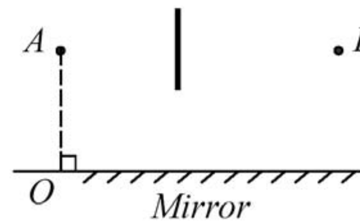
Massachusetts Institute of Technology OpenCourseWare

8.03SC **Fall 2012**

Problem Set #8

Problem 8.1 – Nature is in a hurry - Fermat's Principle

Light travels from A to B (A and B are in the plane of the paper in the same medium). An opaque screen prevents travelling along the straight line AB. The journey now takes place via a mirror perpendicular to the plane of the paper (see figure).



a) Construct B', the mirror image of B and draw the line AB'. The intersection of this line with the mirror is C.

The light beam that travels from A to B via the mirror follows route ACB.

b) Show that the angle of incidence is the angle of reflection.

We now introduce a coordinate system with the center at O (see figure). Let the coordinates of A be $x = 0,\ y = d$, of B $x = l,\ y = d$ (we take $y_A = y_B = d$, to simplify your math) and let the coordinates of a point C' on the mirror be $x = x'$ and $y = 0$.

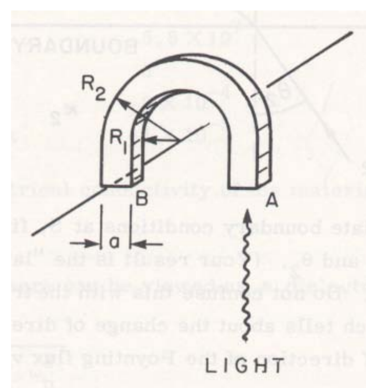
c) Calculate the distance $s = AC' + C'B$ in terms of $x',\ d$ and l and calculate for what value of x' the distance $AC' + C'B$ is a minimum.

Fermat's Principle dictates that your solution under (c) is the route that nature will choose.

d) Show that the position of C' (result under (c)) is the same as that of C (result under (a)).

Problem 8.2 (Bekefi & Barrett 7.4)[1] – Fiber optics

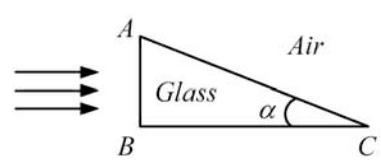
A glass rod of rectangular cross section is bent into the shape shown to the right. The inner and outer contours of the curved portion of the rod have fixed radii R and $R + a$, respectively. The index of refraction of the glass $n = 1.5$.



For a given $R \gg \lambda$, what is the maximum value of "a" for which all the light entering at A will emerge at B?

Problem 8.3 (Bekefi & Barrett 7.5) – Total reflection

A plane light wave is incident normally on the face AB of a glass prism as shown in the figure. The index of refraction of glass is 1.50.



a) Find the value of the angle α such that the wave will be totally reflected at the surface AC.

b) Is this the smallest or largest permissible angle for total reflection?

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[1] The notation "Bekefi & Barrett" indicates where this problem is located in one of the textbooks used in 8.03 in 2004: Bekefi, George, and Alan H. Barrett *Electromagnetic Vibrations, Waves, and Radiation*. Cambridge, MA: MIT Press, 1977. ISBN: 9780262520478.

Problem 8.4 – Light under water

a) Red light ($\lambda \approx 650$ nm) is incident on the surface of the water in a swimming pool. What is the wavelength λ' of the light inside the water? If you swim under water and look up at the refracted light coming from the surface, what color do you see? Justify your answer in a few words. Assume the refractive index of water is 1.33.

b) Using the Fresnel equations at normal incidence, show that the sum of the power in the reflected light and the power in the refracted light is the same as the power in the incident light. Thus, energy is conserved.