PROFESSOR: I will summarize what we have learned about the normal modes of a string fixed at both ends, which is very relevant today for musical instruments. Suppose I have a string with length L, mass per unit length mu and tension T. And I know that the speed of propagation is the square root of T divided by mu, and the wavelength is always the speed of propagation divided by the frequency-- speed of propagation times the period of one oscillation, but I avoid capital T's for period, so I will just write it down as frequency.

So in its lowest mode-- we call that the fundamental-- first harmonic, you will get, then, this situation, n equals 1. And then lambda 1 is then clearly 2L if this length is L. And the frequency is then v divided by 2L, so that's the frequency for the lowest possible mode, first harmonic. If we go to the second harmonic, then the picture changes. We get a node in the middle, and so we have here n equals 2. So lambda 2 is now L, and so f2 is now, again, v divided by the wavelength, so it becomes now v divided by L.

So we can write down, now, the general equation for the n-th mode, n being Nancy. Lambda of n is then 2L divided by n, and the frequency in the n-th mode-- that is, the n-th harmonic-- is then nv divided by 2L. And this v is given there.

So what you see here, that if the fundamental here were, for instance, 100 hertz, then the second harmonic would be 200 hertz, the third harmonic 300 hertz. They come in series 1, 2, 3, 4, 5, and so on.

Now clearly, there is the possibility that I would have one end of that string open. We call this, then, closed-closed or fixed-fixed. But suppose I have the same length L, but here I have this infamous rod, which is frictionless, with the infamous ring, which has no mass, and it is fixed here, and now I want to know what the normal modes are. And then in the lowest mode, you've got this.

The angle is 90 degrees. dy/dx must be 0 there, and then it oscillates like this, back and forth, So now, you have that n equals 1, which is now the fundamental first harmonic. Lambda 1 is now 4L-- lambda 1. And therefore, the frequency that you now generate is twice as low, is now v divided 4L.

So you can go one step further. You can ask, now, for the second harmonic. So here, again, is that rod. So now, I introduce another node here. So now, the string would look like this. This is, again, 90 degrees. And as it oscillates, it would move like this.

So now this is n equals 2. This is the second harmonic. You can just look at it. Lambda 2 is 4/3 times L. You need to make it longer. And therefore, f2 is now 3 divided by-- 3v divided by 4L. And so now, we can write down the general recipe for the n-th mode, n being Nancy. So we'll find, then, that lambda n is now 4L divided by 2n minus 1. And that, now, changes the picture quite dramatically, because if you follow here that you go from lambda 1 to lambda 2, the change is not by a factor of 2, but it changes by a factor of 3.

So the frequency in the n-th mode, which is the velocity divided by lambda n, is 2n minus 1 times the velocity divided by 4L. So what you see now, if we take both systems-- and suppose L were the same, mu were the same, and T were the same, then f1, in this mode, this instrument is half the frequency of f1 there, because there, you have downstairs a 2L, and here, you have a 4L.

So for instance, if that were 100 hertz, with the same length here, you would get 50 hertz if all other things are the same, and then the second harmonic would be 150 hertz, because if n becomes 1, you get a 3 upstairs. If n becomes 2, you get a 5 upstairs. So now the ratio is 1, 3, 5, 7, whereas there, the ratio is 1, 2, 3, 4, et cetera.

Now, If I take a string in isolation, and I oscillate it, I almost hear no sound. There's not enough air that is displaced by the string. So what we do is we have to mount it on a surface that starts to vibrate with it. And you see that on all musical instruments. I will demonstrate that to you in a simple way with a tuning fork. I have here a tuning fork, and if I just excite this tuning fork-- it is 440 hertz-- you will hear practically nothing. I hit it now. I can hear it. I'm very close. But most of you cannot hear it.

However, the moment I put it on a surface, the surface starts to vibrate with it. So I drain the energy faster out of the tuning fork, but you get more volume, more sound. The larger surface oscillates, so the pressure wave is stronger. And I will demonstrate that. You hear it now? Do it here. Big difference, right? Huge difference when I put it on the surface.

I have here a music box, which I bought many years ago in Austria. And it has these prongs, and when I rotate it, I may be able to hear it, but you won't hear it-- maybe some of you very close. Can you hear it? Good for you. Now listen. Big difference.

And now the whole surface oscillates. Again, you drain the energy faster, of course, but you get more sound. I can put it on here. Big difference. And of course, you'll see that in the design of all musical instruments. Needless to say that the design of the sounding boards, which are connected to the strings, are, of course, secrets that the company are very fond of and not telling you.

I am not aware of musical instruments-- of string instruments-- whereby one end of the string is attached to a frictionless rod and whereby that end is attached to a massless ring, so I will restrict myself in discussing musical instruments-- string instruments-- to the one whereby both ends are fixed. So therefore, the frequency f1-- that is the one that I'm interested in-- is the speed of propagation, which is T divided by mu, and then 1 divided by 2L. So these are the key players in the design of instruments. If you make L longer, you get a lower pitch. If you make L shorter, you get a higher pitch, and we all know that. If you make L shorter, you get a higher pitch.

If you make the tension higher, you get a higher pitch. If you make the mass per unit length lower, you get a higher pitch. So that's key in designing musical instruments. If you take a piano, it has 88 keys. The lowest frequency is 28 hertz. The highest is 4,000 hertz. It covers seven octaves. And when you hit a key, a hammer actually comes down onto the string and it excites the string in a combination of various normal modes.

In fact, in many cases, when you hit one key, without you realizing it, you hit more than one string simultaneously. But that's a detail now which I will not further expand on. If you take a Steinway grand piano, even though it has only 88 keys, it has 216 strings. So the idea of a piano, then, is that you change the length of the strings. So that's a player. Shorter the lengths, the higher the frequency.

And you can change mu. And when you open a piano, and you look at the various strings, you immediately see that some strings are as thick as my pinkie-- they have a huge mu-- and others are very thin. They have a very low value for mu. And that will give you, then, a higher pitch.

The tension of the strings in a piano are approximately all the same. They're quite high. They are near 200 newtons per sting. So a grand piano, then, the total force-- all the strings together-- is something like 45,000 newtons. It's an immense force, when you think about that.

Now, if you go to violins and cello and a bass, you have four strings, then the length is a given. But if you look at the various strings-- and we will see a violin later-- you will see that one string is much thicker than the others. So you play with mu. You change the mu. So you only have four fundamentals, then.

If you go to the Museum of Fine Arts in Boston, they have a wonderful collection of musical instruments. You will see musical instruments with just two strings, and you will also see musical instruments with one string. But I have never seen one, as I said earlier, whereby one end of the string can freely move. But maybe they do exist.

So now comes the issue, if you have built an instrument, how do you tune it? Well, with a piano, you ask the piano tuner maybe once a year or once every year to come and tune the piano for you. And what the piano tuner does is simply change the tension in the string, which is a major job, of course, if you have 216 strings.

With a violin and a guitar and a cello and a bass, it is the player herself or himself who is doing the tuning. We will see that later today. They actually change the tension in the string, and then they listen carefully to get just the right tone. And they do that before they start playing.

If you play the piano, with all due respect for piano players, all they have to do is hit the right keys in the right sequence. That's all there is to it when you're a piano player. Now, think about a violinist or a guitar or a cello. They cannot just hit one string. They have to change the length of the string all the time to change the fundamental. And that's the whole idea when you have the violin, and you strike it with the bow, and you rub the string, and the string will start to oscillate in the fundamental and the higher harmonics. By making the string now shorter with your hands, you increase the pitch.

So playing there is that you, as a player, continuously have to move the length of the string and just hit the right length. It goes beyond my imagination that anyone can do that. I cannot. I tried it when I was young. I took violin lessons. I was a total disaster.

If you take a harp, it is, in a way, like a piano, except there's no hammer, but you pluck the string. And now there is something very special about the harp. You can decide where you pluck the string. And that makes a difference in the percentage of higher harmonics and which higher harmonics you excite. And we will deal with that in 8.03 when we do Fourier analysis.

You will see, then, that if you pluck a string in the middle that you get a different series of higher harmonics than when you pluck it, say, a certain distance, 10 or 20 centimeters from the end. It's a very interesting part. In fact, the piano is designed in such a way that the hammer hits the string about 1/7 of its length from one end.

And that is done to suppress the seventh harmonic. Now, why this seventh harmonic has to be depressed, that is up to you to decide. Maybe we don't like seventh harmonics. I don't know. It beats me, but in any case, it's purposely done to suppress the seventh harmonic.

There is a musical instrument that has only one string. One was designed by my daughter when she was in nursery school. That's a long time ago. And I have it here. I'm very fond of it. I've demonstrated it many times. It's called a wash top base, I think. So you see here the one string. And you see here, then, this surface that is necessary to make you hear the string. In this case, it is, I think, a box from Kentucky Fried Chicken or something.

So I need three hands to play this, so the only thing that I can do now, I can't change mu. It's a given. I can't change the length. That's a given. So what is the only thing I can do?

AUDIENCE: Change the tension.

PROFESSOR: I can change the tension, and that's the way you play it. And for that, I need three hands, but I will try to do it with my mouth. And then you listen. When I increase the tension, you will hear the frequency go up. When I decrease the tension, it goes down. You could hear it, right? Clearly different in frequencies.

Pythagoras, who lived in the sixth century BC, discovered that musical notes are very pleasing when the length of the strings come in simple ratios, which is quite remarkable when you think of that. 2:1 give you an octave, 3:2 give you a fifth, and 4:3 give you a fourth.

And the evolution of Western music is based on that, where you take a piano, the piano has the octave-- 1:2-- it has the fourth-- 4:3-- and it has the fifth-- 3:2. And somehow, these are very pleasant intervals, when you play them in combination, for our Western ears. Doesn't mean that it is also true for other cultures.

The ancient Greek astronomers, even all the way up to Kepler in the 17th century, believed that the musical instruments could also explain orbits of planets. And this was known as the "music of the spheres." It was believed that the movement of the planets produces music, but our ears were just not sensitive enough to be able to hear that music. And of course, it shouldn't surprise you that all this was mixed up with religion. The fact that it was pleasing to hear music that comes in simple ratios must show the hand of God, if you want to believe that.

I now want to turn towards wind instruments. And we're going to put a wind instrument here, which is nothing but a tube, length L. Let's start with one that is closed on both sides, length L. And if it is closed on both sides, then the pressure here on this side and this side can build up, as we discussed last time, so there are going to be pressure anti-nodes. And there are going to be nodes for the motion of the molecules.

If I have here a sound cavity, which is open on both sides, then I have here a pressure node, and I have here a pressure node. The pressure can never be higher than the ambient pressure, because it's connected with the universe. When I say "pressure," I always mean overpressure-- over and above one atmosphere.

So it's immediately clear that the solutions that we worked out here for closed-closed system must be exactly identical for this. So you do get precisely here that lambda n is, again, 2L divided by n. There is no difference. And f of n is nv divided by 2L.

But of course, the enormous difference is that v is a god-given. That is the speed of sound in air, the air column, begins to oscillate. So v, now, is approximately 340 meters per second, which is non-negotiable. So there is no way that you can manipulate mu and T to change the speed of propagation.

The speed of sound, in a gas, something that I really wouldn't want you to remember-- I will return to that at the end of my lecture-- is given by this equation. But you may forget it, as far as I'm concerned. R is the gas constant. T is the temperature of the gas in degrees kelvin, and gamma, if any of you remember that from 8.01, is the ratio of specific heat at constant pressure divided by specific heat at constant volume. But if you've never had that, that's fine, too. Just take my word for it. I'll give you some numbers for gamma later. And then M is the molecular weight.

So this is the speed of sound in a gas. So it's a given for air. There's nothing you can do about it. Room temperature, it's given. It changes with temperature, which is interesting.

So the way you get the system going, you must somehow blow air past-- maybe in this direction or so-- and then your column starts to excite. It's very hard to see why that happens, but then the column gets into normal modes, and you may get a whole collection of various normal modes, the superposition of many normal modes.

Now, there is, in the case of sound cavities, what is very easy to make a system which is closed at one end and which is open at the other. That is very easy, which you cannot do here. You can do that here. So therefore, in this case, that one side of the cavity is closed, and the other one is open, there are musical instruments-- the clarinet comes very close to being almost closed at one side and open at the other. But there are many musical instruments which are open on both sides.

So then you would have here that f of n equals 2n minus 1 times v divided by 2L. So then you have, indeed, this case, which we dismissed for string instruments, divided by 4L. And the easiest way to demonstrate that to you, to take my pen, is that the cover of my pen, it is closed on this side-- believe me-- and it's open here.

And it is about 3 centimeters long. You may not like the frequency. It's a very high-pitched frequency, and it may be a cocktail of more than one mode. That is a musical instrument. Not the best, but it is a musical instrument, closed and open on one end and the other.

So now it is important that we get some feeling for the frequencies that you can produce with the various wind instruments. So here, I have listed for you the length, L. And this is, then, an open-open system. Of course, it's the same for a closed-closed system. But a closed-closed system doesn't make a very nice musical instrument, because the sound is not coming out. So that's not the idea.

So therefore it's open-open, or it is closed-open. And you see here the length, and of course, the speed of sound is a given, so I can't change that. And this is, then, the fundamental, the first harmonic, covering the entire range of your hearing, all the way from 17 hertz-- so you would take a 10-meter-long tube, open on both sides, to get the fundamental to 17 hertz. And for a closed-open system, that would then be easier.

So you could get away with half the length to get the same-- you'll get half the frequency. That's basically what a closed-open system does. So we'll keep that on so that we can understand when we hear the various frequencies why the frequencies are as high or as low as they are.

Now here, we have a tuning fork, which is 256 hertz, which is mounted on a sound cavity, which is closed on one side and open on the other. And that is done in order to get perfect matching between the resonance frequency, the fundamental of the box, and the 256 hertz. You can do that, of course, if you're only interested in one particular frequency.

So for this case, then, you would have the frequency, f, is the velocity. So we go to this case now, to the case of this equation. So the frequency-- this is the first harmonic, now-- is v divided by 4L. So that is 256. For this, it's 340, approximately, divided by 4L, and then L comes out to be about 33 centimeters. And indeed, if you measure the length of this, you will find that it is roughly 33 centimeters. So now, you have a case where you have perfect matching.

So when I hit this tuning fork, then the box starts to-- loves to oscillate exactly at that frequency. So you get a very large sound, so you drain the energy very quickly. You do see often tuning forks which are mounted instead this way. You can also have a box like this, open and open on both sides.

And it has to be twice as long. So for very high-frequency tuning forks, you often see it open-open. If we have to do it for this one, it would have to be twice as long, and that is just impractical and not necessary, of course.

Now, with musical instruments, you produce an infinite number of tones. So there is no way that you can, of course, design your sounding board that it resonates with every single frequency. So this is where the secrecy of the manufacturer comes in. And they're not going to tell you how they do that, and therefore, some instruments are better than others, and you pay for that.

If you have a barrel filled with liquid-- I remember that in France, they did this technique that I'm telling you about. And the wine was up to here, say. And they wanted to know-- you couldn't look into the barrel-- they wanted to know where the level of the wine was, they would knock on this.

And the sound here would be very different from the sound here, because here is air, and here is liquid. So it's obvious that where you have liquid that the resonances are very different from the resonances here. So that's one way that you can tell what the level is of the wine.

When you have a cold, which I happen to have today, I remember as a kid, I would go to the doctor, and he would ask me to inhale. And then he would knock on my chest. And then I would exhale and inhale and knock on my chest. And then he would be able to tell whether there was fluid in my lungs or not. I don't know how sensitive that is. It's not my field. But that's the way it was done, and maybe you have experienced the same thing. They sometimes also-- I don't know why-- they knock on your back. Fine.

Now, I have developed-- in fact, I can even say invented-- a method to test for the presence of brains. And the way I do that is that I strike a tuning fork and put the tuning fork on someone's head. And you can imagine, if it's empty here, then you get the same effect as you have with the wine barrel. And then you hear a very clear sound.

But if there are brains here, like the liquid, you get nothing. And I have patented that, so you can't try that at home. But I can demonstrate it to you. You hear much? Oh, you hear much?

AUDIENCE: Yes.

AUDIENCE: A little bit.

PROFESSOR: Maybe I shouldn't be lecturing 8.03, then. Now, of course, in my case, I think I passed the test within reason. It is not so clear, of course, that all students would pass that test. And I was wondering whether anyone has the courage that I may try that. Any one of you? You're afraid, right? You are worried that it will all come out. You mind?

AUDIENCE: No.

PROFESSOR: You don't mind. You're a strong man. You have to be quiet. Otherwise, we can't tell the difference.

[LOUD PITCH]

[LAUGHTER]

I say no more. I think it's better that I teach 8.03 than you do. So when you design a wood instrument, the only thing you have to play with is L, because v is no longer negotiable. So with organ pipes, when you go to churches, and you see organ pipes, you see a whole zoo of these organ pipes-- open-open and closed-open. And for every fundamental that you want to excite, you need one pipe. So it's a huge number of pipes. It's, in a way, like the piano.

If you take a flute, then you make it longer and shorter simply by drilling holes in it. And if you hold your hands on both holes, then this is the length of the flute, which gives you, then, a lower frequency than when you take one hand off, because now it's shorter. This is connected with the universe, so here, no pressure can build up. So this becomes a pressure node, so it's shorter.

And if you take this one off, it's even shorter, and so the pitch will go even up. That's the basic idea behind a flute. And I can demonstrate that to you. Did you hear the pitch go down? I have here a flute-like instrument, which is open on this side, and it's also open here where we pass the air by. So you could consider this, to a very good approximation, as open-open. It is 16.6 centimeters, and so if you want to know what the fundamental is, well, then you apply this equation-- n equals 1-- and then you get something like 1,024 hertz. But if you make it closed at one end, and you apply this equation, you would get half that. So you only get 512 hertz. Big difference-- factors of 2.

So this is, then, the fundamental for open-open. And now comes open-closed. Factor of 2 difference. I have here a very special tube. It is open and open on both sides, and if you don't believe me any, I can see you. Can you see me? Can you? Can you see me? It's open and open on both sides.

And it is corrugated, which is, of course, very important why it works so well. And this has a length of 77 centimeters. So you can calculate now, using this equation--n equals 1-- what the fundamental is. And the fundamental is about 220 hertz, the first harmonic.

And since it's open-open on both sides, the second harmonic will be 440 hertz, and the third harmonic, 660 hertz. And by twirling this around, with a little bit of luck, you can get a wind flow past here, which only excites the lowest mode. Sometimes, you hear the lowest one and the second. But when you twirl it faster, you get higher harmonics. And I want to demonstrate this to you.

I'll first try to excite the 220, which is the lowest mode possible, which is the first harmonic-- the fundamental-- and I'll try to make it clean, just only the fundamental. That's it. It's about 220 hertz. 440. 660. 880! 880! 660. 440. I can't get above 880.

There are ways you can change the length of a wind instrument. One is by making holes in it. Another one is by really physically changing the length. And there is one instrument which is well known for that, which is the trombone. So you actually change the-- this is a system which is open, and it is closed at the end, in this case. You can see that. Just like a piston, it's always closed. So now, by changing the length of the cavity, you can change the fundamental. So we get it this way.

[PLAYING "JINGLE BELLS"]

We recognize instruments by the cocktail of the harmonics that they generate and depending upon how you excite them. I mentioned already the plucking of the harp. But also, the way, for instance, you blow on musical instruments. I don't whether any of you have ever tried to play on a trumpet, but if I gave you a trumpet, chances are that you will get no sound out of it at all.

You have to hold your lips in a special way. You have to know how to spit in there the right way. It goes like [SPUTTERS], something like that. So that is also a way that you can excite certain harmonics in relation to the fundamental. And that, then, determines the sound quality of your instrument.

And I'd like to demonstrate this to you now, in various ways, that these different instruments have different sound quality. What it comes down to is that if you ask each instrument to play, for instance, 440 hertz tone, you will see that a violin cannot just simply produce 440, but it will automatically also, at the same time, generate 880 and make that as the second harmonic, and maybe higher harmonics. And that's different for different instruments. And that's, of course, the idea behind the tone quality.

And the way that we're going to demonstrate that to you is as follows. We have here a microphone. And I will first make you listen to a tuning fork, for instance, 440 hertz, which will give you, then, this signal, times amplitude of the membrane of the microphone. If, however, an instrument were to generate 440 in addition higher harmonics, you would get the sum of the signal, and of course, the higher harmonics. So you will see no longer a nice sinusoid, but you see on top of the sinusoid the higher harmonics.

So let me now first show you the 440 hertz. Here is the microphone. Boring. Just one frequency. Nothing rich about it. 256. There's no indication for any higher harmonic. That's the way that tuning forks are designed. If I take this flute, as far as I can tell, only the fundamental, no higher harmonics. Now, maybe if you blow in a very special way, maybe I can excite higher harmonics. But this is now-- oh, I want to show you 4,000 hertz so that you can easily hear. This is 4,000 hertz.

So you get the idea. So you know now what you're going to see, but you don't know yet what you're going to hear. Neither do I. We have six students who are very brave, I would like all of them to come down now. And they're going to demonstrate their musical instrument. So will all of you come down, please. Yes, you've got your instrument there. Just come here. That's fine. Don't worry. Come here. All right?

Where is the violinist. Ah, there is the violinist. Yeah. Oh, boy. Oh, boy.

The first person who is going to demonstrate the violin is going to be [? Mark Viro. ?] And I'm going to ask Mark first to only produce what he thinks, then, is 440 hertz. Now, we have for you a special chair. We will bring the chair very shortly.

So let me remind you, then, of the 440. This is 440. Come a little closer and produce 440. Now, look. Did you see that you could really see the 440 in there. It was the same spacing as my tuning fork. But there were many higher harmonics there. Show it once more. And they, then, give you all these wiggles. There. You see?

STUDENT: Shall I try a harmonic?

PROFESSOR: Now the audience is yours. And now play-- I'll give you 20 or 30 seconds-- anything you want, anything you love. Go ahead. You are fantastic.

Is Sharon here? Sharon Chu? Ah, so Sharon is going to demonstrate to us the flute. And maybe you can show it to the class. See, her flute has more holes than mine. I only have two. She has quite a few more, and she opens and closes them with valves. Sharon, you may not be able to exactly get the 440 with this instrument, but that's OK. Come close, and show us your 440.

Excellent. You see? You could really see the 440, but you could see more than that. Separates it from the violin, of course. The audience is yours. Make us happy. Terrific.

Now, we have a very special guest, which is Shauna Jin, who was so kind to go through the trouble of bringing her cello. Now, you think, Sharon, that you can come close to 440? You can try that. And if you can do it just in a fundamental, you may have to tell us whether you have to shorten the string with your finger. You do have to do that? Go ahead. Try once more. I can see the underlying 440. And you shortened the string?

STUDENT: No.

PROFESSOR: There was no finger on the string.

STUDENT: No.

PROFESSOR: So this is the fundamental of that string.

STUDENT: Yes.

PROFESSOR: The middle A of the piano.

STUDENT: Yes.

PROFESSOR: The show is yours.

STUDENT: Oh, I don't practice anything. I just wanted to bring my cello in.

PROFESSOR: You practiced on it?

STUDENT: No, I didn't

PROFESSOR: Oh, you didn't. Good for you. So you prefer not to play? That's fine. Thank you very much. Now we have a Chinese clarinet from Yiwen Chu. And she told me that the nice thing about it is that it's not really a clarinet. So can you make a 440?

STUDENT: I can try. It'll be a little off. [INAUDIBLE].

PROFESSOR: Well, since it's not a clarinet, I think you have good reasons to be off.

STUDENT: OK.

PROFESSOR: OK, try it. Now, you should show the class. By the way, you see all these holes in here? One, two, three, four. It even says F there.

STUDENT: Right. [INAUDIBLE].

PROFESSOR: Very nice. I do see overtones. The audience is yours.

STUDENT: Sorry. I'm not very good. I don't really practice.

PROFESSOR: It's all right. I did also poorly on the trombone. That's the way it goes.

STUDENT: OK, hold on.

PROFESSOR: You're doing great. Wonderful. Where is the saxophone? There is the saxophone. Colin Johnson. Colin, 440. Man! Can you go all the way down there. 440. Very nice. You can't see it, but we can. It's very nice. If you stay there, the audience is yours. Terrific. Now we get the hero, Aston. Aston is going to play a 440 hertz tone on his percussions. The show is yours.

STUDENT: Actually, so it's not going to be technically possible for me to do.

PROFESSOR: But we had email exchange about it, and you said you could produce 440 hertz. Can you show the percussions for one thing?

STUDENT: Well, I can do these.

PROFESSOR: Oh, no, we're going to do percussion, right? OK, go ahead.

STUDENT: I'll do these first. These are smaller, so they're--

PROFESSOR: Small percussions.

STUDENT: Well, these are finger cymbals, so they almost get sort of near a tone that you could see on this. So I'll do this.

PROFESSOR: I think it's nowhere near 440. What do you think?

STUDENT: It's got 440 overtones.

PROFESSOR: Oh, it has 440 overtones. It can never have a 440 overtone. The overtones are always higher. It has maybe an undertone 440. Go ahead. Well, let's call it 440.

STUDENT: And then these are just for fun.

PROFESSOR: Well, that's why you're here, right? So man, we can't wait. 20 seconds. It's your show. That's closer to 440 than what I've seen.

STUDENT: It's actually got a really nice low frequency.

PROFESSOR: Ah. you can see it.

STUDENT: Yeah, it's pretty low frequency and not very loud.

PROFESSOR: We need one more bang.

STUDENT: One more?

PROFESSOR: Thank you, all. Great. If there is any singer in the audience who wants to try, be my guest. I think this is a great moment for the break. While we have a break, feel free to play around with the wind organs and also with the tuning forks. Be a little careful. You may also try the trombone and the flutes. And then five minutes from now, we will reconvene.

STUDENT: Which one's the 440 again? Which one's the 440?

PROFESSOR: All right. So what we have been looking at are one-dimensional standing waves, normal mode solutions to the famous wave equation. In the case of transverse motion, we have d2y dx squared is 1 over v squared times d2y dt squared. So this is the wave equation, y now in this direction.

So for instance, if we have a string, and we have string with length L, and it is fixed on both ends, then the boundary conditions determine the precise allowed solutions. We call them eigenstates in quantum mechanics. And I can write down, now, the displacement y in the n-th mode as a function of x and t. And we give the amplitude of the n-th mode, say, a value that can differ from mode to mode. So we get a of n, and then we get the sine of n pi x divided by L. And then we get the cosine of omega n times t.

So this is the time domain, which makes it shake with that frequency, and this is now the part that makes sure that the ends are always fixed. If you make x equals 0, y is 0. And if you make x equals L, y is 0. And that gives you, then, a whole zoo of possible frequencies, because omega n is always v times k of n. And so that then becomes v times n pi divided by L. So you see the ratios, 1, 2, 3, 4, and so on, and vL. That's right.

Now, if we go the route of longitudinal, then if we have a system which is open-open, like we discussed before, and it has length L, then you get exactly the same solutions, provided that you replace what we have y there, you have to replace that by p, p being the overpressure. Because at the end of an open-open system, the pressure can never become higher or lower than ambient, and so you have pressure nodes. So you get exactly the same solution that you have there, provided that you replace this by p and this you replace, maybe, by some capital P, which is then the amplitude. But this and this is the same, except, of course, that omega, this v is non-negotiable in the case of sound.

If you have an open-closed system-- so now you close one end-- then you want the boundary conditions that p is 0 at the open end and p has to be an anti-node at the closed end where the pressure can build up. That gives you, then, different values for this k of n. The sine may even have to be changed to a cosine. Depends on where you define x equals 0.

That's not so important now, but what is important now that your k of n in that equation, now, gets the form 2n minus 1 times pi divided by 2L. So everything comes from boundary conditions. And now I want to make the step from one dimensional to two dimensional.

If we go two dimensional, which is easy to do-- and we can even demonstrate that-- so this has length Lx, and this has length Ly, could be a frame with a soap film, a membrane, which can oscillate. And we'll assume that the membrane is attached everywhere to the frame. So that is the boundary condition. It's everywhere attached.

So we want to know, now, what the normal modes are for this system. So now our one-dimensional wave equation becomes a two-dimensional wave equation. So if this is my coordinate system, x, y, z, and now we get that d2z dx squared plus d2z dy squared is now 1 over v squared times d2z dt squared. That, now, is the two-dimensional wave equation.

And when you put in your boundary conditions, which in this case, suppose we call this 0, so for x equals 0, and for x equals Lx, you want z to be 0, because the membrane cannot move. And for y equals 0, and for y equals Ly, you also want z to be 0. Then you can now, in analogy with that, you can write down now the solution for z as a function of x, y, and t. Now you have to introduce two quantum numbers, if I call n Nancy and m a quantum number.

So now you can get z, and now I have two integers that can change. So m, now, goes 1, 2, 3, et cetera. Nancy is 1, 2, 3, et cetera. And this is now a function of x, y, and t. It is two dimensional. So I have to introduce some kind of an amplitude, which can be different for the different modes Mary Nancy. And now I get something extremely similar to this, except I get two sines. So I get here sine of m pi x divided by Lx times the sine of Nancy pi y divided by L of y times the cosine of omega m, n times t.

And you see that it meets the boundary conditions. If you put in x equals 0 or x equals L of x, you get a 0. If you put y equals 0 or y equals L of y, you get your 0. And if you substitute this solution, which we clearly, intuitively, know has to be correct, if you substitute that in that wave equation, then you get the connection between the k values and omega.

And then you will find that omega m, n, which is always the velocity times k value of m of n, becomes now v times the square root of Mary pi divided by Lx squared plus Nancy pi divided by Ly squared. This, now, sets the entire range of all possible normal mode frequencies. So now you can play with Mary-- m-- and you can play with Nancy-- n-- and you can cocktail that in all combinations that you want to.

And for instance, if we ask this membrane to oscillate in the m equals 1, n equals 1 mode, then it means that the entire membrane comes to you and goes back and comes to you and goes back. That's the lowest possible mode. All points are in phase, and they all have the same frequency. That's the characteristic for a normal mode.

If, for instance, we had Mary equals 2 and then Nancy equals 1, then we would have in the direction of Mary, we would have plus and minus. This part would come to you, but this part goes away from you. So a plus and a minus, and this, now, is a nodal line.

So we get a nodal line. So it's not anymore a nodal point, which you get in one-dimensional solutions, but you get a nodal line. This stands still, and then the membrane oscillates like this and is attached to the edge.

And if, for instance, you wanted to know the Mary equals 2, Nancy equals 2 mode-- so m equals 2, n equals 2, then you would get two nodal lines, plus, minus, minus, plus. If this comes out of the blackboard, this comes out of the blackboard. This goes into the blackboard, this goes into the blackboard. And in French, you will see some remarkable, wonderful pictures of a guy with beautiful hair from the '50s whereby you see the oscillations of the soap film in these various modes. It's really very, very exciting, very interesting.

And I want to demonstrate this to you in a way that I cannot possibly calculate. I doubt whether there are many people who can, although maybe some civil engineers can do that, because this system is way more complicated than a membrane that we have here that is attached to the frame. What we have is a square plate, and it is only fixed in the middle. That's where it's held. That's the only point that is not moving. That is here.

But what we do, we drive the thing-- actually, there's a piston below here, which drives that middle point, and it oscillates it. And then you get an infinite number of frequencies whereby the system goes into normal modes. We can change the frequency. And the way we can make you see these frequencies is by putting a powder on it, which we will do. And then the places where there are nodal lines, the powder stays. And the places where there are anti-nodes, the powder is literally thrown off, and it accumulates at the nodes.

And that's the idea of the next demonstration. We call these the Chladni plates. And after the lecture, I would invite you to actually play here with this violin bow. So here, we have similar Chladni plates, which have different boundary conditions. This is a square, a triangle, and this is a circle. And you can put stuff on there, and then you can try to hit these resonant frequencies with the bow. You get remarkable patterns.

So let us first go to our system, which is driven so that it's very controlled. We can adjust the frequencies almost any way we want to. And I will start with a low frequency. Sometimes the sound may be very high. I may have to adjust the sound. You can hear it. We'll start somewhere, I think, in the range of about-- we are close to 300 hertz. And then we'll increase the frequency, and I will add powder as necessary.

So what you're looking at now, at two-dimensional normal modes, which give you nodal lines. Wow, I'm right on resonance. That's excellent. We didn't try that. That's a beautiful resonance. Who on earth could have predicted these nodal lines? I will go to higher frequency. I will lower the volume a little. Wow! Look at that. You see where the anti-nodes are. It's thrown off, and it accumulates at the nodes. Hey, that's a weird one coming up. My goodness.

I've done this many times. Every time I do this, I see different ones, because you can so easily-- no, the normal mode frequencies are often so close together that you can very easily skip one and go over one. Isn't that amazing? Look at these lines here. It's completely ridiculous! This was a different one, right? Different from the previous one.

I'll increase the volume. When you're coming up to a resonant, you can even hear it, because the sound volume increases. Holy smoke! Isn't that amazing? You try to calculate that. I would like to think, though, that aero and astro people, who design airplanes, that have wings, and people who build bridges, that they should be able to calculate these resonant frequencies, because you may get destruction.

Look at that one. I've never seen this one in my class. This is completely bizarre. All right, so you get the idea. And I really would like you to play with the ones there at the end of the lecture, see whether you can excite them.

So now I want to make the step to three-dimensional wave equations. This was a problem that Professor Mavalvala, who's sitting in the back, put on the final exam of 8.03 this spring, this year. If you don't like it, there she is.

So this problem, who had suggested that problem to you, Nergis? Don't tell them. Here we have a sound cavity in three dimensions. a, b, c are the lengths. And this is our coordinate system, x, y, z. And it is open in this direction, and it is open there.

So it is a box which is open on both sides in the z direction. And the rest is all closed. So now, the three-dimensional wave equation, d2p dx squared-- p is the overpressure-- plus d2p dy squared plus d2p dz squared equals 1 over v squared times d2p dt squared. That is the three-dimensional wave equation.

So it should really be no problem for you or me to write down immediately the solution of the normal modes. You must make sure that you have pressure nodes in the z direction at z equals 0 and at z equals c, and you must make sure that you have anti-nodes in the x and the y direction where the pressure can build up. That's the boundary condition for a closed system.

So the solution, then, for the pressure in terms, now, of three quantum numbers, l as in lion, m as in Mary, n as in Nancy, is now some kind of an amplitude, which can be chosen differently for the different modes. So I give them, then, the index l, m, n. And then you get the cosine of pi times l times x divided by a, which is my l of x. And then I get the cosine of pi times Mary times y divided by b. And then I get the sine of pi times Nancy times z divided by c. And then I get the cosine of omega l, m, n times t.

So you see, when you look there, convince yourself that you meet all the boundary conditions. This is the one that is the open end. That means if you substitute z equals 0, this one will make it 0. That's where you have the pressure node where it's connected with the universe. No pressure can build up. But if you substitute in here z equals c, you also get 0. And you get the anti-nodes here. So this is a solution that immediately presents itself.

If you have followed what we did 1D and what we did 2D, this is the obvious solution. So the system can oscillate in a combination of all these normal modes. But what is interesting now is that l can be 0, 1, 2, 3, because there's nothing wrong with making l 0. The cosine remains 1, then. Nothing goes to 0. And Mary, Mary can go 0, 1, 2, 3. It's only Nancy that cannot be 0, because if Nancy is 0, then there is no pressure everywhere. There is no wave.

So you see here that you now allow for 0, 0, 1 modes, which will give you certain frequencies. And the frequencies that you will get, you will find by substituting this solution back into the differential equation into the wave equation, which gives you, then, always the connection between omega and k.

So that connection is then that omega l, m, n becomes v, which is the speed of sound times the square root-- if you're ready now-- we get pi l divided by a squared plus pi m divided by b squared plus pi Nancy divided by c squared. So the lowest possible frequency for this system is the 0, 0, 1 mode.

If I assume that c is the largest dimension in the system, so that's the length L-- I didn't put the length in there, but c is sort of the length of my musical instrument-- so you'll get, then, that 0, 0, 1 mode would give me, then, omega, 0, 0, 1. So this is not there, this is not there. I called this, now, L to make the connection with my musical instrument. So I get v times n times pi divided by L. And that is precisely what we had before for a musical instrument for a sound cavity, which is open and open at both sides, or which is closed and closed on both sides.

So what you see now, that if you have a musical instrument, even though it has dimensions a and b, that the lowest frequency that you get is always dictated, then, by L. But not only that, if a and b are much, much smaller than c, then the second harmonic is almost certainly going to be 0, 0, 2. So by making this one go to 2, you're still at a lower frequency than by making m go to 1.

And the third harmonic is very likely to be here. And then there comes a time, of course, that the next frequency comes in when you change the l and the m's. And you can get a whole zoo of frequencies, and that is what that problem was all about.

The Spring 2004, it's on the web, and the solutions are also on the web. It was the final exam. It was one of several problems.

If I return, now, to the speed of sound in gas, R is the universal gas constant. You may have had that in 8.01. T is the temperature in degrees kelvin. What I want to mention, though-- what is interesting-- a friend of mine was a flute player in an orchestra. And I asked him whether they have to take into account the change in temperature when they start playing.

And he said yes, they actually do. They tune their instruments to adjust it to the temperature by making the instrument a little longer or a little shorter. So that temperature, which changes the velocity of sound, must, of course, be taken into account if you're in an orchestra, and you have the flutes, and you have the wind instruments together with the violins. So the violins can change the tension in the strings, but the wind instruments also adjust it to the temperature.

Now, if we have air, and gamma, air is a diatomic molecule-- nitrogen and oxygen-- for those of you who've taken 8.01, and when this was covered, for a diatomic gas, gamma is about 7/5, which would make it roughly 1.4. And the molecular weight, well oxygen 2 is 32. Nitrogen 2 is 28. You have 80% nitrogen, and you have 20% oxygen, so anything you take-- you take 29 or 30, it's fine with me. So that is then the molecular weight for air.

So now suppose we do the same for helium. Now, helium is a monoatomic gas. And a monoatomic gas, gamma is about 5/3, which is about 1.66. But the molecular weight, which in this case, its atomic weight-- helium is just a single atom-- is 4. And consequently, the speed of sound in helium is roughly 3 times the speed of sound in air. You can just substitute those numbers in there, and you will find that it's about 3 times higher.

Now, my voice is some kind of a musical instrument, not unlike a violin. It has strings, which are my vocal cords, which oscillate. And these strings are connected to a sounding board, which is my larynx, which is my throat. And that creates a voice which you recognize. You say, yeah, that is Walter Lewin.

My sounding board, just like the sounding board of the violin, resonates at certain frequencies better than at others, and that gives you, then, the characteristic tone of that particular violin, of that particular person. However, when I inhale helium, my poor throat has no idea about that, and so it starts to resonate at different frequencies. And therefore, my voice will be very different because of the difference in speed of sound. Because remember, the resonance frequencies of sound cavities, their resonance, their frequency depends directly on the velocity v. I have here omega, but the same is true for f, of course.

Now, there's only one problem. And the problem for me-- not for you-- is that there is no oxygen in helium. And if I take in too much helium, I'll be on the floor. If I take in too little, it won't work. So that is always a very fine line. But I'm not joking about it, that this is really an experiment that can make me extremely dizzy, because I will be without oxygen for some time.

And I'm actually not aware of it, because you just inhale that helium, it feels great. It's like-- the person who is going to talk to you next you will think is not Walter Lewin.

(HIGH-PITCHED VOICE) I wasn't joking. Really, I'm telling you. I hope you're going to have a good weekend. See you.