Physics 8.321, Fall 2002
Homework #2

Due **Wednesday, September 25** by 4:30 PM in the 8.321 homework box in 4-339B.

The operator measuring the spin of a spin-1/2 particle along the axis parallel to a general unit vector $\hat{\mathbf{n}}$ is given by

$$S_{\mathbf{n}} = \mathbf{S} \cdot \hat{\mathbf{n}}$$

where $S_i = \sigma_i \, \hbar/2$ for $i = 1, 2, 3$, and

$$\sigma_1 = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

These operators are used in the following problems.

1. (a) Measurement of an electron's spin along the z-axis (S_z) using a Stern-Gerlach apparatus gives the eigenvalue $\hbar/2$. What is the probability that a subsequent measurement of the spin in the direction $\hat{\mathbf{n}} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ yields $\hbar/2$?

(b) Measurement of an electron's spin along the axis $\hat{\mathbf{n}}$ gives the eigenvalue $\hbar/2$. What is the probability that a subsequent measurement of the spin along the z-axis yields $\hbar/2$?

2. Show that it is impossible for an electron to be in a state such that

$$\langle S_x \rangle = \langle S_y \rangle = \langle S_z \rangle = 0 \, .$$

3. A beam produced by a Stern-Gerlach filter contains electrons that are all in the same spin state, which can be written as

$$|\alpha\rangle = s_+ |+\rangle + s_- |-\rangle$$

where $|+\rangle, |-\rangle$ are eigenstates of S_z with eigenvalues $\pm\hbar/2$.

Part of the beam is passed through an analyzer oriented in the z direction, giving

$$\langle S_z \rangle = 0 \, .$$

The other part of the beam is passed through an analyzer oriented in the x direction, giving

$$\langle S_x \rangle = \hbar/4 \, .$$

(a) Calculate $\langle S_y \rangle$.

(b) What are the possible directions along which the original Stern-Gerlach filter may have been oriented?

4. Sakurai: Problem 19, Chapter 1 (page 64)

5. Sakurai: Problem 20, Chapter 1 (page 64)

6. Sakurai: Problem 24, Chapter 1 (page 65)

7. Sakurai: Problem 26, Chapter 1 (page 66)