MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department
Earth, Atmospheric, and Planetary Sciences Department

Astronomy 8.282J-12.402J February 22, 2006

Problem Set 3

Due: Wednesday, March 1 (in lecture)

Reading: Zeilik & Gregory Chapters 8 & 9 (over the next week and a half).

Problem 1

"Eccentric Orbits"

a. The Earth's distance from the Sun varies from 147.2 million km to 152.1 million km. What is the eccentricity of its orbit?

b. The orbit of Mars has an eccentricity of 0.093. What is the ratio of its closest approach to the Sun compared to its greatest distance?

Problem 2

"Doppler Effect"

a. A nearby galaxy has a recessional velocity of 1500 km/s. What do we observe for the wavelength of the hydrogen alpha line (rest wavelength = 6563 Ångströms) in its spectrum?

b. Doppler radar can be used to determine the rotation rate of a planet by measuring the maximum speed of the reflecting surface. The equatorial radius of Mars is 3400 km, and its rotation period is 24 hours and 37 minutes. If we transmit a monochromatic (single frequency) radar signal toward Mars with a frequency of 1000 MHz, what will be the range of frequencies received in the reflected signal?

Problem 3

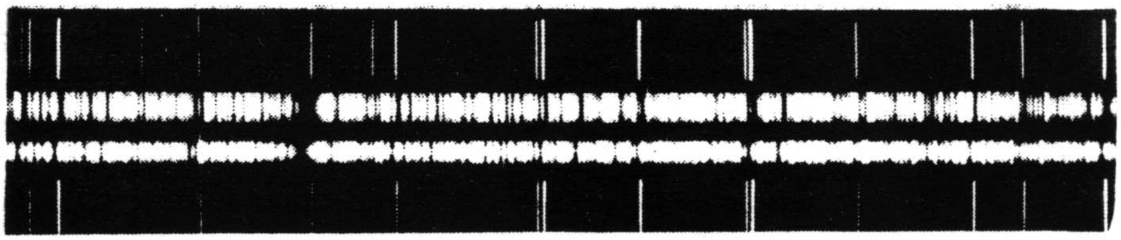
"Determination of the AU from Doppler Shifts"

The photocopied figure below displays two spectrograms of the star Arcturus taken six months apart (these are the two dense spectra). The spectrogram at the very top and the one at the very bottom (with only a relative handful of lines) are of a reference light source at rest with respect to the telescope. For each spectrum, the central region is in the visible (near 4300 Ångströms), and the scale is 1.0 mm = 0.95 Ångströms.

a. Find the wavelength shift between the two spectra. (Because of the crude presentation of the spectra you should not expect a very accurate value.)

b. Compute the velocity of the Earth in its orbit around the Sun under the assumption that Arcturus is near the ecliptic plane, and that these spectra represent the extreme velocities of this star that can be recorded during the year. (In actuality, Arcturus lies 33° from the ecliptic plane.)

c. Use the results of parts a & b to determine the astronomical unit (in cm). You may use the fact that the length of the year is 365 days.



Spectra (λ4200Å to λ4300Å) of the constant velocity star Arcturus taken about six months apart.

Problem 4

"Determining the Mass of a Neutron Star" – (with apologies to those who did this in 8.01T)

A binary system known as 4U0900-40 consists of a "neutron star" and a normal "optical star". You are given two graphs of actual data obtained from observations of this system. The top graph shows the time delays (in seconds) of X-ray pulses detected from the neutron star as a function of time (in days) throughout its orbit. These delays indicate the time of flight for an X-ray pulse (traveling with the speed of light) to cross the orbit of the binary in its trip to the earth. (Ignore the heavy dots scattered about the "x" axis.)

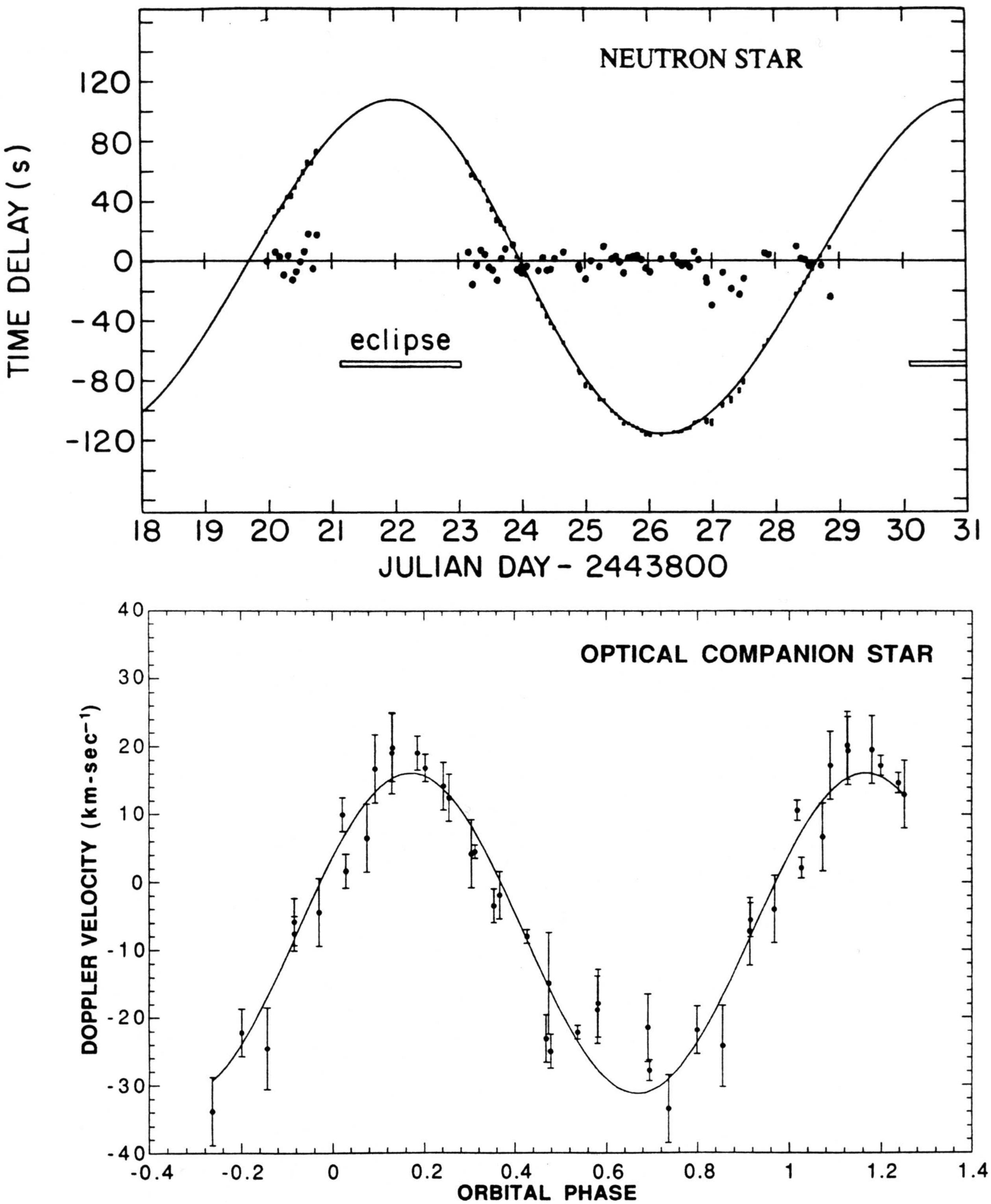
The bottom graph displays the velocity component of the optical star toward (or away from) the Earth as a function of its orbital phase. The velocities (in km per second) are determined from Doppler shifts in its spectral lines. Time on this plot is given in units of orbital phase, where the time between phases 0.0 and 1.0 corresponds to one orbital period.

Assume that the orbit of 4U0900-40 is circular and that we are viewing the system edge on, i.e., the Earth lies in the plane of the binary orbit. In each graph, the solid curve is a computed fit to the individual data points.

Suggested procedure (you will need a ruler for the first three parts):

a. Determine the orbital period, P, in days.
b. Estimate the velocity, v_O of the optical star in its orbit around the center of mass.
c. Use P and v_O to find the radius of the orbit of the optical star, a_O, around the center of mass.
d. Estimate the size of the orbit, a_N of the neutron star around the center of mass. First express your answer in light seconds, then in centimeters.
e. Find the ratio of the mass of the neutron star, M_N, to the mass of the optical star, M_O. [Hint: Recall that in any binary $m_1 r_1 = m_2 r_2$]
f. Use Kepler's law to find the total mass of the binary system.
g. Find M_N and M_O; express your answers in units of the mass of the Sun (2×10^{33} gm).
Optional: Use the length of the eclipse of the neutron star (indicated on the top graph) and simple geometry to compute the physical size of the optical companion star. Compare your answer to the radius of the sun which equals 7×10^{10} cm. ($G = 6.7 \times 10^{-8}$ in cgs units and 6.7×10^{-11} in mks units.)

BINARY X-RAY PULSAR SYSTEM



Doppler Curves for 4U0900-40.

Problem 5 (Optional)

"Planetary Orbital Periods"

The semimajor axes of the 9 solar system planets are listed below in units of the AU.

Mercury	0.387
Venus	0.723
Earth	1.0
Mars	1.52
Jupiter	5.20
Saturn	9.54
Uranus	19.2
Neptune	30.1
Pluto	39.4

Use Kepler's third law to find the orbital periods of the planets about the Sun, expressed in years.

Problem 6

"Refraction"

a. The index of refraction of water is 1.33. Find the angle of refraction of a beam of light in water, emerging into air, for angles of incidence of 20°, 30°, and 40°. Sketch these rays on a diagram.

b. For what incidence angle is the angle of refraction equal to 90°? (This corresponds to "total internal reflection", a phenomenon used in the propagation of light in fiber optics.)

Problem 7 (Optional)

"Optics Problem"

Suppose you set out to build your own small telescope. You obtain an objective lens that is 10 cm in diameter and has a focal length of 1 meter.

a. What is the diameter (in mm) of an image of the Moon at the focus of the objective?

b. If a simple lens is used as an eyepiece, what would its focal length have to be to yield a magnification of 200?

c. If this telescope were used to observe a bird at a distance of 100 meters, by how much would the eyepiece have to be moved to refocus the telescope from infinity?

For Those With a Spectrometer Kit
Suggested sources for study include:

a. incandescent light bulb - with clear bulb
b. incandescent light bulb - viewed though transparent colored objects
c. incandescent light bulb - with frosted glass bulb
d. fluorescent light - indoors
e. fluorescent lamp - outdoor street light
f. the Sun - note the absorption lines!
g. the Moon
h. candle flame
i. match flame
j. red or green light emitting diodes (LEDs)
k. chemical light stick
l. any other light sources that appear interesting