

**Topics:** Undriven LRC Circuits

**Related Reading:**

Course Notes (Liao et al.): Sections 11.5 – 11.6

Serway & Jewett: Sections 32.5-32.6

Giancoli: Sections 30.5-30.6

**Experiments:** (10) *LR* and Undriven *LRC* Circuits

## Topic Introduction

Today we investigate a new type of circuit – one which consists of both capacitors and inductors. We will see that the resulting current in these circuits will oscillate, in a fashion completely analogous to the oscillation of a mass on a spring, and we will do a lab to measure the properties of this oscillation.

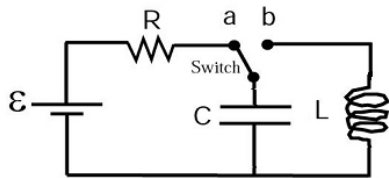
### Mass on a Spring: Simple Harmonic Motion

Consider a simple system consisting of a mass hanging on a spring. When the mass is pulled down and released it oscillates up and down. How do we understand this? One way is to look at the forces on the mass. When it is extended past its resting point the spring will want to pull it up. If compressed the spring will want to push it down.

This leads directly to a second way of thinking about it: a differential equation for the motion of the mass,  $F = m\ddot{x} = -kx$ , where  $\ddot{x}$  means two time derivatives of the displacement,  $x$  (in other words, acceleration). The solution to this differential equation is simple harmonic motion:  $x = x_0 \cos(\omega t)$  where  $\omega = \sqrt{k/m}$ .

A third way of thinking about this is to consider the energy in the system. As the mass moves, energy oscillates between kinetic energy of the mass and potential energy stored in the spring. If there is no damping in the system (no friction) to dissipate the energy of the oscillation it will continue forever.

### LC Circuits



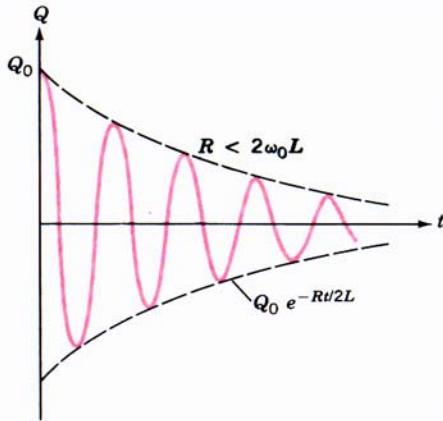
Each of these ways of thinking can be applied to the circuit at left: an LC circuit. Imagine that the switch is left in position a until the capacitor is fully charged and then the switch is thrown to position b. This is analogous to pulling down a mass and then releasing it. Why? Remember our

first way of thinking about the mass-spring combination above. The mass wants to keep moving at a constant velocity, but the spring eventually gets extended or compressed as much as it can and manages to force the mass to come to a rest and move in the opposite direction. Here the capacitor will want to discharge and hence will start to drive a current through the inductor. Eventually all the charges will have run off of the capacitor, so it won't "push" anymore, but now the inductor will want to keep the current flowing through it that it already has (this is what inductors do – they have inertia). It will keep the current flowing, but that will eventually fill up the capacitor which will stop the current and send it back the other direction. That is, the inductor is the mass (the current is the velocity of the mass) and the capacitor is the spring. Instead of position we talk about charge on the capacitor  $q$ . Our

differential equation is the same,  $V = -L\ddot{q} = q/C$ , and has the same solution:  $q = q_0 \cos(\omega t)$  where  $\omega = \sqrt{1/LC}$ .

We can also think about energy here, where it oscillates between being stored in the electric field in the capacitor and the magnetic field in the inductor. As long as there is no dissipation (resistance) in the circuit the oscillations will continue forever.

### LRC Circuits



If we add a resistor in series with the capacitor and inductor we will provide a method of energy loss in the system. Whenever current flows some energy will be lost to heat in the resistor, and hence the oscillations will eventually damp out to zero. The exact path the charge will take as it oscillates to zero depends on the relative sizes of  $L$ ,  $R$  and  $C$ , but will typically look something like the curve to the left, where the oscillations are bounded by an “envelope” which is exponentially decaying to zero as a function of time.

### Important Equations

Natural Frequency of LC Circuit:  $\omega_0 = \frac{1}{\sqrt{LC}}$

### Experiment 10: LR and Undriven LRC Circuits

**Preparation:** Read lab write-up.

This lab consists of two parts. In the first you will measure the inductance of a solenoid by putting it in an LR circuit and measuring the time constant  $\tau = L/R$  of the circuit. In the second you will use that inductor in an LRC circuit and measure the frequency of the

resulting oscillations, determining that it is  $\omega_0 = \frac{1}{\sqrt{LC}}$ .