possible.

a) In terms of your circuit parameters, L, R, and C, what is the amplitude of your current in your circuit as a function of the angular frequency of the input signal?

b) What is the angular frequency of the input signal at the desired resonance?

c) What values of L and C must you use?

d) What is the quality factor for this resonance?

e) Show that at resonance, the ratio of the amplitude of the voltage across the inductor with the driving signal amplitude is the quality of the resonance.

f) Show that at resonance the ratio of the amplitude of the voltage across the capacitor with the driving signal amplitude is the quality of the resonance

g) What is the time averaged power at resonance that the signal delivers to the circuit?

h) What is the phase shift for the input signal at $89.5\,MHz$?

i) What is the time averaged power for the input signal at $89.5\,MHz$?

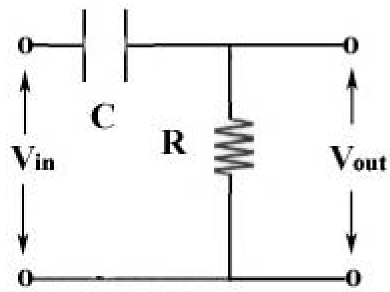
j) Is the circuit acting capacitatively or inductively at $89.5\,MHz$?

Problem *7:*

Consider a circuit consisting of a resistor and a capacitor with an ac sinusoidal input,

$$V_{in}(t) = V_0 \sin(\omega t)$$

and two output terminals.



a) What is the total impedance for this circuit?

b) What is the amplitude and phase of the current $I(t) = I_0 \sin(\omega t - \phi)$ in the circuit?

c) What is the amplitude and phase of the output voltage $V_{out}(t)$ across the resistor?

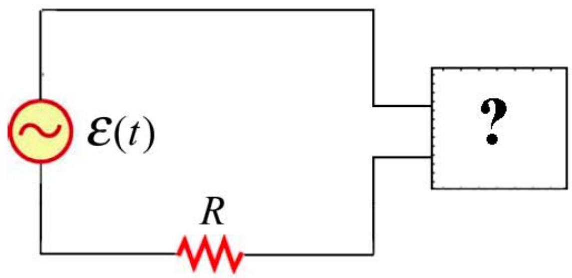
d) What is the ratio of the magnitudes of the output signal amplitude to the input signal amplitude $\left|V_{out,0}\right| / \left|V_{in,0}\right|$?

e) Explain why this type of circuit element is referred to as a *'high pass filter'*.

Problem 8:

The circuit shown below contains an AC generator which provides a source of sinusoidally varying emf $\mathcal{E} = \mathcal{E}_0 \sin \omega t$, a resistor with resistance R , and a "black box", which contains either an inductor or a capacitor, but not both. The amplitude of the driving emf, $\mathcal{E}_0$, is 200 volts, and the angular frequency $\omega = 5 \text{ rad} \cdot \text{s}^{-1}$. We measure the current in the circuit and find that it is given as a function of time by the expression:

$$I(t) = (10 \text{ A}) \sin(\omega t - \pi / 4) .$$



a) Does the current lead or lag the emf?

b) What is the unknown circuit element in the black box – an inductor or a capacitor?

c) What is the numerical value of the resistance R ? Include units.

d) What is the numerical value of the capacitance or inductance, as the case may be? Include units.

e) What value of C or L should be connected in series for the circuit to be in resonance with the given driving angular frequency ? What would be the amplitude I_0 of the new current?