PROFESSOR: So today we're going to talk about the interactions of electromagnetic waves with conductors. No electric fields can exist inside an ideal conductor. So when electromagnetic waves are incident to an ideal conductor, somehow that electromagnetic wave must be reflected. And so we're going to look this morning at the boundary conditions and at the consequences of those boundary conditions.

There were four Maxwell's equations, so we expect four boundary conditions. The key is that inside an ideal conductor the electric field must be zero. The magnetic field does not have to be zero. A static magnetic field is possible inside a conductor. However, you cannot have a changing magnetic field inside a conductor, because the curl of E is minus dB dt and so if E is 0 inside, dB dt is also 0 inside.

Now before we work this out, the consequences, I have to be fair to you and tell you that, in reality, no conductors are ideal. This is what Bekefi and Barrett discuss on page 442 of the book. The electric field falls off exponentially in a very thin layer, which we call the skin depth. And that skin depth, delta, is defined in such a way that the electric field falls off by a factor of E. And that skin depth, which I will not derive here, is the square root 2 divided by the frequency, omega, times mu 0, times the conductivity, sigma.

And the conductivity for an ideal conductor, sigma, that is the conductivity for an ideal conductor would be infinitely high, so that the skin depth is 0. But even if you take something like copper, which is an extremely good conductor, then sigma, of course, is not infinitely high. But sigma is something like 5.8 times 10 to the 7, 5.8 times 10 to the 7, and the units of conductivity are ohm meters to the power minus 1 in SI units.

So if you use this sigma, and you take 1,000 megahertz radiation, which has a wavelength of 30 centimeters, then the skin depth is only 2 microns. Very, very small. If you take optical light which has a frequency of 5 times 10 to the 14 hertz and you put that in here, then you'll find the skin depth is only 3 nanometers. But it is 30 atomic distances that the E field decays into the layer, and reduces them by a factor of E. The wavelength of visible light is 500 nanometers roughly, so the skin depth is only something like three nanometers.

So I want to start now deriving the boundary conditions. And I will start with first Maxwell's equation, which is the divergence of E equals rho divided by epsilon zero whereby rho, this rho is the volume charge density. So is rho is in coulombs per cubic meter. So you should remember this from your 8.02 days.

I will write it in integral form. If I have the integral of E dot dA, I have a closed surface, and you will see it at work very shortly. Closed surface, very essential. And this is the electric flux that emerges from that surface. It's a dot product. There's a dot here. Then this is the volume is 1 over epsilon 0 times the volume integral of all the charge that is inside. You have called that earlier, q inside divided by epsilon zero. But this is a more mature way of writing it. So you integrate it over that whole volume.

Whenever you deal with divergences, you make pillboxes or depends a little bit on your geometry of course. So let's assume that this is the boundary between the conductor where the E field is everywhere zero, and lets this be vacuum. And so we have an electric field, electromagnetic wave comes in, and the electric field, say, is in this direction. It's a changing electric field vector.

I'm decomposing this now in a component which is tangential, that is, in the plane of the surface, and I call that the E tangential, and a component which is normal to the surface, and so I give than an n, which stands for normal. So now I'm going to make my pillbox. So here is my pillbox.

This height of the pill box is dl, and in my calculations I'm going to go to the limiting case that dl goes to 0 so that I get the conditions at the surface. And this has an area, dA. I draw the vector perpendicular to this surface. By convention we always have the vector from the inside of the box outside, so the dA is in this direction.

And so now I can apply Gauss's law, which is what we have there. This surface here, only E of n contributes, because E of t is in the plane, so the dot product between this vector and this vector is 0. So I get as my first contribution E of n times dA. That is the flux leaving that surface.

Then I have an electric flux which I will call just phi of E, this is also an electric flux. But I call it phi of E, shorthand notation for everything that goes through the cylindrical surface. So it is what I call the curved surface. Let's call it the cylinder.

Well, when I'm going to make dl 0, that cylindrical surface goes to 0, so no electric flux can escape there. So this one then equals 0. Now there is no electric field here, so this is the entire electric flux as it escapes from this surface. From this box. And so that is now 1 divided by epsilon 0, times the volume charge density that I have in here, so that is rho, times the volume, which is dl times dA. So this is now coulombs per cubic meter.

So you will say now when dl goes to 0, that that goes to 0, but that's not true. Because there can be, and there will be charge right at the surface of a conductor, which is not coulombs per cubic meter, but it is the surface density in coulombs per square meter. Totally different dimension. And that is rho of s. How much charge there is per square meter on that surface.

And so in the case when dl goes to 0, this does not go to 0, but this becomes rho s, divided by epsilon 0, times that dA. And this rho s now is coulombs per square meter. So that is the surface charge density. And so you see that we have derived now our first boundary condition, which I will write down on the blackboard there. And that is that the normal components of any changing electric field that is due to the electromagnetic radiation equals the surface charge density divided by epsilon 0.

Now when you see this result you will say, ah, that's nothing new. We've seen that also in 8.02 for static charges. That's true. It is also true for static charges. But here, there's something special, and that is that this E vector is changing all the time with the frequency of the incident wave, and therefore, rho s must also change all the time. That is, all the time, rearrangements of that surface charge density to make sure that this boundary condition is met. But yes, it's also holds for static charges.

So the next one is the divergence of B. You're going to do that. You're going to make a pillbox, and you're going to demonstrate to me that if you use the divergence of B, which is the second equation Maxwell, that now you'll find that the normal component all the changing magnetic field at the surface is 0. Keep in mind, it's time variable, the magnetic fields, as it comes in and it's the time variable component, the normal one, that has to be 0.

The next one in line is the curl of E minus dB dt. The famous Faraday's law. And then we have the curl of B. I will do the curl of E and I will leave the curl of B up to you. So the curl of E is minus dB dt. This equation runs our economy. This is why we have light. This is why we have energy. This is Faraday's law.

And I'm going to write this in integral form, because you're probably used to that. This is a closed loop now. Any closed loop that you may choose of E dot dl-- this is a dot product, it's a line integral-- equals minus d phi B dt. And this is a magnetic flux that goes through the surface that you attached to that closed loop. It is an open surface.

Any surface that you may choose is fine. It is an open surface. I cannot stress that enough. You choose a closed loop. You can do it right here in space. You can attach to that an open surface which could bulge out into the audience at any moment in time. This will hold.

And so whenever you deal with curls, you do line integrals. And so let's do a line integral, and then see what boundary condition that leads to. So we do the same thing that we did there. So we have here a conductor, and here we have vacuum. And so the E field here is everywhere 0. And here is your electric field vector coming in from your electromagnetic wave. So here is your Et and here is your En, exactly in the way that I defined it. So this is the normal component to the surface.

Now I'm going to make a closed loop. This is my closed loop. It shouldn't surprise you that I choose it this way. And I'm going to march around this closed loop in any direction that I want to, but I chose this direction. It makes no difference which direction you choose. And let this be dl, and let this be db. And I'm going to do exactly the same thing. I'm going to go in the limiting case when dl goes to 0, so that I get the boundary condition at the surface.

So if you're ready for this, I start here and I move in this direction. So the E of t, the E of t has no impact, because it's perpendicular to my direction of motion, so the dot product is 0. So all I worry about is this one. And so I get E of n times 1/2 of dl. Let's suppose that this is 1/2 of dl, and this is 1/2. So that's when I go from here to here.

So now I go from here to here, and now, of course, E of n has no effect, because it's perpendicular to dl. dl is a vector in the direction that I'm moving. So now I only deal with E of t. So now I get plus E of t, times db. Now I'm here and I go down. So now E of t is up, but I go down, so the dot product creates a minus sign. So now I get minus E of n times 1/2 dl. I said E of t, but I meant E of n. E of n is in this direction, and you're moving in this direction. And so that's why you get a minus sign. And E of t has no effect here, because it's perpendicular to my direction of motion. And the electric field here is 0.

So this now becomes minus the change of the magnetic flux through that surface, this surface if you wish, or a surface that bulges out in the audience, I don't care. It is minus d phi B dt. And that goes to 0, because I'm going to make dl 0, so there is no surface left. So that's one goes to 0. And notice that this one cancels out against this one. And so now we find the third condition. And that is that at the boundary condition, E tangential is 0. So that's the third one.

I want to remind you though there is the skin depth. So in reality, there is an exponential decay of this E vector that is in the surface at the case exponentially over the skin depth delta that we discussed earlier. That is that skin depth.

So now there is the curl of B, which is your turn. And so you're going to do a closed loop integral of B dot dl. And you'll work your way through that. And then you will find that's the magnitude of B of t, that is the component of the magnetic field in the plane of the conductor, the tangential component, that that magnitude is mu 0, times what we call surface current density.

J of s is a surface current density, and it has units, amperes per meter. And so there are actually oscillatory currents on the surface that occur when the electromagnetic wave comes in. These currents change with the frequency, of course, of the incident radiation. So not only is there all the time, a readjustment of rho of s, which is the surface charge density, but all the time as that electromagnetic wave interacts with the conductor, is there going to be a current oscillating in order to make sure that we meet this boundary condition. So weird things are really happening.

So we can use these four boundary conditions now to start looking into some results. And I will start with a very simple example, because I can demonstrate that. I take linearly polarized radiation. This is my z direction. I'm going to make it a traveling wave. This is my x direction, and this is my y direction.

I'm going to linearly polarize radiation so it only has an x component. And I'm going to make it move in the z direction. So at this very moment in time, if it is a traveling wave-- which I don't need that anymore, but I just want to remind you-- so the B vector would be in this direction, so that E cross B is in the direction of propagation.

So I have an incident wave, so I write down E of i. It has a certain amplitude, and then it is a traveling wave, so I get the cosine or the sine, if you prefer that, of kz minus omega t. So it's propagating in this direction. This is the k vector. And that is incident wave and it's in the x direction. So they are plane waves, perpendicular to z, infinite in all directions, and the E vector is then everywhere in those planes. This value changes with z and it changes with time.

But now what I'm going to do, I'm going to put here a perfect conductor, and so the E vector arrives there, there is no normal component to the E vector, so I don't worry about E of n. There is no normal component. I only have E of t. And that E of t must become 0 at the conductor.

So this reminds you of the string that we had. When we were wiggling the string at one side, and we fix that side of the string. Then in order to make sure that there was no motion of the end of the string, the reflectivity was minus 1. A mountain came back as a valley to make sure that this point never moved,

So exactly the same thing is happening now. We must make sure that right here, the electric vector is always at all moments in time, 0. So if I just call this, for convenience, z equals 0 at this location, then the reflective wave must be minus E0i, that is, the mountain becomes a valley. And then it moves in the opposite direction, so you get kz plus omega t in the x direction. And you will see that the sum of these two at any moment in time will make the E vector right here in this entire plane, which is infinitely large, 0. But that's the boundary condition that we have to meet.

The total electric field which is the sum of the two, one moving in this direction, and this one moving backwards, is of course the vectorial sum. So I have the cosine of alpha, I call that alpha, minus the cosine of beta. Your high school days tell you that that is twice the sine of half the sum, times the sine of half the difference. Although I always have to look that up, because I forget that just as well as you do.

And so when I add that up then, then I get that the total electric field, which is the sum of the two, is going to be 2E0i, and then I get the sine of kz, and I get the sine of omega t. And that is a standing wave. All the spatial information is in here, and all the time information is in there. And indeed, if you substitute in there z equals 0, notice that the E field is always 0. This, by the way, is in the x direction.

And so we have a standing wave now. And what that means is that there are surfaces perpendicular to the direction of z where the E vector is always 0. An entire surface. We have now a nodal surface like this. And they're separated by distance 1/2 lambda. So this is a nodal surface, but this is also a nodal surface. So I'm trying to draw the curve of the E vector, the sinusoidal curve.

So the E vector may be up here, and then the E vector would be down here. But it's all in the plane perpendicular to the blackboard, the same. So this is a nodal surface. This is a nodal surface. This is a nodal surface. This is the wavelength, and the E vectors go like this. Nothing is moving anymore in this direction. It goes like this, standing wave with nodal surfaces.

So this standing electromagnetic wave has an electric field which is standing. So there is an associated magnetic field with this, which is a standing magnetic field. And I would like you to work that out. I may even have put that in one of my problem sets. And what you'll find out, much to your surprise-- certainly much to my surprise when I saw this first-- that the magnetic field is now 90 degrees out of phase with the electric field, both in space and in time.

So that means where you have the nodal surfaces in E, you have the anti-nodal surfaces in B. And where the E fields reach the maximum, you will have that the magnetic field is 0. Unlike a traveling wave, where the magnetic fields and the electric fields are in phase, both in space and in time. They are now 90 degrees out of phase. When you think about that, that is perhaps not so weird. Because there cannot be any energy transport, any net energy transport in one direction with a standing wave.

And so the mean value of the Poynting vector must be 0. And if you make E and B 90 degrees out of phase, then that is always met. So the mean vector of the Poynting vector then becomes 0. But in any case, if you use Maxwell's equations, you will see that the magnetic field and the E fields are 90 degrees out of phase and 90 degrees in space and in time.

I have here our famous 88 megahertz transmitter. So this is an antenna where we accelerate charges back and forth. Currents are running at a frequency of 88 megahertz. 88 million oscillations per second. So we are creating, because we went through this that accelerated charges give rise to electromagnetic radiation, we are creating electromagnetic waves. Lambda equals c divided by the frequency. So you will find that the wavelength is about 3.4 meters.

And I have here an antenna, a receiving antenna, which are just two copper wires which are not connected with each other-- well, they are, but they're already through the filament of a light bulb.

And so when the electric fields reaches this receiving antenna, currents will start to flow in this direction, and then the light will go on. I've demonstrated this before but I want to do that again, that if I hold up this receiving antenna like this, then you will see the light bulb go on. If I do it like this then it will not go on, because no currents can flow then in this direction.

But that's something I already showed you before. That's not the reason why I want to do this again. The reason why I want to do this again is that the blackboard here, behind the blackboard is metal. And so the electromagnetic waves that go out in your direction and to the blackboard are going to reflect off here, and they're going to reflect off the walls in a very chaotic way that I could not predict nor calculate. But there will be, in this lecture hall, locations of nodal surfaces where the E field is very low or near 0.

And certainly, that has to be the case near the blackboard, because this whole plane behind here is a conductor. And so the sum of the incoming wave and reflecting wave must be 0 here, so I don't want to see that light on. And then I will walk out in the lecture hall, and see whether there are locations in the lecture hall where we actually receive electromagnetic radiation. Where we will see the light, and where we will not see the light. So, make sure that this is not too close to the transmitter, because otherwise we will blow the bulb.

So let me first show you then, that when I stand here that I'm receiving this electromagnetic radiation. Remember that the E field is proportional to the sine of theta, and so I'm here in an excellent position so I get a maximum electric field. If I rotate it like this, there's nothing and that has to do with the polarization. It has nothing to do with standing waves, simply polarization.

But now look. I claim that there must be here, a nodal surface, there's no electric field. And there it is. Isn't that amazing? This is a standing wave that I'm building up. So I'm now in that surface that I pointed out you on the blackboard. So now I can also try to walk through the audience, and see whether there locations where I see light. Right here, I see light so I am receiving. Now, how it bounces off these walls, I have no idea. That is probably one of the main contributors, but the walls also have metal in them.

So I walk further in, and the light goes out. You may say, yeah, it goes out because you are further away from the transmitter. That's a good argument. But if there are standing waves in this room, perhaps it will go back on again. And when I'm here, I would like to see it go back on again. And if it doesn't want to do, yeah, there it is, there it is, there it is, there it is. Because I tested this out, of course, before you came.

You see the light again? Can you see it? Just tell them that you see it, yeah? It's not very strong, we're very far away, but it's clear. This light is substantially brighter than it is here. Nothing here, but here it is stronger. There it is.

And so there is a huge and incredibly difficult pattern in this room of locations where the E fields are very weak, and where the E fields are very strong, like here. And here it is even weaker. I think the most spectacular one is what I did first, and that is to show you that it becomes 0 at the blackboard, which is really a very well-controlled plane of a conductor. OK.

And now I would like to discuss with you another example, which we give the name, transmission lines. So we're going to try to have travelling waves, just like we have with this system. The reason why I chose this example, because I can also demonstrate this. So here we have a copper wire, you see it right in front of you here. It's 425 centimeters long, and there are two.

And here, I have a power supply. So I'm driving this with a variable voltage V0 times the cosine of omega t. And I'm going to use my same transmitter for that. That's going to be the 88 megahertz transmitter. I call this wire 1, and I call this wire 2. So we're getting a wave of voltage, co-sinusoidal, that is going to propagate in these wires. And what is going to happen is not what you think.

First of all, there cannot be any electric field in the wires, because it's a conductor. So the electric field in this direction does not exist, so there is no potential difference from here to there. But if there is any potential difference, it's going to be between here and there.

If I take this wire and I make a cross-section of that wire-- this is wire 1 and this is wire 2-- then at any moment in time, as the voltage wave passes by, this may be positive, that is that rho of s, and this will be negative, that's that rho of s. And the electric field lines then go like this. So that means if this is, at this moment in time, positive and this is negative, then there is an electric field in this direction between these two wires. Not along the wires, but between the wires.

But then, of course, there are also going to be locations where this is minus and this is plus, and then where this is plus and this is minus, and so then the electric field is in this direction, and the electric field is in this direction. So the potential difference between here and here is positive. The potential difference between here and here is negative, because it's integral E dot dl.

So the potential difference, V1 minus V2, is going from 1 to 2 of E dot dl. And so, if the electric field is in this direction, this has a higher potential than this. If it is in this direction, this has a lower potential than this. So this is how this voltage wave propagates. So the E field is between the wires, not along the wires. So there's only an electric field potential between the two wires.

There are oscillating surface charge densities in order to meet those boundary conditions as the voltage pulse comes by. So it's positive at this moment, but a little later in time, it will be negative and this will be positive. And all of that moves with the speed of light. And then there are also oscillating surface current densities, this J of s.

Let us assume now, I call this z equals 0, that I'm going to short this out. So I'm simply putting there, a wire to totally short it out. That means there cannot be any potential difference between here and there, because I make the ohmic resistance there at the end 0. And what does that mean?

I'm going to make a drawing here, otherwise my figure becomes too cluttered. And so here is the situation, that the end of the lines, which, by the way is there, that the end is shorted out. If it is shorted out, it acts like a conducting wall, just like with what we discussed before. There cannot be any potential difference between here and here, because the E field in this direction must be 0. And so you get a similar situation that the reflectivity is minus 1, that a mountain comes back as a valley, so that the E field of the incident one, and the E field of the reflected one kill each other exactly at this location.

That means you get a standing wave. So that means this here is now a nodal line. Everywhere in this line, the electric factor will always be 0 at all moments in time, but since I get a standing wave here, incident wave and reflected wave right here, there is another nodal line. And this distance then it's 1/2 lambda. And right here, there is another nodal line. And the electric field here along this line is, everywhere, 0.

So there's no potential difference between wire 1 and 2 here. There is no potential difference here, and no potential difference here at all moments in time, because it's a standing wave. And in between, you see the electric field. It may be-- let's assume it is in the down direction here-- so it's smaller here, smaller here, smaller here, smaller here, and then 0 here, and 0 here. And here, the electric field would be like this.

And that whole pattern, then, is a standing wave. And so this is down, and this is up, and then it oscillates like that. But these are always nodal lines.

Imagine now that this end is open. So I just remove the short circuit and I leave it open. Well, I hope you remember the experiments we did with strings. This is really the equivalent of the string whereby the end is attached. So you get a node there. Reflectivity is minus 1, mountain comes back as a valley.

But this now, when the end is open so there is nothing at the end, completely nothing-- that is the situation that it is in right now, by the way. This is completely open. This wire here and this wire are not connected. So when this is open, you expect the reflectivity of plus 1, just like with the string. That the mountain comes back as a mountain. And if that is the case, then here you get the maximum of the E field, you get an anti-node here. But right here in the middle, you get your nodal lines, and right here in the middle, you get your nodal lines.

So you get a shift of 1/4 wavelength. From where you earlier had nodal lines, you get now anti-nodes. And where you had, earlier, anti-nodes, you now get nodes. So if this is open, then these are the nodal lines. And so those are the locations across the wire here where there is no potential difference at all moments in time. And so this is then what we would refer to as closed, and I want you to draw the parallel with our strings. And I even think that in my problems sets that I give you a chance to work a little bit on this.

Remember with strings, we can fix the length of the string in such a way that we get resonance. We can drive a string at one end, just moving it just a teeny weeny little bit-- and the end is fixed-- and we can set that system at resonance. The wavelength that we then get, lambda of n, n being an integer so n equals 1, 2, 3, et cetera, that wavelength is then 2L divided by n.

You can also wiggle it here and leave this end open-- we even discussed that and I even demonstrated that-- in which case, you get resonance for lambda n equals 4L divided by 2n minus 1. And, of course, the frequency at which you reach that resonance, omega of n, is always k of n times the speed of propagation, which in this case is the speed of light. And k of n is then 2pi divided by the wavelength.

And so the way I'm going to do this demonstration for you, we have chosen the lengths of all these wires such that the system is at resonance when it is open. So we have a length which is 4.25 meters, the wavelength is 3.4 meters. So the length is 1 and 1/4 wavelengths. And so when it is open, which it is here, then this is 1/4 wavelength and this is 1/2 wavelength, and so on.

If I leave this open, and that's what I'm going to do, and I have here a light that is connected between these two wires that I can slide along-- so I have light here, a light source, and this is a very special light source. It has an enormously high ohmic resistance, so that when I put it here, that I'm not disturbing the potential difference between the wires. I'm not changing the boundary conditions, really, because it has an enormously high ohmic resistance. And that light bulb lights up, even with a minutely small current.

And so the first thing that I'm going to show you is that if I turn on the transmitter, which I will use the same one that I have here, make sure that I don't get electrocuted now. So we're going to take this off. And it may help, actually, if we give you a nice light situation so that it's not completely dark, but--.

So now I'm going to move the light across the two wires, and you're going to see that there are locations where the potential difference is very large because we are at resonance. And therefore, you will see a lot of light. And there are locations where there are nodal lines and you will see no light.

Let's first go here, that's the open end. The open end is clearly where the electric field is very high. It's like the situation of a string with an open end. You have an anti-node there. You have an anti-node right here. But look, when I move the light a quarter wavelength, it goes out. I must be now near a nodal line, maybe not exactly on the nodal line.

And now I move it another quarter wavelength and the light goes on again. Now I'm going to move it back to the open end and you see the light is on. I move it a quarter wavelength, light goes off. I move it another quarter wavelength, light goes on. I moved to another quarter wavelength, light goes off. Isn't this amazing? And now I think I'm going to move it another quarter wavelength, and the light goes on again.

And so it is an amazing thing that you see in front of your eyes. That there's a standing voltage wave between these two wires. And it is across the wires, not along the wires.

Now what I'm going to do, I'm going to short this out here when I put the light bulb right in the middle here. So I'm going to put the light bulb at quarter wavelength. Now when I short it out at the end, I am no longer at resonance. You no longer meet the resonance condition, which I meet now at an open end. Something that you can very easily verify.

Now the moment that I'm no longer at resonance, I no longer have very high potential difference at the anti-nodes, which I had before when I was at resonance.

But for sure, the moment that I short out these lines at the end, the locations, which are now nodal lines, must become anti-nodes. Now, maybe not super-duper anti-nodes with a huge potential difference, but probably high enough for that light to go on.

OK. Are you're ready for this? Now we're going to short this out. And as you can see, the light goes on. Now, it may not be as bright as you've seen it before but this is, again, an anti-node, and the light is on.

And so you see a remarkable demonstration here that you see the standing waves, you see the nodal lines, you see the anti-nodal lines, and you also see that it depends on the boundary conditions. That if you make this open, then 1/4 wavelength from the open end is an anti-node. If you have it open, 1/4 wavelength from the open end is a nodal line. And if you short circuit it, then it becomes an anti-nodal line.

I have a second demonstration which is also very interesting. Let me turn this off before some of you touch those wires and we get some problems. I have another very nice demonstration which is a cable which is 127 meters long. You see it here. It starts there, goes all the way here, to make you see that it's really big, and then it goes back there. And in this cable, I'm going to send in a voltage pulse.

It is a coaxial cable. And a coaxial cable has a wire which is at the core, metal. And then there is a cylinder around it, that's why it's called coaxial. But this is a conductor, and this is a conductor, and it is filled here with dielectric material to insulate the two conducting surfaces. And what we're going to do, we're going to send in here voltage pulses which have a length of 100 nanoseconds, 100 times 10 to the minus 9 seconds. 100 nanosecond length of the pulse.

In other words, if you look at the wire from the side, so here is the wire, and here is the shell, just take a cross-section here, so you see only that shell. Then as the voltage pulse moves in this direction, then you will see locally, between the outer conductor and the inner conductor, locally, you will see that E field. And it goes to 0 here and here. And that marches, then, with a certain speed along the wire.

And so as that pulse passes, there will be an E field like this. But when there is no pulse, there is no E field between this and that. So in that sense, it's sort of similar to this transmission line, except the geometry is different here. It is a coaxial cable.

Now comes the punchline. I have an option to keep this end open so that the line at the very end, so this is the inner and the outer, is completely open. And I have the option to shortcut it. And when I short it out, a mountain comes back as? When I short it out a mountain comes back as a valley. When I leave it open, a mountain comes back as a mountain.

We're going to show you this pulse at three locations. Where the pulse enters the system right at the beginning, which is here. Then we'll show you the pulse at the end of the line here. And then we show it to you again as it comes back. So you're going to see it three times, and you do. And the system now, is closed at the end. All right. Thank you, Markos. Very thoughtful.

SPEAKER 2: Yeah.

PROFESSOR: So here is the pulse that goes in. And 1.3 microseconds later, between this line and this line, there is a marker here which is 1.3 microseconds. The mountain has returned as a valley. So this is the pulse when it enters, and then when it has come back.

But now, I'm going to make it an open end. All right, so here we see the incoming pulse. And you see the one that comes back is now 1.29 microseconds, big deal. A mountain comes back as a mountain, but now there's something very interesting. At the end of the cable where it is open, you see twice the voltage. Do we understand that? I see someone saying yes. Why do we see twice?

Remember the open string. Now the reflecting wave and the incident wave have the same polarity. So at the end, they add up together, and they give you double the amplitude. That was exactly the same situation with the string. When we drove in a pulse on the string when the end was open, remember, the ends moved up by twice the amplitude of the individual pulse. And you see that too here. It has twice the amplitude of the-- so this is right at the end.

But now there's even more, which I discovered purely by accident. Well, not so much of an accident. I always do some consistency checks to see whether physics works. And so I said to myself, okay, 1.3 microseconds. That's what it takes to go make a round trip through the wire. So with all the confidence that I have in physics, I said, well, 2L divided by c, L being 127 meters, should therefore be 1.3 microseconds, because that goes with the speed of light.

But when I calculated this, I found that this is only 0.84 microseconds. So then I said to myself, why is it slower, the speed, than the speed of light? And I decided to ask you? I see Amanda. Why is the speed substantially lower? This is only-- remember, we observe 1.3 microseconds. That's what we observe. So this is only 65% of the speed of light.

Amanda knows the answer. Who else knows the answer? I don't blame you. Took me also 2 seconds before I realized it. You also know, Dan? OK. Amanda?

AMANDA: The wire's not ideal, so it should have, like, internal resistance.

PROFESSOR: Nice try. But it's the wrong answer. But it's better that you tried than that you don't try yeah?

AUDIENCE: Because the transmission's not going to be through a vacuum, so what's happening is one electron [INAUDIBLE] to another electron, and there's some delay between.

PROFESSOR: I think you mean well. You may not express it in the most effective way. Do you remember, or do you not, that we discussed that the speed of propagation, v, is c divided by the square root of the index of refraction of the dielectric constant? Remember that? I did, at one point. I even showed you how this dielectric constant is a strong function of frequency for water.

Well, when I looked at the dielectric constant of the material that is in here, I found that the dielectric constant, kappa e, is about 2.3. And if you calculate 1 divided by the square root of 2.3, that happens to be 0.65. And so that's why the speed is 65%

of the speed of light. So you see also the effect of the dielectric constant here, which is quite wonderful. Remember the square root of kappa e was also called the index of refraction.

OK. This is a nice moment for a break. We will reconvene in four minutes.

I would like to bring to a test now what we have learned. And so you get a chance to do some thinking for change. I have here an electromagnetic wave, linearly polarized, E vector in this direction, going to propagate in this direction. And it's set up here 10 gigahertz radiation, radar, 10 gigahertz. So lambda is about 3 centimeters.

And it's going to be polarized in this direction, you have to take my word for that. And as it arrives here, this receiver is set so that it receives the radiation in this direction. Now if I rotate it 90 degrees then, it doesn't receive it. But that's a different demonstration which we did before.

So now I have a peculiar comb. This is a comb of metal. These bars are not connected in here. They're just metal bars. And I'm going to put this here at the end. And I'm going to do it in two different ways. In one situation, the comb is like this, and in the other situation, the comb is going to be like this.

So in both cases, will the electric field of the wave that comes in be in this direction? And here, it will be in this direction. In one of those two cases, the radiation will go straight through. So I will hold it in there, and the receiver, happy. In the other situation, the radiation will not go through. It will be reflected.

And now I'm asking you to make a prediction. Do you think that if I hold the comb like this in the beam, that means like this, that the radiation will go through, or will the radiation be reflected? Or do you think that in this case the radiation will go through, or will it be reflected? So let's have a vote on that.

So first I want to ask the class if I hold the comb in the beam like this, do you think that the wave will be reflected? Who thinks that the wave will be reflected? Who thinks that the wave will go straight through? Who thinks that in this case the wave will go straight through? And who thinks that it will be reflected in this case?

I'm actually quite impressed by your answer. Most of the time I get more wrong answers than correct answers. Indeed, it is in this case that the wave is practically for 100% reflected, and it is in this case that the wave will go through. And if you have a good understanding about the currents that are running at these surfaces to make sure that you meet the boundary conditions, you will be able to give the answer for yourself. But if you can't, then see me, or write me email. But I want you to think about it a little bit for yourself.

So I will demonstrate it now. So we have this 10 gigahertz signal. It's modulated with 550 hertz sound so you can hear it. That's the only reason why we modulate it. You can hear it. It's a nasty sound. Here it is. So this is the transmitter, and this is the receiver. Now, here is the comb. I'm going to first stick it in like this, which is this situation. And I kill it. And now I rotate it, and now it goes through.

So it in this situation that it goes through, and in this situation, not. My hand in not a bad conductor. All of your bodies are very good conductors. Look at my fingers. I kill it. Look now with my fingers. Go through. So you don't even need a comb, which is a conductor. You can just do it with flesh. Blood and flesh. Go through. And I kill it.

This experiment, to give you some insight into the secret behind Edwin Land's linear polarizers for light, which we never discussed. Imagine that the radiation that hits this plate is not linearly polarized, but is completely unpolarized. Then the component in this direction will be reflected, but only the component in this direction will be allowed through. That becomes a linear polarizer. That's the whole idea behind the linear polarizer.

So this, in a way, is for radar, a linear polarizer. Shine on here unpolarized radar, and all components in this direction will be gone. Only this component will go through. So linearly, radar light will come out. And so it may give you some thoughts about how the optical polarizers work. What they do is they align strings of molecules in such a way that you get a behavior not unlike these metal bars.

So now I want to change gears, and I want to talk for the remaining time about radiation pressure. In modern physics, we don't think of electromagnetic radiation as plane parallel waves which are infinite in all directions. But we think of them as bullets, as packages of waves.

We call them photons. Photons can be produced by, for instance, atoms or molecules. If they are in an excited state, and then they decay to a low energy state, they can radiate just one photon. But photons, in our modern physics, have momentum.

If you throw a tomato at me and it's a rotten tomato, [SQUISH] it goes [SQUASHED TOMATO SOUND] I feel a push. Another tomato. So another push. Another tomato, another push. So I feel a force. And in that same way, the light, when it hits me, also push on me. And that's what I want to discuss with you.

First of all, energy-- and we have so many E's today. We even had an E of n, and we had an E which was the E vector, and now we have E for energy, so I'll write the n here. This stands now for energy. The energy in a photon is Max Planck's constant times the frequency of the photon. And Max Planck's constant, which of course plays a key role in quantum physics, is 6.63 times 10 to the minus 34 joule seconds.

So if you tell me what the frequency is, I will tell you what the energy is. Which is very different from the classic idea that we always said, well, the energy is proportional to the electric field strength squared. And we have plane waves, were infinite in this direction, infinite in this direction. So we always had infinite energy into a wave of a plane wave. Not for photons.

So take an example. Take radio waves. I'll do only one example for you. So we take 10 megahertz, 10 to the 7 oscillations per second. So that would be a wavelength of about 30 meters. One photon contains energy, and each radio photon with this frequency has exactly the same energy. It's a quantized amount of energy. And if you substitute for this frequency and you put multiplied by h then you get that the energy is about 6.6 times 10 to the minus 27 joules.

In physics, we don't like to work with such very small numbers, so we often convert into what we call the electron volts. One electron volts is 1.6 times 10 to the minus 19 joules. And so that energy then becomes a more digestible number. It's still small. It's about 4 times 10 to the minus 8 electron volts. So I did this exercise only for radio waves, but you can do it for any other frequency that you choose. And we call that then, a photon.

So in modern physics, photons have momentum. And the momentum of the photon, p, is the energy divided by c. In other words, it is also hf divided by c. But you remember from 8.01 that the force is dp dt. So let's return to our tomatoes first, so that you feel comfortable that you know what we're talking about.

So I'm throwing rotten tomatoes at you. And those tomatoes come in like this. And then they lose all their momentum in this direction, which, of course they go [SQUASHED TOMATO SOUND] Momentum is conserved. So the momentum that they lose must become my momentum, because momentum is conserved. And so suppose I throw at you 10 tomatoes per second, that the mass of each tomato is 1/4 of a kilogram. And I throw them at you at a speed of 40 meters per second, which I can do because I hire a baseball pitcher, and the baseball pitcher can throw them at 90 miles per hour, which is 40 meters per second.

And now I can calculate what is the average force that you will experience when all those tomatoes hit you. You'll feel a force, because the momentum of the tomato is mv. And the force that you will feel in the x direction, momentum transfer, is now going to be 10 times 1/4 times 40, which is about 100 newtons. That is a substantial force. It may actually throw you over. It's a huge force.

So let's now turn to photos. And so we have a beam of light that comes in. And let's give the beam of light a cross-sectional area, A, so the light comes in this direction. And this light strikes you, and you absorb it. Hits your face, and it's absorbed. So the momentum in this direction is destroyed. Momentum is conserved so you must have a momentum in this direction. You must feel a force.

The pressure that you feel is obviously the force that you experience divided by the area. The force is the dp dt divided by the area. But that is also, if this is the energy divided by c, I take dp dt so I get 1/c, then I get my 1/A, and then I get dE dt, E being the energy now. So I get here d of energy, dt. This is joules per second. How many joules per second hit me? That's what that means.

And this is the area of the cross-sectional beam. And this is what we earlier called, in this course, the Poynting vector. We called that the mean value of s. This is joules per second. So this is joules per second per square meter, which is exactly what the Poynting vector was.

And so we can write now that the average pressure that you will experience if you absorb that light, or the radio emission, or radars, or whatever it is, is given by this relationship. Namely, that the average pressure is the average Poynting vector divided by c. You no longer have to think of S being E cross B divided by mu zero.

In fact, the whole idea of individual photons, which is perfectly fine, that's where it all came from, can simply be replaced now by how much energy per second hits you. And whether this is UV, or infrared, or radio, or radar, as long as you absorb it-- if it goes straight through you, of course, then nothing happens-- but as long as you absorb it, that is then the pressure that you will experience.

Remember that we calculated the Poynting vector at Earth due to the radiation from the sun. And I still remember that number, because it's called the solar constant. It's the famous 1.4 kilowatts per square meter. So that is the Poynting vector from the solar radiation as it reaches earth, joules per second per square meter.

So I can calculate now, when I expose myself to the sun, what the pressure is. The radiation pressure. Because I absorb it, you better believe it. None of that radiation goes through me. I absorb it, and I feel a pressure, and I can calculate that pressure by taking this number and divide it by c.

That pressure is very, very low because c is huge. In fact, I can even calculate, if I hold my hand up in the direction of the sun, what the force will be on my hand. Well, the force is the pressure times the area. And when I do that, then I find that the force on my hand is only 5 times 10 to the minus 8 newtons.

So I hold my hand here, there is the sun. And there is this force on my hand of 5 times 10 to the minus 8 newtons. It's not going to knock me over. It's not going to be like the tomatoes. In fact, I don't even notice it. It's completely insignificant. So the force due to the radiation pressure from the sun is, for you and me, insignificant.

If I hold a mirror in my hand, and I reflect the light back to the sun, then of course, the momentum change of the light doubles. Comes in like this, and it goes back like this. So you have twice the momentum transfer to you, and so then the force on you, and the radiation pressure on you doubles. So with 100% reflection, the radiation pressure is twice what it is for 100% absorption.

It is clear that you and I will never experience, in our lives, radiation pressure. For all the light sources that we deal with, and the distances involved, the Poynting vectors are always so small that it will not be noticeable for you and me. However, in astronomy that is different.

There are stars 20, 30, 40 times more massive than the sun which radiate so much energy, that the atmosphere of the stars is floating on the radiation pressure. So if the star would radiate more, it would blow its atmosphere off, and if it radiates less, the atmosphere would come down. So it is very important in astronomy.

Comets have two tails. I will show you shortly a picture of comet. It has two tails. In general, you and I can only see one of the tails, which is the white tail, but there's also blue tail. The blue tail is the result of solar wind. The sun, apart from emitting UV and electromagnetic radiation, it also loses hydrogen. Protons and electrons. And when they reach the earth in large quantities, we see northern lights at night, because they interact with the atmosphere of the earth.

Well, they also interact with the carbon monoxide and the carbon dioxide of the comet tail. And when they excite those molecules, and the molecules de-excite, they emit blue light. So that's where the blue tail comes from. But independently, a comet has dust. And it is the radiation pressure on these dust particles that blows the particles away from the sun, purely radiation pressure. These particles are a few microns in size.

And so you get a huge tail of dust, and now you get reflected sunlight off that tail of dust. The dust tail is formed by radiation pressure. And the reason why that light is not very bluish, which you might expect if you accept the Rayleigh scattering, is probably that those particles are a few microns in size. Whereby the preferential scattering for blue light is no longer very effective.

So that's why these tails, that can be 100 million kilometers in length, why they are white. The bright white ones are due to the radiation pressure, and the blue one, which is very faint, is due to the solar wind. And I would like to show you Hale-Bopp, which was very prominent in the sky in 1997.

I hope that many of you have seen. It was extremely easy to see. And I saw it 20, 30, 40 nights in a row. I couldn't get enough of it. Every night I was staring there for hours at the sky to see this gorgeous comet. But I only saw one tail. That was my problem, of course.

So let's take a look at this beautiful comet, Hale-Bopp. Who has seen it in 1997? You we're all born? OK. Good. Good. So there you see Hale-Bopp.

So you see the blue tail-- that's the result of the solar winds-- which is very faint and not easy to see, unless you have a long exposure like this picture. And then you see the white tail, which is the result of radiation pressure. So the sun is pushing onto these small particles, and then you see the reflected sunlight. You would expect that if the dust particles are small enough and, if you see it at the right angle, that you would actually sometimes see this tail, which is mostly white, that you might see it actually a little bluish. Which we understand why that have to be bluish.

So now I want you to test your knowledge. Call it a brain teaser. I have here a radiometer. And many of you may have a radiometer in your rooms. They're very cheap. It's a little glass bulb. There's low pressure inside, it's not a vacuum, but it is low pressure.

And it has an arm here, and an arm perpendicular to that like windmills, like this. And they can rotate with very low friction in this direction. There are two. One perpendicular and one like this. And here is a little surface, no larger than, you will see it, a square centimeter or so. And this side is white, and the backside is black, and this side is black, and the backside is white. And then there's another one like this.

And we're going to radiate light on this. So the light will interact with this whole thing. And now I'm testing you. Not very fair what I'm doing, because I know the answer. Do you think that as the light strikes, that this white surface will start to move in the board, because it's going to rotate? You're going to see that.

Will this go in the board, and this come out of the board, or will it be reversed? Let's have a vote. I first want to know who thinks that this white surface as it rotates goes in the board, and that this will come out. Who thinks that, in fact, this white surface will come to you, and this will go in? It's about 50-50, it's interesting. Yeah.

Well I can demonstrate it to you. The one thing I can tell you though-- well, let me first demonstrate. Then we can test it. It's nicer. There it is. So we'll make it a little darker. So try to remember what your prediction was. And now I'm shining onto it. You can clearly see the white. This part is white and this is black.

And so the white surface is coming to you, and the black surface is going away from you. So half of you had it right, and half of you had it wrong, and the other half were not even thinking, so they had no answer. How can it be that the white is coming to us? Because radiation pressure would predict would predict that the force on the white surface would be about twice as high as the force on the black surface, right? Because the white surface reflects, and the black surface absorbs.

Well, the answer is very clear. This has nothing to do with radiation pressure. If you calculated the force on these surfaces due to radiation pressure, it is insignificant. It's nothing. It is even less than the force on my hand from the sun. So what makes this rotate has nothing to do with radiation pressure. That's why it was not quite fair when I said I'm going to test you.

I like idea that you may have sleepless nights, you know that. And so, I would leave you with this, and think about why is it rotating. And if you can't sleep, send me email, and we will discuss it.

Have a good weekend, and see you next Tuesday.

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