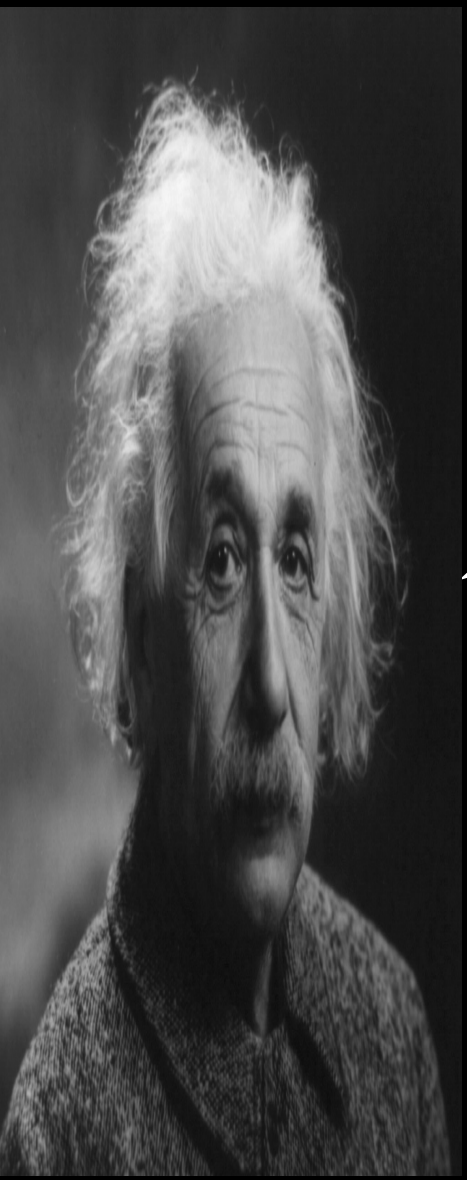


Welcome back to 8.033!

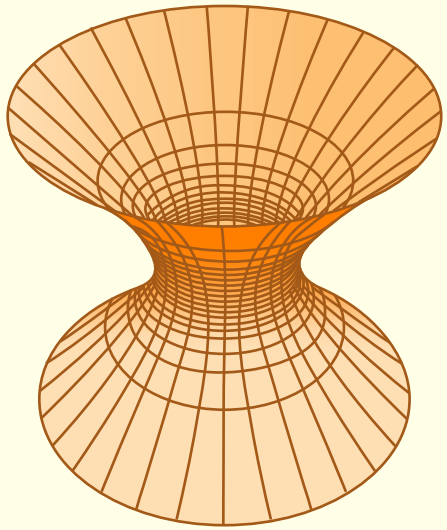
Image Courtesy of Wikipedia.

Summary of last lecture:

- Space/time unification
- More 4-vectors: **U**, **K**
- Doppler effect, aberration
- Proper time, rest length, timelike, spacelike, null

The Three Types of 4-Vectors:

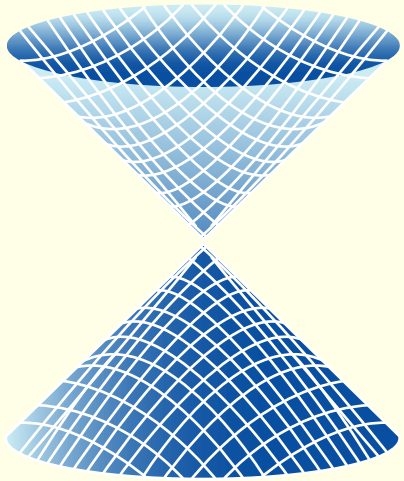
SPACELIKE



$\Delta X^t \eta \, \Delta x > 0$

$|\Delta x| > c\Delta t$

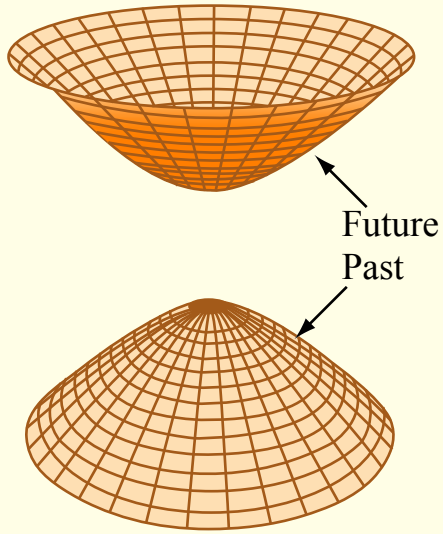
NULL



$\Delta X^t \eta \, \Delta x = 0$

$|\Delta x| = c\Delta t$

TIMELIKE



$\Delta X^t \eta \, \Delta x > 0$

$|\Delta x| > c\Delta t$

Figure by MIT OCW.

MIT Course 8.033, Fall 2006, Lecture 7

Max Tegmark

Today: Relativistic Kinematics Wrapup

- Doppler effect: intuition & applications
- Aberration: intuition & applications
- Superluminal travel?
- Twin paradox

$$
\begin{aligned}
\phi' &= \phi \\
\cos\theta' &= \frac{\cos\theta - \beta}{1 - \beta\cos\theta} \\
k' &= k\gamma(1 - \beta\cos\theta)
\end{aligned}
$$

DOPPLER EFFECT

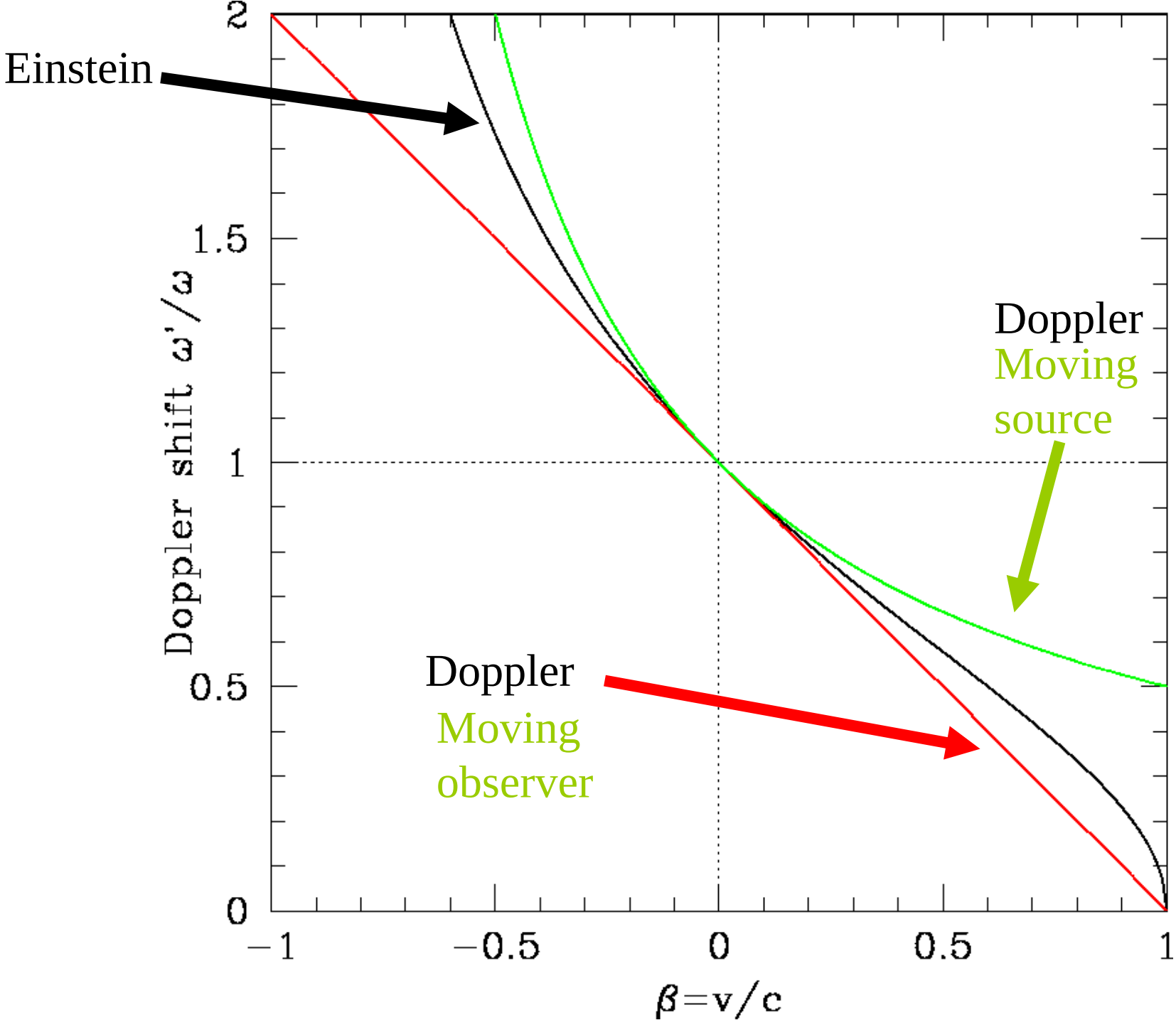
- This matches equations (1)-(4) in the Weiskopf et al ray tracing handout
- The change in the angle θ is known as *aberration*
- The change in frequency ck is known as the Doppler shift — note that since $k = 2\pi/\lambda$, we have $\lambda'/\lambda = k/k'$.
- If we instead take the ratio $\sqrt{k'^2_x + k'^2_y}/k'_z$ above, we obtain the mathematically equivalent form of the aberration formula given by Resnick (2-27b):

$$\tan\theta' = \frac{\sin\theta}{\gamma(\cos\theta - \beta)}$$

- Examine classical limits
- Transverse Doppler effect: $\cos\theta = 0$ gives $\omega' = \omega\gamma$, *i.e.*, simple time dilation

Doppler demos

Doppler applet


Einstein
Doppler
Moving
source
Doppler
Moving
observer
Doppler shift ω'/ω
β=v/c
2
1.5
1
0.5
0
−1
−0.5
0
0.5
1

Three interpretations of Doppler effect: In terms of

1. Momentum
2. Energy
3. Pulses

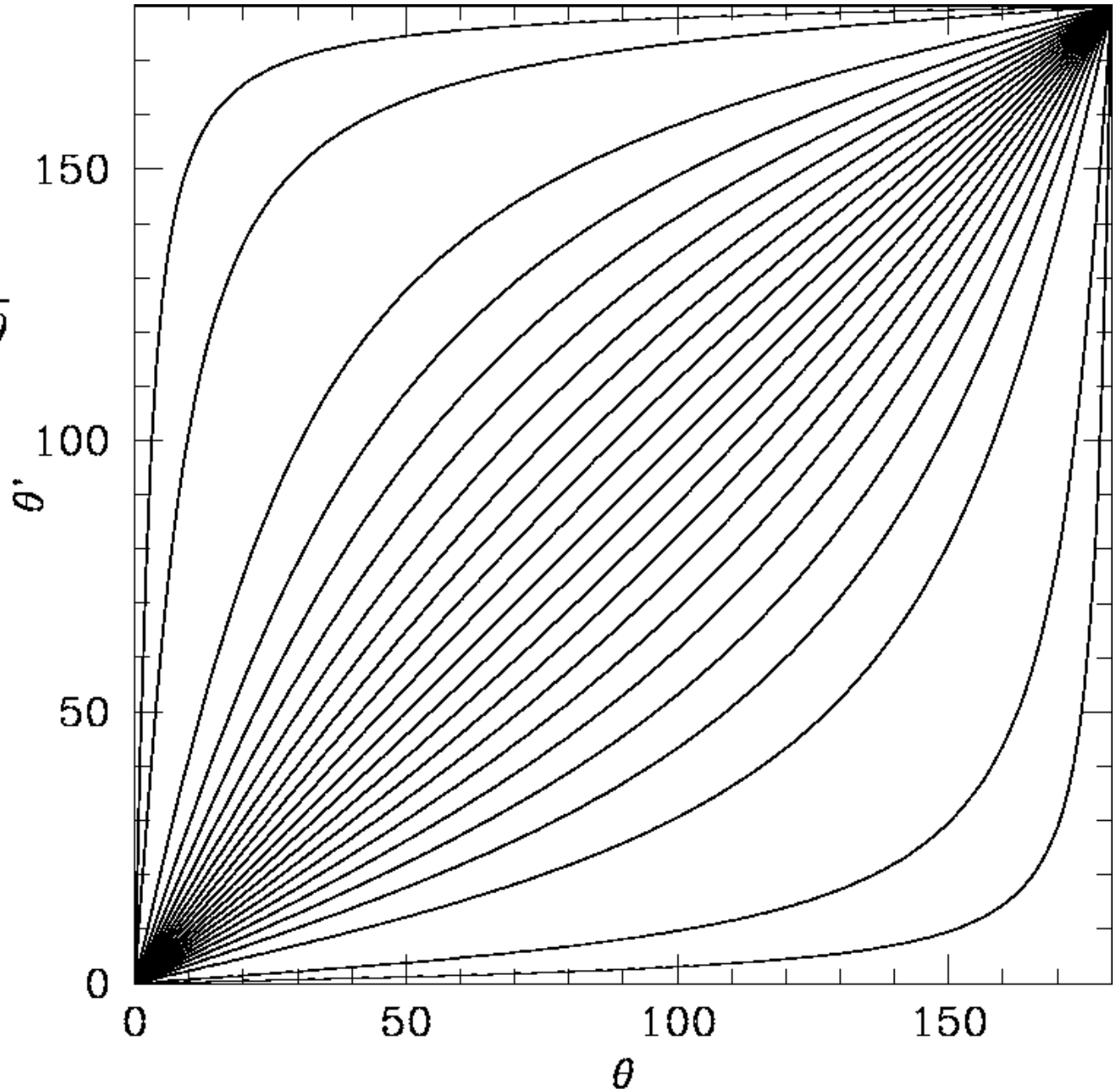
- This matches equations (1)-(4) in the Weiskopf et al ray tracing handout
- The change in the angle θ is known as *aberration*
- The change in frequency ck is known as the Doppler shift — note that since $k = 2\pi/\lambda$, we have $\lambda'/\lambda = k/k'$.
- If we instead take the ratio $\sqrt{k'^2_x + k'^2_y}/k'_z$ above, we obtain the mathematically equivalent form of the aberration formula given by Resnick (2-27b):

$$\tan\theta' = \frac{\sin\theta}{\gamma(\cos\theta - \beta)}$$

- Examine classical limits
- Transverse Doppler effect: $\cos\theta = 0$ gives $\omega' = \omega\gamma$, *i.e.*, simple time dilation

ABERRATION

$$\cos\theta' = \frac{\cos\theta - \beta}{1 - \beta\cos\theta}$$



Production of particles in the first CERN liquid hydrogen bubble chamber

Image removed due to copyright restrictions.

A photograph of production of particles in the first CERN liquid hydrogen bubble chamber.

CHANDRA OBSERVES COSMIC TRAFFIC PILE-UP IN ENERGETIC QUASAR JET

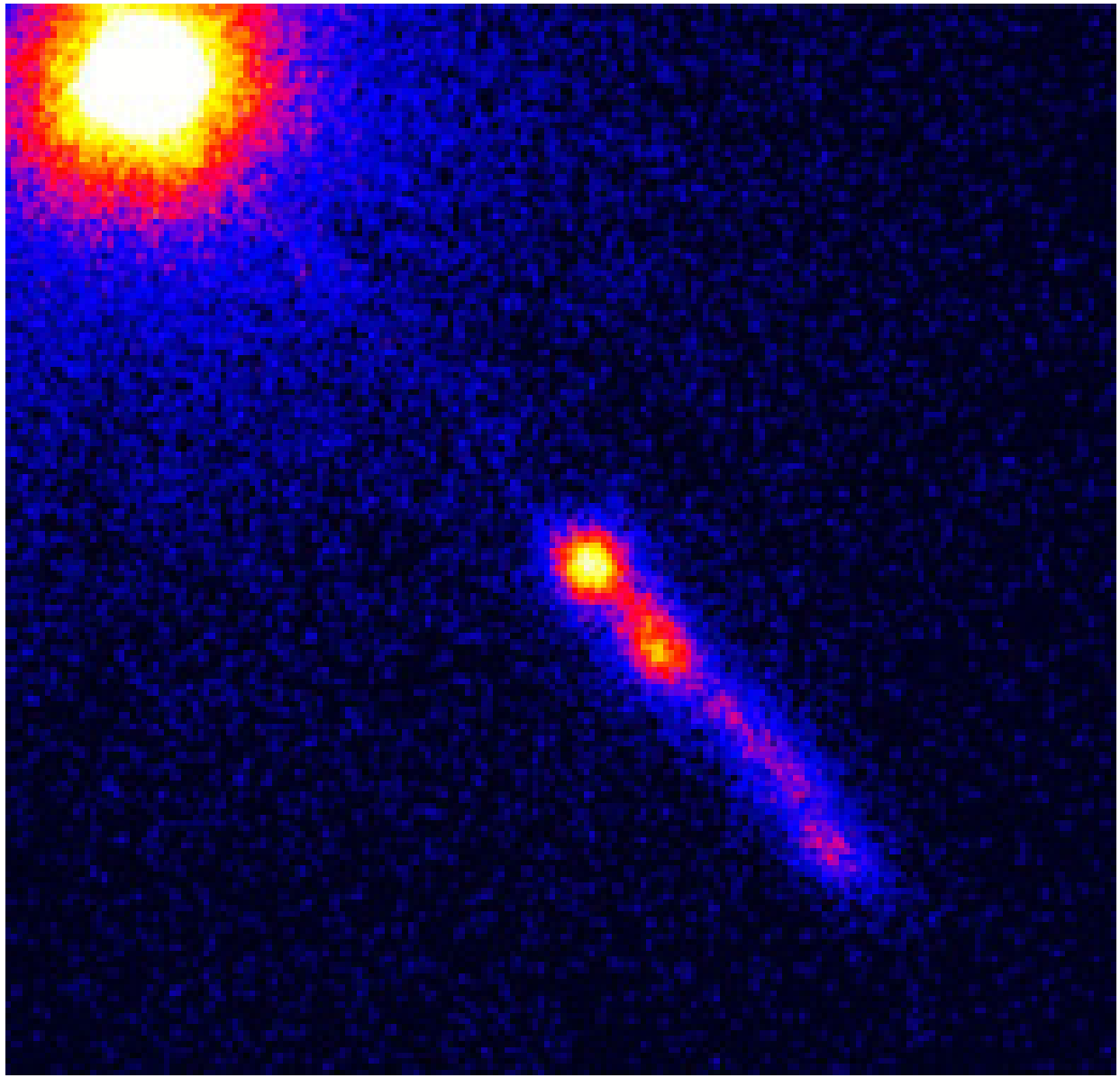


Image courtesy of NASA.

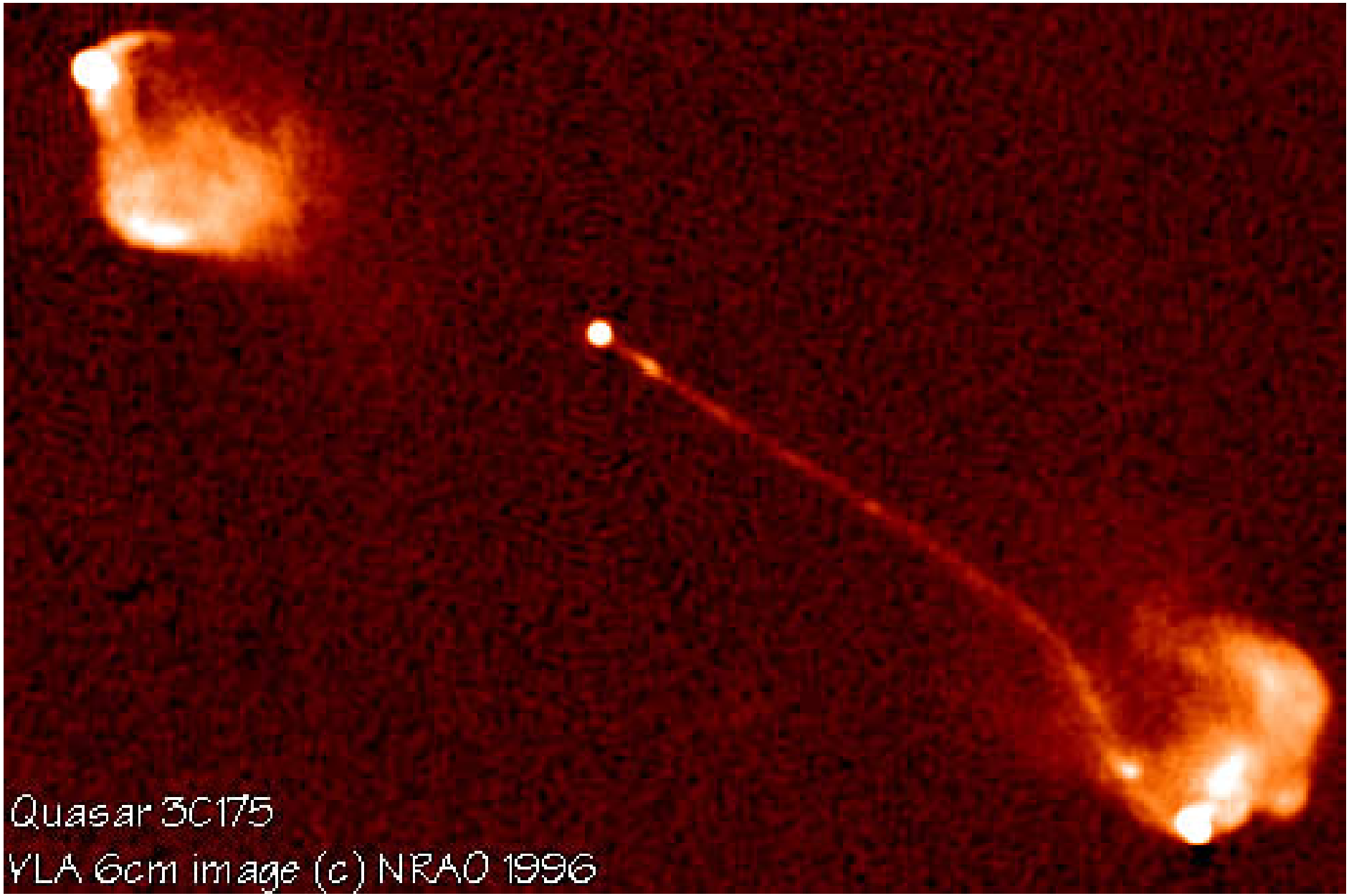


Image courtesy of the National Radio Astronomy Observatory / Associated Universities, Inc. / National Science Foundation

The Three Types of 4-Vectors:

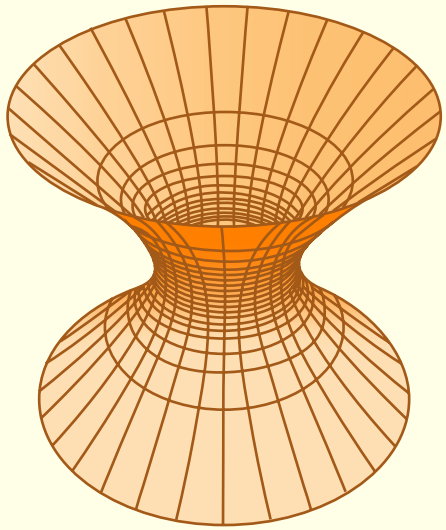
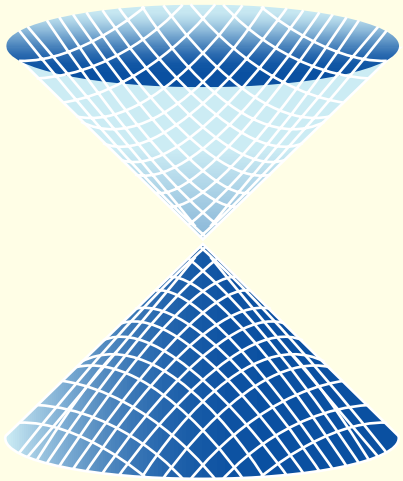
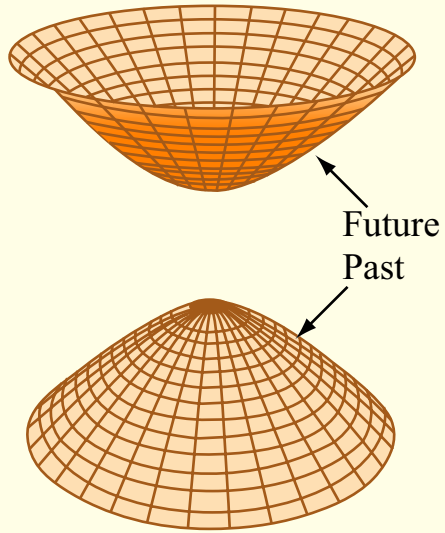
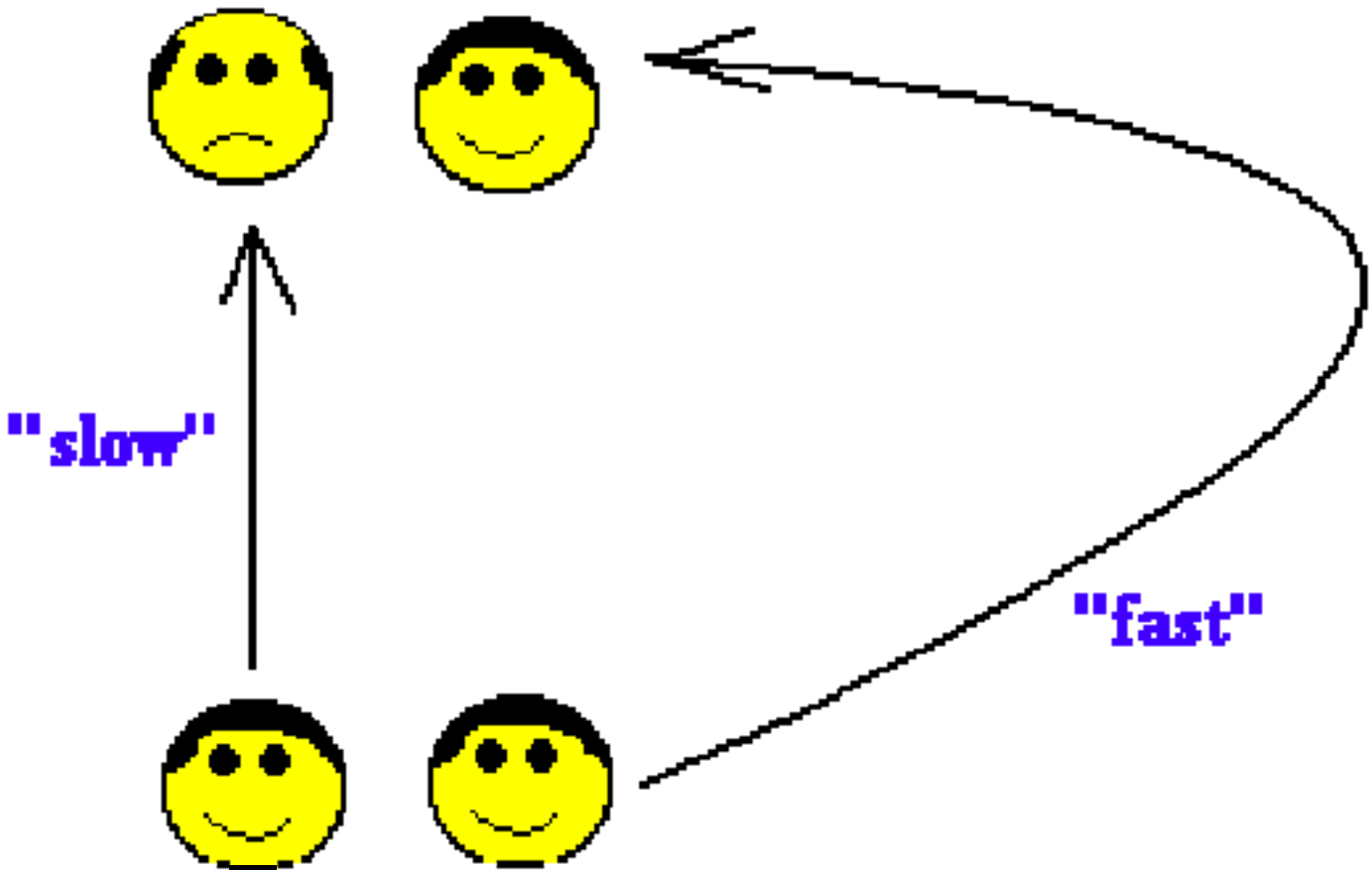
SPACELIKE	NULL	TIMELIKE
		
$\Delta X^t \eta\, \Delta x > 0$ $\lvert\Delta x\rvert > c\Delta t$	$\Delta X^t \eta\, \Delta x = 0$ $\lvert\Delta x\rvert = c\Delta t$	$\Delta X^t \eta\, \Delta x > 0$ $\lvert\Delta x\rvert > c\Delta t$

Figure by MIT OCW.

Superluminal communication?

- Velocity addition formula shows that it's impossibe to accelerate something past the speed of light
- But could there be another way, say a type of radiation that moves faster than light?
- Can an event A influence another event B at spacelike separation (hence transmitting information faster than the speed of light)?
- There is another frame where B happened before A! (PS3)
- Draw Minkowski diagram of this
- By inertial frame invariance, B can then send a signal that arrives back to A before she sent her initial signal, telling her not to send it.
- Implication: c isn't merely the speed of light, but the limiting speed for *anything*

THE TWIN PARADOX



IS IT RIGHT?

Time dilaton examples:

GRB’s

SUPERNOVAE

CLOCKS ON PLANES

GPS

GPS uses a constellation of 24 “NAVSTAR” satellites that are 11,000 miles above the earth's surface.

How GPS receivers calculate your location:

The positioning process:

1. Satellite 1 transmits a signal that contains data on its location in space and the exact time the signal left the satellite.

2. The GPS Receiver collects and interprets this signal and is able to determine the distance from the satellite to the receiver. This creates a circle of possible locations of the receiver.

3. The process is repeated for satellites 2 &3.

4. Your position is where the three circles meet. This process is called trilateration.

5. A fourth satellite is required to obtain the elevation of your current position. Coordinates are displayed on the GPS receiver.

6. More satellites may be used to create a more accurate position.

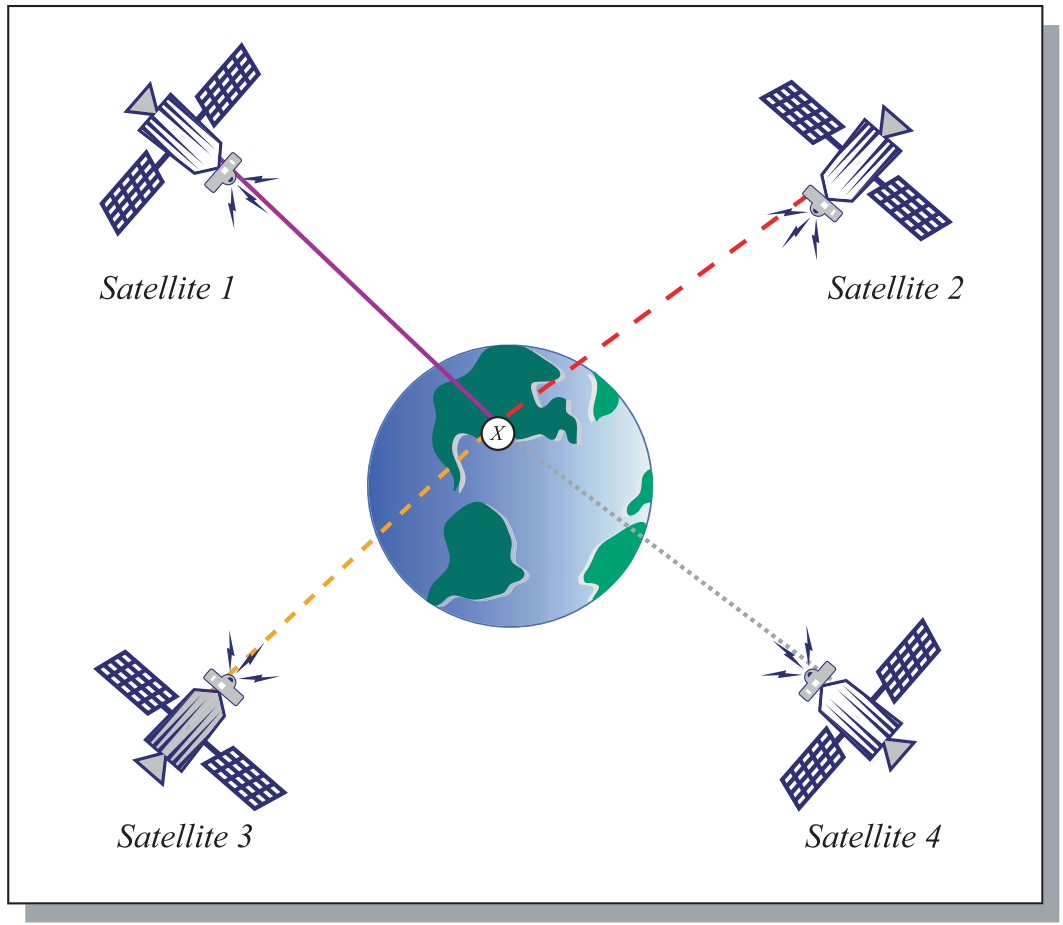


Figure by MIT OCW.

The Distance Calculation

Rate = Speed of Radio Waves (~ Speed of Light) 299,792,459 m/s
Time = amount of time for signal to reach the GPS receiver
Rate * Time = Distance Traveled