MIT Department of Physics

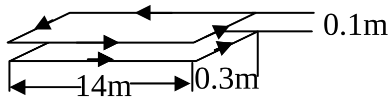
Physics 8.02X **Spring 2005**

Solution to Problem Set #9

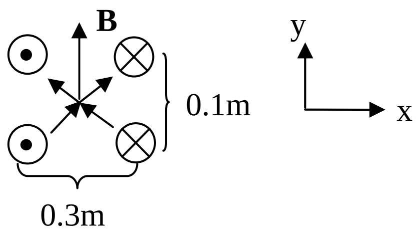
Problem 1 (20 points) The 1200 dipole magnets of the LHC accelerator at CERN will have a field of 8 Tesla. Approximate the magnet as two square loops of dimension 14m x 0.3 m, with a distance of 10cm between the two loops (similar to the arrangement in question 4 of quiz 3).

(a) What direction should the field have to keep protons going clockwise on a circular trajectory?

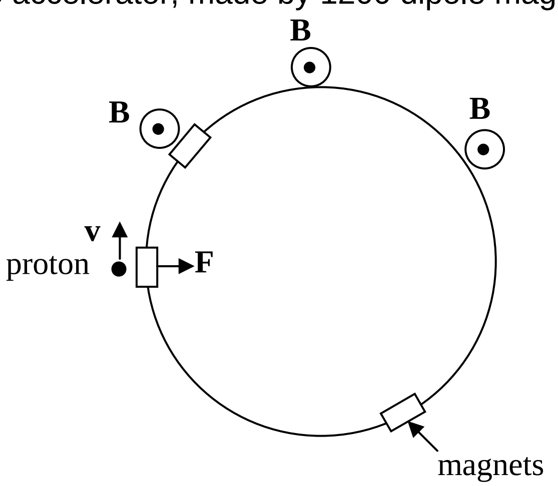
Magnet:



End view:

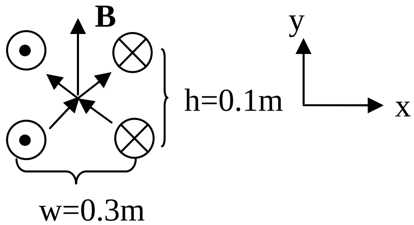


At the center, the four fields add up to for a net vertical field.
The accelerator, made by 1200 dipole magnets, is as the following:



$\vec{F} = q\vec{v} \times \vec{B}$. The direction of $\vec{B}$ is shown in figure above.

(b) What is the current in each loop needed to create a field of 8T in the midplane of the two loops?



For each wire,

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 I}{2\pi\left(\sqrt{h^2+w^2}/2\right)}$$

And the y component:

$$B_y = B\frac{w}{\sqrt{h^2+w^2}} = \frac{\mu_0 I w}{\pi\left(h^2+w^2\right)}$$

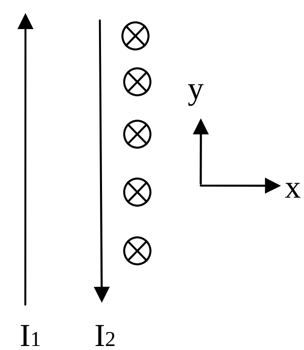
By symmetry, all wires contribute the same B_y, therefore in total

$$B_{yTotal} = 4B_y = \frac{4\mu_0 I w}{\pi\left(h^2+w^2\right)} \Rightarrow$$

$$I = \frac{B\pi\left(h^2+w^2\right)}{4\mu_0 w} = \frac{8\pi\left(0.1^2+0.3^2\right)}{4\times 4\pi\times 10^{-7}\times 0.3} = 1.7\times 10^6\,\text{A}$$

(c) Again using pictures and words show that you get repulsion if the currents in the two wires run anti-parallel.

The magnetic field created by wire 1 at wire 2 is shown in figure.
The force on wire 2 is given by
$\vec{F} = I\vec{L}\times\vec{B}$
Which is in the +x direction: repulsion



(d) . Assume that the field is homogenous at 8T over a cross-section of 200 cm^2 . What is the total energy stored in the magnetic field of all dipoles combined?

The energy density of magnetic field is

$$u = \frac{B^2}{2\mu_0}$$

For each magnet, the energy stored is
$U = uV$

For the total 1200 magnet, the energy stored is

$$U_{tot} = 1200uV = 1200\frac{B^2}{2\mu_0}V = 1200\times\frac{8^2}{2\times4\pi\times10^{-7}}\times(14\times0.3\times0.1) = 1.3\times10^{10}\,\mathrm{J}$$

Problem 2 (10 points) Explain why power is transformed to high voltages to be transported over power lines.

Energy loss (per unit time) in transmission line is due to the resistance of the wires, i.e.

$$P_{loss} = I^2 R_{loss}$$

The power delivered is given by

$$P_{del} = IV$$

For a given delivery power P_{del},

$$P_{loss} = \frac{P_{del}^2}{V}R_{loss}$$

Therefore, the higher the voltage, the lower the loss.