Massachusetts Institute of Technology OpenCourseWare

8.03SC **Fall 2012**

Notes for Lecture #20: Interference

The lecture begins with a discussion of Isaac Newton's (1643-1727) espousal of the particle view of light and its refutation by Thomas Young (1773-1829) in 1803. Newton's work started in the 1670's and was collected his book *Opticks* in 1704 (viewable on Google books). Christian Huygens' (1629-1695) wave theory of light was formulated before Newton's work. Newton's particle theory explained reflection very simply and (with many elaborations) refraction. Such was the influence of Newton, and the scope of his experimental work, that his view prevailed for nearly 100 years. We now know that not only does wave-particle duality apply to light, but also to particles. While the naïve view of the world that allows successful analysis at the introductory level is based on particles, in some sense they do not exist, except as an abstraction or limiting case!

Huygens' principle (also known as the Huygens-Fresnel principle in recognition of the work of Fresnel building a theoretical framework after Young's experiment) states that every point on a wave can be considered as a "source point" from which new waves are emitted. This can explain, for example, how water waves passing through a narrow gap spread out, creating a circular pattern radiating out. The scientific basic of this principle was questioned in the past (**5:30**). Although not discussed in this lecture, one can argue that Huygens' idea does *not* "give the right answer for the wrong reason." Instead, it has a very sound basis in QED or quantum electrodynamics, specifically the fundamental postulate that wavefunctions propagate over any and all allowed (unobstructed) paths between two points. It is the result of interference (addition) of all path integrals that defines the amplitude and phase of the wavefunction of the object at any given point. Not only light quanta (photons), but electrons, neutrons, protons, atoms, molecules, and all other objects obey this simple principle. This lecture is concerned with interferences of a somewhat less complicated nature.

Consider a plane wave impinging on a screen perpendicular to its direction which has two small gaps. Looking at a point P beyond the screen and at an angle from the original wave direction, light from the two gaps will travel a different distance to get to P and therefore these two waves will have a phase difference. *Destructive interference*, or light+light=darkness, results when the phase difference is $\lambda/2$. A fixed difference in distance to two points occurs along a hyperboloidal surface. Similarly, constructive interference occurs for phase differences equal to integral multiples of 2π (path differences of $\pm n\lambda$ for integer n) (**8:30**). Very far away, the hyperboloidal surfaces are nearly straight and a simplified analysis can proceed on that assumption.

For two slits separated by a distance d and looking at an angle θ with respect to the normal to the screen, the difference in distance is approximately $d \sin\theta$. This physical difference in path length corresponds to a phase difference (one full wavelength equals 2π in phase) of $\delta = \frac{2\pi}{\lambda} d \sin\theta$. Constructive interference, where the two waves add to give a higher amplitude wave, requires $\delta = 2\pi n$, where $n = 0, \pm1, \pm2, ...$ (**11:40**). Another way to write this condition is that $d \sin\theta_n = n\lambda$. For destructive interference $\delta = (2n-1)\pi$ or $d \sin\theta_n = \frac{2n-1}{2}\lambda$ (**13:20**). A demo uses red laser light with λ=600 nm going through two slits separated by 0.25 mm, and viewed on a screen 5 m away. The angles for interference can be used to calculate distances along the screen from the point at $\theta = 0$, denoted $x = 0$. Constructive interference is predicted to occur first (away from $x = 0$) at $x_{\pm1} = 1.2$ cm, with the 10^{th} maxima at $x_{\pm10} = 12$ cm (**19:00**). A sketch of light amplitude (intensity, in units of W/m^2) versus $\sin\theta$ has maxima spaced by λ/d, so that the maxima will be spaced closer together for smaller wavelengths (**22:30**).

The electric fields of the waves from slit 1 and slit 2 are $E_1 = E_0 \cos\omega t$ and $E_2 = E_0 \cos(\omega t - \delta)$, where δ is the phase difference discussed above. Their sum, using the familiar "cos half the sum times cos half the difference" trigonometry identity, is $E_{tot} = 2E_0 \cos\left(\omega t - \frac{\delta}{2}\right) \cos\left(\frac{\delta}{2}\right)$. The intensity (Poynting vector) is the square of the amplitude. The first term has a very rapid variation (of order 10^{15} Hz), which will average to 1/2. The second term gives an intensity proportional to $\cos^2(\delta/2)$ (**25:00**). The two slit interference pattern is shown using red laser light, along with some discussion of the wave-particle duality detection of photons. In an experiment where one could determine which slit each photon went through, no interference pattern would be seen (**29:25**).

Sound interference also occurs but with a much longer wavelength, for example 11.3 cm for 3000 Hz. An example is shown with two speakers separated by 1.5 m and driven by the same source. There should then be 26 surfaces of interference (both minima and maxima), found using the fact that $\sin\theta = n\lambda/d$ must be less than 1. At a distance of 5 m, the angle of 4.3° from maximum to maximum corresponds to 38 cm. So, the students should be able to detect the sound interference by moving back and forth, but only if they cover one ear since the separation of maxima and minima is about the width of a human head! (**38:20**)

Path difference so far has been obtained through simple geometry. Now a slightly more complex way of getting path difference, through transmission and reflection in a medium, is examined. Consider a thin, horizontal, oil film of thickness d and with index of refraction of 1.5 (that of the air above and below is 1.0) (**41:00**). The speed of light in a medium is reduced by a factor of the index of refraction so light travels 1.5 times slower in oil than in vacuum (or air). The fraction of the light intensity reflected and transmitted at an interface was discussed in Lecture 18. Using the results found there for light hitting perpendicular to a surface between materials with indices of 1.0 and 1.5, 4% of the intensity is reflected at the first air-oil interface and another 4% (of the 96%

that goes through) is reflected at the second oil-air interface. Again, 96% of this light from the second reflection emerges back into the air, for a net intensity of $0.96 \times 0.04 \times 0.96 = 3.7\%$ of the incoming light (**44:00**). So, the light from the first and second reflections have very similar (but not exactly identical) intensities. These two waves can interfere constructively or destructively, although the intensity differences mean that the latter can never give exactly zero.

These two waves have a difference in total path length as was the case for two slit interference. In this case, the *distance* difference is exactly twice the thickness of the oil layer but that cannot be converted to *phase* difference as easily as was done previously. This is because the first reflection causes a phase shift of 180° (recall the result "$E_r/E_i = -0.2$" in Lecture 18) while the second does not. So, the full phase difference between the two waves is $\delta = 2\pi\left(\frac{2d}{\lambda_{oil}}\right) + \pi = 2\pi\left(\frac{2dn_2}{\lambda_{air}}\right) + \pi$. Note that if the second interface was with a 3rd medium with an index of refraction *higher* than that of oil, the second reflection would also introduce a phase shift of 180°, and the $+\pi$ would become $+2\pi$ which is equivalent to a phase shift of 0 (**49:00**).

As an example, consider blue light with a wavelength of 400 nm in air (267 nm in oil). Because of the phase shift at the first reflection, the oil "thickness" required for the first destructive interference ($n = 1$ in the equation $\delta = (2n-1)\pi$) is zero. The next one occurs at $d = 133$ nm, which one could get trivially from the fact that this is 1/2 of the wavelength in oil (**52:30**). For a layer of this thickness, the relative intensities for green light ($\lambda = 500$ nm) and red light ($\lambda = 650$ nm) are 0.35 and 0.90, respectively. So, the reflection of white light from such an oil layer will appear reddish. For thick layers of oil (large d), essentially every wavelength will have a constructive interference for some value of n so this effect of "colored" reflection cannot be observed (**58:00**). A demonstration of generating colors is done using large soap bubbles, something everyone has seen at some point.

A similar effect can be seen by reflecting light off a soap film suspended from a metal frame. Because of gravity, the film will be thicker at the bottom than at the top, having a sort of convex-triangular shape. As a result of the thickness variation, different colors are maximum at different vertical locations (**1:03:00**). Over time, the film slowly gets thinner and thinner so the color bands move. Eventually, the film is actually much thinner than the wavelength of light ($d \approx 0$) and so no light of any color is reflected. There is also some glycerin in the soap mixture which causes some impressive and chaotic color displays (**1:06:00**).

Path length differences purely in air can be generated by two almost-parallel glass slides with a very small angle between them or a curved piece of glass (a lens for example) lying on a surface. These effects are demonstrated using monochromatic green light for the two plates to make the maxima/minima pattern clearer, and white light for the lens which results in clear color rings.

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These viewing notes were written by Prof. Martin Connors in collaboration with Dr. George S.F. Stephans.