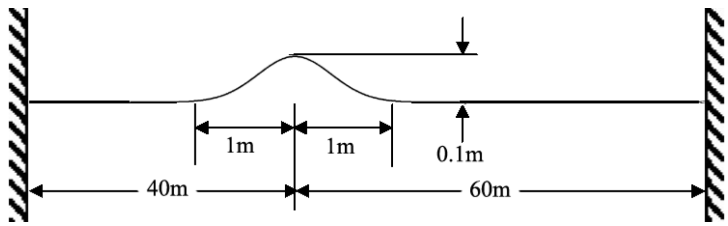
Massachusetts Institute of Technology OpenCourseWare

8.03SC **Fall 2012**

Problem Set #4

Problem 4.1 (French 7-12)[1] – Traveling pulse

The figure shows a pulse on a string of length 100 m with fixed ends. The pulse is traveling to the right without any change of shape, at a speed of 40 m/sec.



a) Make a clear sketch showing how the transverse velocity of the string varies with distance along the string at the instant when the pulse is in the position shown.

b) What is the maximum transverse velocity of the string (approximately)?

c) If the total mass of the string is 2 kg, what is the tension T in it?

d) Write an equation for $y(x,t)$ that numerically describes sinusoidal waves of wavelength 5 m and amplitude 0.2 m traveling in the negative x direction on a very long string made of the same material and under the same tension as above.

Problem 4.2 (French 7-13) – Traveling pulse

A pulse traveling along a stretched string is described by: $y(x,t) = \dfrac{b^3}{b^2 + (2x - ut)^2}$.

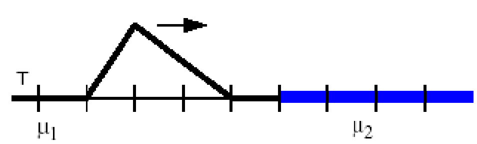
a) Sketch the graph of y against x for $t = 0$.

b) What are the speed of the pulse and its direction of travel?

c) The transverse velocity of a given point of the string is defined by $v_y = \dfrac{\partial y}{\partial t}$. Calculate v_y as a function of x for the instant $t = 0$, and show by means of a sketch what this tells us about the motion of the pulse during a short time Δt.

Problem 4.3 – Pulse reflection at a boundary

Two strings with mass per unit length $\mu_1 = 0.1$ kg/m and $\mu_2 = 0.3$ kg/m, respectively, are jointed seamlessly. They are under tension $T = 20$ N. A traveling wave of a triangular shape shown in the figure is moving to the right along the lighter string. The tick marks set the scale of the pulse width.



a) Find the reflection and transmission coefficients at the interface (including the signs).

b) Make a careful sketch of the *total* deformation of the string when the incident pulse has its peak exactly at the interface. Indicate how you arrived at your answer on your sketch.

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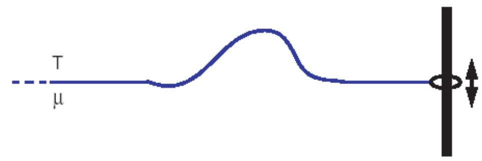
[1]The notation "French" indicates where this problem is located in one of the textbooks used for 8.03 in 2004: French, A. P. *Vibrations and Waves. The M.I.T. Introductory Physics Series.* Cambridge, MA: Massachusetts Institute of Technology, 1971. ISBN-10: 0393099369; ISBN-13: 9780393099362.

c) Make a careful sketch of the *total* deformation of the string when both the reflected and transmitted pulses have moved away from the interface.

d) What is unphysical about the shape of this pulse? (Be quantitative)

Problem 4.4 – Boundary conditions on a string

A very long string of mass density μ and tension T is attached to a small hoop with negligible mass. The hoop slides on a vertical rod and experiences a vertical force $F_y = -b\dfrac{\partial y}{\partial t}$ when it moves.



a) Apply Newton's law to the hoop to find the boundary condition at the end of the string. Express your result in terms of the partial derivatives of $y(x,t)$ at the location of the rod.

b) Show that the boundary condition is satisfied by an incident pulse $f(x - vt)$ and a reflected pulse $g(x + vt)$. Find g in terms of f.

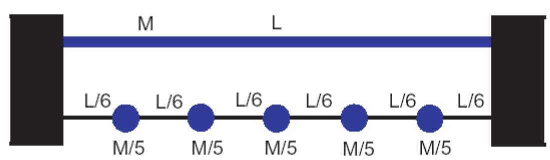
c) Show that your result has the correct behavior in the limits $b \to 0$ (the string is free to slip) and $b \to \infty$ (the string is firmly clamped).

Problem 4.5 – Boundary conditions in a pipe

Pressure oscillations in a hollow pipe of length L are described by the wave equation $\dfrac{\partial^2 p}{\partial z^2} = \dfrac{\rho_0}{\kappa}\dfrac{\partial^2 p}{\partial t^2}$ where p is the over-pressure (over and above the one atmosphere ambient pressure), ρ_0 is the density of the gas in the pipe, κ is the bulk modulus, and z is the longitudinal direction along the pipe. Assuming a solution of the form $p(z,t) = [A\cos(kz) + B\sin(kz)]\cos\omega t$ find all the unknowns $(A,\ B,\ k$ and $\omega)$ for the case where the pipe is open at both ends and $p(z = L/2, t = 0) = p_0$.

Problem 4.6 – Normal modes of discrete vs. continuous systems

Referring to the diagram, you are given a uniform string of length L and total mass M that is stretched to a tension T. You are also given a set of 5 beads, each of mass $M/5$, spaced at equal intervals on a massless string with tension T and total length L.



a) Use boundary conditions to derive a general expression for the frequencies of the normal modes of oscillation of the string. Give the frequencies in terms of $n,\ T,\ L$ and M.

b) Write down the frequencies of the five lowest normal modes of transverse oscillations.

c) Compare the numerical values of these normal mode frequencies with the normal mode frequencies of five beads on the massless string. *Hint: You do not have to solve for the frequencies of the beads. You may use Eqs. (5-25) and (5-26) on page 141 in French which are:*

$$\omega_n = 2\omega_0 \sin\left[\frac{n\pi}{2(N+1)}\right] \quad \text{and} \quad A_{pn} = C_n \sin\left(\frac{pn\pi}{N+1}\right)$$

d) Sketch the five lowest normal modes you found for the massive string. Sketch also the five normal modes of the massless-string-with-five-beads.

e) In a sentence or two, discuss the differences, if any, in the normal modes of the two systems considered here.

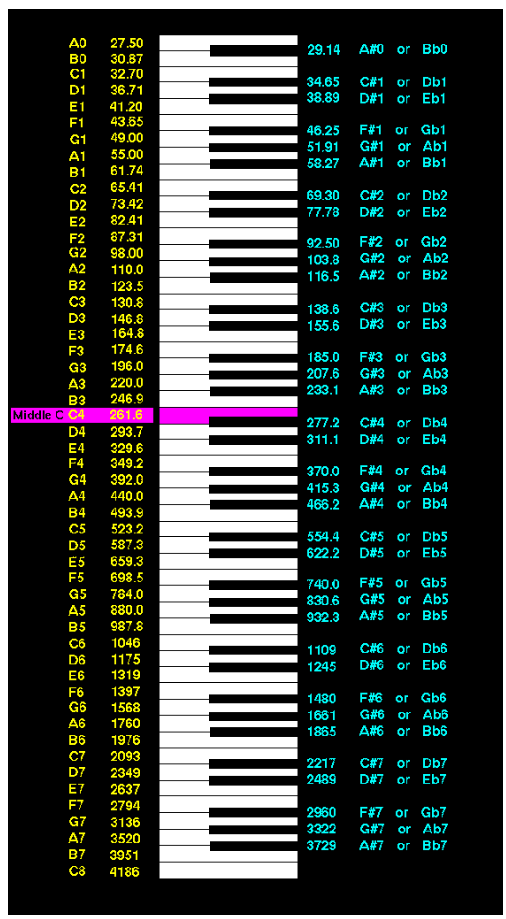
Problem 4.7 – Piano galore

For this problem you need a piano. The picture labels the keys to which we will refer.

Let 256 Hz equal one unit of frequency, $\nu = 1$. The harmonics of this note are then $\nu = 2,\ 3,\ 4$, etc. Middle C on the piano is $C256$ (if the piano is tuned that way). We call it C_4. The subscript refers to the octave; it increases by one at each higher octave of C. Thus the fundamental of C_3 (called a subharmonic of C_4) is 128 Hz ($\nu = \frac{1}{2}$). To avoid confusion, I will always refer to the fundamental as the first harmonic. Thus the first harmonic of C_3 is 128 Hz and the second harmonic is 256 Hz.

Suppose that piano strings behave ideally. Then the mode frequencies of a given string would consist of the harmonic sequence $\nu_1,\ 2\nu_1,\ 3\nu_1$, etc. The names and frequencies of the first 16 harmonics of string C_4 and also its first two subharmonics ($\nu = 1/3$ *and* $\nu = 1/2$) would be as follows (we underline C_4 and its octaves):

$$\begin{array}{rcccccccccccccccccc} Names: & F_2 & C_3 & C_4 & C_5 & G_5 & C_6 & E_6 & G_6 & Bb_6 & C_7 & D_7 & E_7 & F\#_7 & G_7 & G\#_7 & Bb_7 & B_7 & C_8 \\ \nu: & \frac{1}{3} & \underline{\frac{1}{2}} & \underline{1} & \underline{2} & 3 & \underline{4} & 5 & 6 & 7 & \underline{8} & 9 & 10 & 11 & 12 & 13 & 14 & 15 & \underline{16} \end{array}$$



We will start this experiment by determining whether your piano belongs in a bar or a concert hall. Strike the notes from C_3 up (one at the time) and listen for beats. Many keys (not all) activate two or three identical strings simultaneously. A Steinway grand piano has a total of 216 strings (88 keys). If the three (or two) strings which make-up one note are not properly tuned, you will hear beats.

Suppose you hit C_5 and you hear maximum sound at 1 second intervals.

a) What then is the difference in tension between the strings of C_5? The tension in each string in the piano is about 250 Newtons.

b) What is the approximate total force on the frame of the piano that holds all the strings?

Steadily hold down various keys (one at the time) so as to lift their dampers without sounding the notes. Then, while you are still holding down a key of your choice, strike the G_5 sharply, hold it for a few seconds and release it (you still hold the other key down). Listen carefully. You clearly hear sound in case you had chosen C_4 or G_6.

c) What frequencies do you hear in these two cases?

d) Which other notes might be excited by G_5? (G_5 also produces higher harmonics!) Verify your predictions.

To demonstrate the presence of higher harmonics, we will make you "hear" the 6th harmonic of C_3. Hold the C_3 key down (keep your finger on it), strike G_5, hold it for a few seconds and let it go. Now listen carefully to the sound produced by your C_3 strings. This sound is the 6th harmonic of C_3.

Clearly if your piano is out of tune things may sound quite different. But even if it is in tune you may notice by listening carefully that the G_5 does not sound exactly like the 6th harmonic of the C_3. It seems that our piano strings do not behave as "ideally" as we earlier assumed.

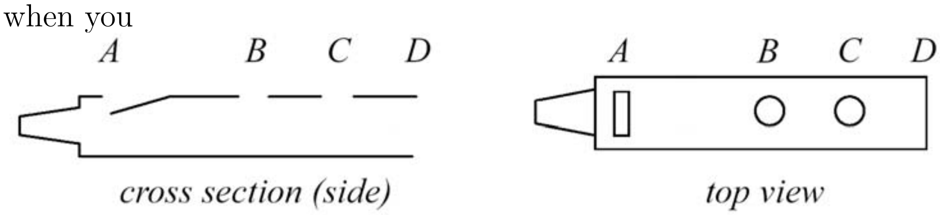
e) How would you explain that?

The lowest two notes on the piano are A_0 27.5 and $A\#_0$ 29.1. Their beat frequency is thus 1.6 Hz, which is easily detectable. Hit both notes together, gently. Once you think you hear beats, let one key up, but not the other.

f) Do the beats go away?

Problem 4.8 – Holes in woodwind instruments

A simplified "flute" as shown in the figure is open at D. There is also a large opening at A (near the mouth piece) and there are two holes at B and C. [$AB = BD$, and $BC = CD$]. The distance AD ~37 cm. The speed of sound is ~ 340 m/sec. What frequency do you expect to hear when you blow and when you



a) hold both holes at B and C closed?

b) hold only hole C closed?

c) hold only hole B closed?

d) do not close either one of the holes B or C?

Keep in mind that wherever the air inside the "flute" is in "open" contact with the air outside, no pressure can build up (pressure nodes). Now read pages 204 and 205 (see reproduction on next page) of Horns, Strings and Harmony by Benade[2] and reconsider your answers. If you play any woodwind instrument, we recommend that you read Chapter IX of that book. Very enjoyable!

Problem 4.9 – Pianos can talk back

Revisit your piano (it does not have to be in tune!). Open the cover so that you can see the strings. Hold down the damper pedal. Shout "heyeyeyey" (hold it for a few seconds) into the region of the strings and sounding board. If you have a grand piano, that would be super! Shout "oooooooh". Try all vowels. The piano strings are responding to your sound. They sort of "Fourier analyze" the sound, and they produce your sound for several seconds.

[2] Benade, Arthur H. *Horns, Strings, and Harmony.* Garden City, N.Y.: Anchor Books, 1960.

a) Explain how this remarkable process of "Fourier analysis" takes place. Why is it not necessary that the piano be in tune?

All components (with frequencies $\omega_1,\ 2\omega_1, \ldots n\omega_1$) in a real Fourier analysis are either in phase or out of phase. However, you won't succeed in making all piano strings that participate in the "analysis" of your voice vibrate in phase (or out of phase).

b) Why not? Give a quantitative answer.

In spite of the complete absence of the phase relations we clearly hear the piano produce our sound.

c) What does that tell you about the importance to your ears and brains of the relative phases of the Fourier components that make up the sound?

d) How would you explain that?

Reproduction from Horns, Strings and Harmony by Benade[3]

HORNS, STRINGS, AND HARMONY

rally be pretty much the same as that of a bore that extends only to the position of the hole. On the basis of this sort of thinking we can guess the correct general behavior that is observed in experiments, a behavior that is illustrated in Figure 46 for a pipe of length *L* provided with a side hole at a distance *D* from the closed end.

The top row in the diagram shows this pipe with different sizes of holes, increasing in diameter from left to right, while the second row gives the lengths of shortened pipe which would sound in unison with the pierced ones directly above. For future convenience, let us call these shortened pipes "equivalent" to their pierced mates. We can then speak of a pierced pipe as having a certain "equivalent length" L_e, which is that of its corresponding simple pipe. Sometimes it is useful to speak of the "hole correction" *C*, which is the difference between the equivalent length and the distance from closed end to the hole. We can summarize our thoughts on holes by saying that the equivalent length L_e always lies between the pipe length *L* and the length *D* from closed end to the hole. A minute hole leaves *L* and L_e almost alike, while a huge hole makes *D* and L_e nearly equal. Similarly, the hole correction is very close to *L* minus *D* for small holes, and falls to zero for large holes.

For the purposes of making a first visit to side holes as they are used in woodwind instruments, we can, on the basis of our deductions, think of the hole as a means for shortening the pipe. The extent to which the pipe is thus effectively shortened depends on the relation of the area of the hole to the cross-sectional area of the bore at that point, and it depends also on the thickness of the wall through which the hole

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WOODWINDS

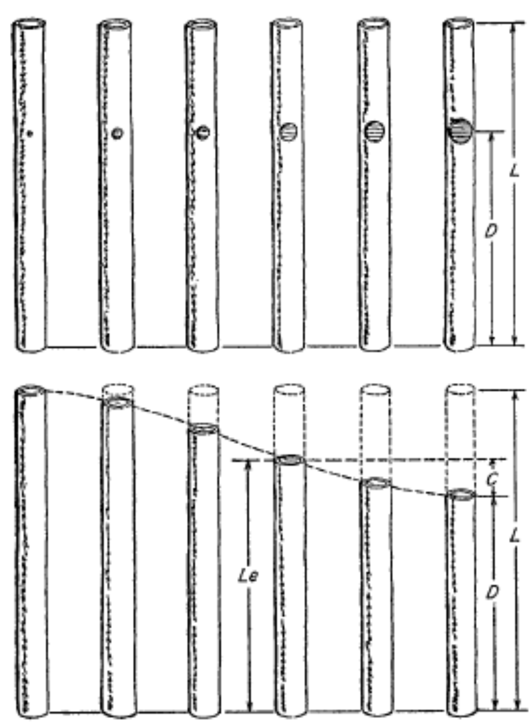

Fig. 46. The size of a side hole affects a pipe's vibrational frequencies. The extent of the effect is illustrated here by comparing different hole sizes with pipe lengths that give matching frequencies. Each pipe in the lower row has a frequency matching that of the holed pipe immediately above it.

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[3]Benade, Arthur H. *Horns, Strings, and Harmony.* Garden City, N.Y.: Anchor Books, 1960.

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