MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.282 March 10, 2003

Quiz 1

Name Solutions **(please print)**
Last **First**

1. Be sure to attempt all problems.
2. The point values of the problems are indicated at the top of each page.
3. Closed book exam; you may use one page of notes.
4. Wherever possible, try to solve the problems using general analytic expressions. Plug in numbers only as a last step.

Problem	Grade	Grader
1	25	
2	25	
3	35	
4	15	
Total	100	

Problem 1 (25 points)

Orbital Dynamics

It has long been suspected that there is a massive black hole near the center of our Galaxy. Recently, a group of astronmers determined the parameters of a star that is orbiting the suspected black hole. The orbital period is 15 years, and the orbital radius is 0.12 seconds of arc (as seen from the Earth). Take the distance to the Galactic center to be 8 kpc. Compute the mass of the black hole, starting from $F = ma$. Express your answer in units of the Sun's mass ($M_\odot$). (Assume that Newton's law of gravity is applicable for orbits sufficiently far from a black hole, and that the orbiting star satisfies this condition.)

$$F = -\frac{GM_{BH}M_*}{R^2} = -\frac{M_* v^2}{R}$$

$$v = \frac{2\pi R}{P}$$

$$\frac{GM_{BH}}{R^2} = 4\pi^2 \frac{R}{P^2}$$

$$GM_{BH} = 4\pi^2 \frac{R^3}{P^2}$$

likewise —

$$GM_\odot = 4\pi^2 \frac{(1\ AU)^3}{(1\ yr)^2}$$

$$G = \frac{1}{M_\odot} 4\pi^2 \frac{(1\ AU)^3}{(1\ yr)^2}$$

$$\frac{M_{BH}}{M_\odot} = \frac{(R/1\ AU)^3}{(P/1\ yr)^2}$$

$$R = \frac{0.12}{200{,}000}(8)(1000)(3\times10^{18}\ cm) = 1.4\times10^{16}\ cm = 900\ AU$$

$$\frac{M_{BH}}{M_\odot} = \frac{(900)^3}{(15)^2} = 3\times10^6$$

$$\boxed{M_{BH} = 3\times10^6\ M_\odot}$$

Problem 2 (25 points)

Comparison of Radio and Optical Telescopes.

(a) The Very Large Array (VLA) is used to make an interferometric map of the Orion Nebula at a wavelength of 10 cm. What is the best angular resolution of the radio image that can be produced? Note that the maximum separation of two antennae in the VLA is 36 km.

$$\Theta = \frac{\lambda}{D} = \frac{0.10\,m}{36 \times 10^3\,m} = 2.8 \times 10^{-6}\ rad.$$

$$200{,}000\Theta = \boxed{\Theta'' = 0.6''}$$

(b) Estimate the diameter ~~and focal length of t~~he primary mirror (or lens) needed in an optical telescope to achieve the same angular resolution. Take the wavelength of optical light to be 500 nm (5000 Å $= 5 \times 10^{-5}$ cm $= 5 \times 10^{-7}$ m.) If you are unable to do part (a), simply adopt an angular resolution of 1″.

$$0.6'' = 2.8 \times 10^{-6}\ rad. = \frac{\lambda}{D}$$

$$D = \frac{5 \times 10^{-5}\,cm}{2.8 \times 10^{-6}} = 18\ cm$$

(c) What is the size of a 10 arcminute square portion of the nebula on the detector on the optical telescope if its focal length is 200 cm?

$$s = 0.01745 f$$
$$= 3.49\ cm/^\circ$$

$$10' = \left(\tfrac{1}{6}\right)^\circ = (0.17)^\circ$$

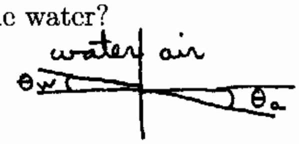
width $= 0.58$ cm

area $= 3.4\ cm^2$

Problem 3 (35 points)

Short Answer Questions

(a) A collimated light beam propagating *in water* is incident on the surface (air/water interface) at an angle θ_{water} with respect to the surface normal. If the index of refraction of water is $n = 1.3$, find an expression for the angle of the light once it emerges from the water into the air, θ_{air}. What is the critical angle, i.e., $\theta_{\text{water}} = \theta_{\text{crit}}$, such that the light will not emerge from the water?



$1.3 \sin\theta_w = \sin\theta_a$

$\theta_a = \sin^{-1}(1.3 \sin\theta_w)$

$\theta_a = 90^\circ$ if $\sin\theta_a = 1 \Rightarrow \sin\theta_w = \frac{1}{1.3} = 0.77$

$\theta_{crit} = \sin^{-1} 0.77 = 0.88 \text{ rad} = 50^\circ$

(b) Use the Bohr model of the atom to compute the wavelength of the transition from the $n = 100$ to $n = 99$ levels. [Useful relation: the wavelength of $L\alpha$ (n = 2 to n = 1 transition) is 1216 Å.]

$\lambda^{-1} = R\left(\frac{1}{99^2} - \frac{1}{100^2}\right) = 2.03 \times 10^{-6} R$

$\lambda_{L\alpha}^{-1} = R\left(1 - \frac{1}{4}\right) = \frac{3}{4}R$

$\lambda = \lambda_{L\alpha} \quad \left(\frac{3}{4}\right)(2.03 \times 10^{-6})^{-1}$

$\lambda = 4.49 \times 10^{8}\ \text{Å} = 4.49 \text{ cm}$

(c) What basic physics laws or principles lie behind Kepler's 1st and 2nd laws (elliptical orbits and equal areas swept out in equal times, respectively.)

1st — cons. of ang. momentum & $\frac{1}{r}$ potential

2nd — cons. of ang. momentum only

(d) A galaxy moves directly away from us with a speed of 3000 km s^{-1}. Find the wavelength of the Hα line observed at the Earth. The rest wavelength of Hα is 6565 Å.

$\frac{v}{c} = \frac{(3000)(1000)}{3 \times 10^8} = 0.01$

$\lambda_{obs} = (6565\ \text{Å})\left(1 + \frac{v}{c}\right) = 1.01(6565\ \text{Å}) = 6630\ \text{Å}$

(e) A star has a measured parallax of 0.01″. How far away is it?

100 pc

Problem 4 (15 points)

Magnitudes

(a) The Sun's effective temperature, T_e, is 5800 Kelvin, and its radius is 7×10^{10} cm (7×10^8 m). Compute the luminosity (power output) of the Sun. Treat the Sun as a blackbody radiator.

$$A = 4\pi R^2 = 6 \times 10^{22} \text{ cm}^2$$

$$L = \sigma A T_e^4 = (6\times10^{-5})(6\times10^{22})(5800)^4 = 4\times10^{33} \text{ erg/s}$$

(b) If the Sun's absolute magnitude is +5, use the results of part (a) to find the luminosity of a star of magnitude 0. If you did not do part (a) use the value of $L_\odot$ given in the table of *Useful Constants.*

$$5 = 2.5 \log\left(\frac{L_*}{L_\odot}\right)$$

$$2 = \log \frac{L_*}{L_\odot}$$

$$L_* = 100 L_\odot = 4 \times 10^{35} \text{ erg/s}$$

(c) If a star cluster is made up of 10^6 stars whose absolute magnitude is the same as that of the Sun (+5), compute the combined magnitude of the cluster if it is located at a distance of 10 pc.

at 10pc, the magnitude is just the absolute magnitude.

$$L_{TOT} = 10^6 L_\odot$$

$$\Delta m = 2.5 \log\left(\frac{L_{TOT}}{L_\odot}\right) = 2.5 \log 10^6 = 2.5 \cdot 6 = 15$$

$$\therefore m = -10$$