



Bivariate Relationships

17.871

2012




Testing associations (not causation!)

■ Continuous data

- Scatter plot (always use first!)
- (Pearson) correlation coefficient (rare, should be rarer!)
- (Spearman) rank-order correlation coefficient (rare)
- Regression coefficient (common)

■ Discrete data

- Cross tabulations
- χ^2
- Gamma, Beta, etc.



Continuous DV, continuous EV

- Dependent Variable: DV
- Explanatory (or independent) Variable: EV

- Example: What is the relationship between Black percent in state legislatures and black percent in state populations

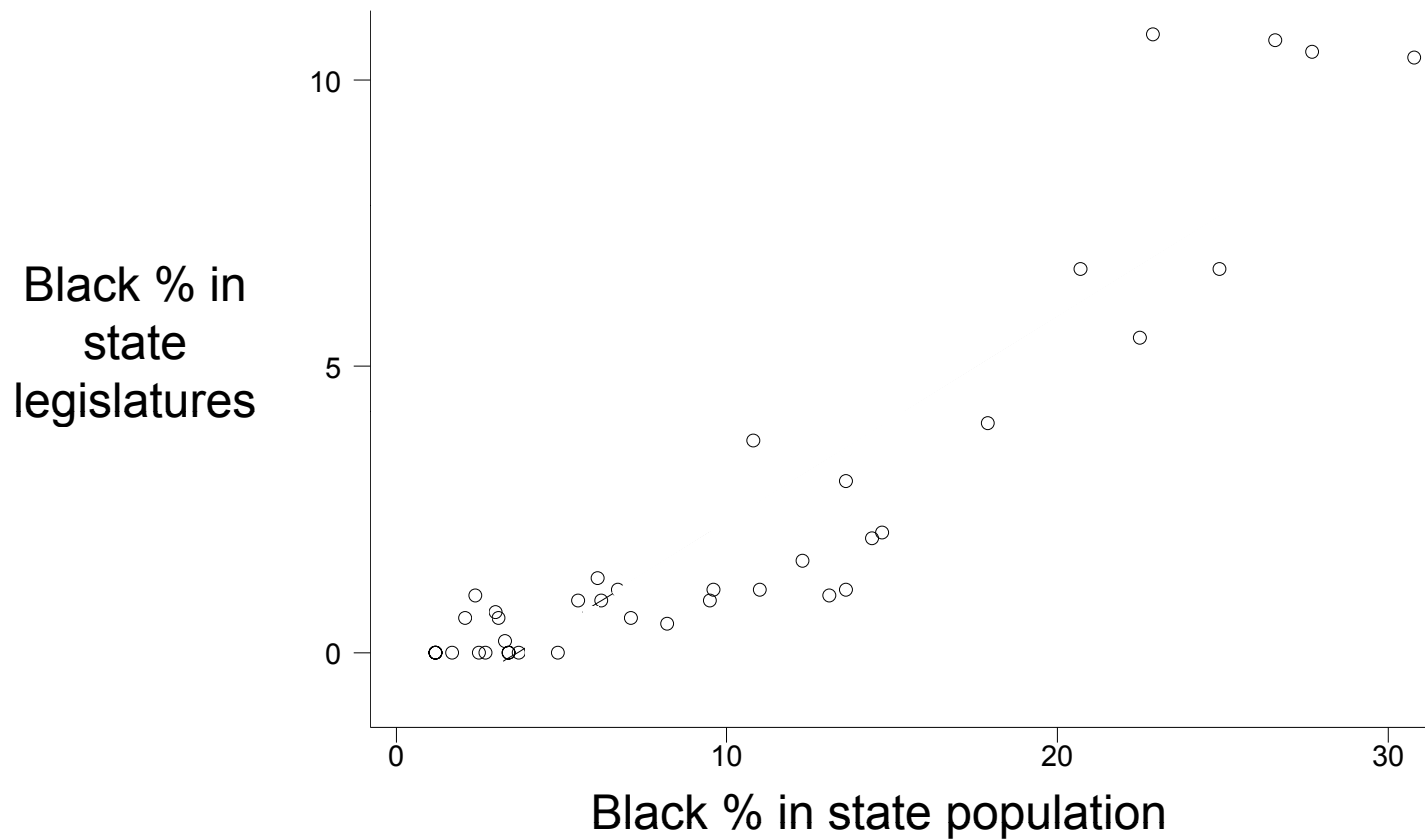


Regression interpretation

Three key things to learn (today)

1. Where does regression come from
2. To interpret the regression coefficient
3. To interpret the confidence interval
 - We will learn how to calculate confidence intervals in a couple of weeks

Linear Relationship between African American Population & Black Legislators





The linear relationship between two variables

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$$

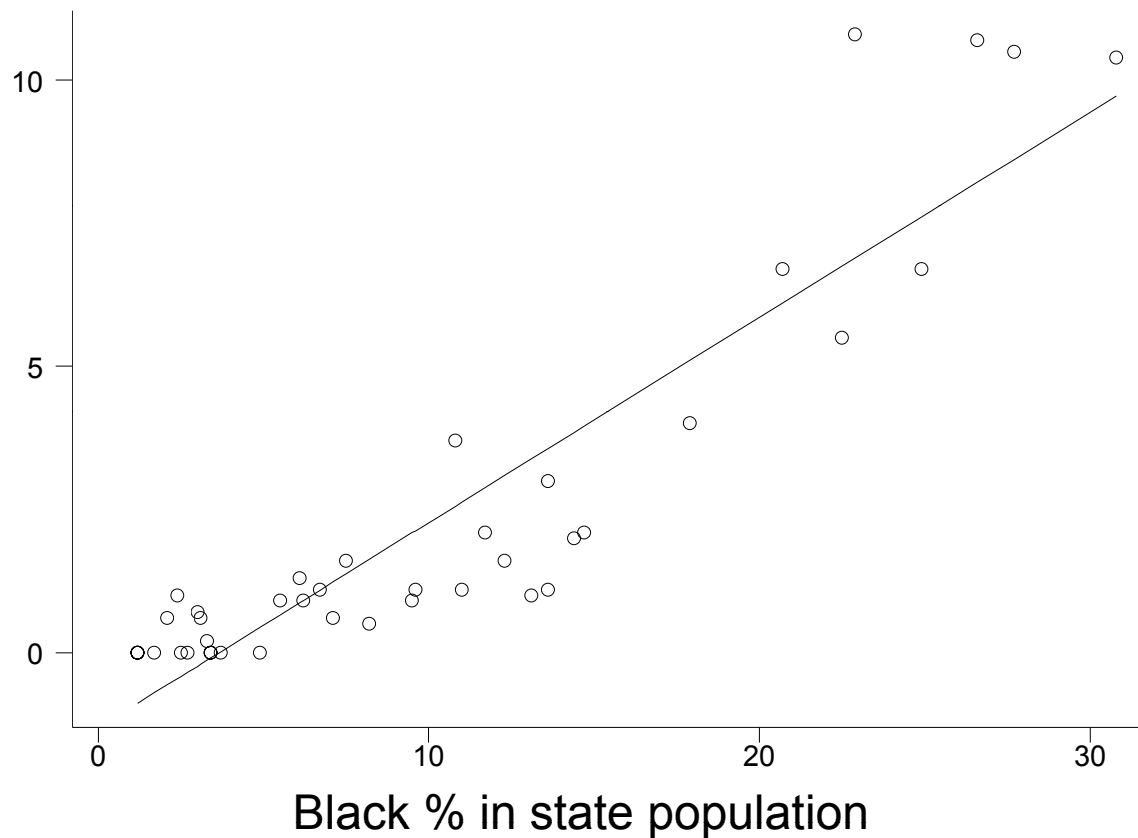
Regression quantifies how one variable can be described in terms of another

Linear Relationship between African American Population & Black Legislators

Black % in
state
legislatures

$$\hat{\beta}_0 = -1.31$$

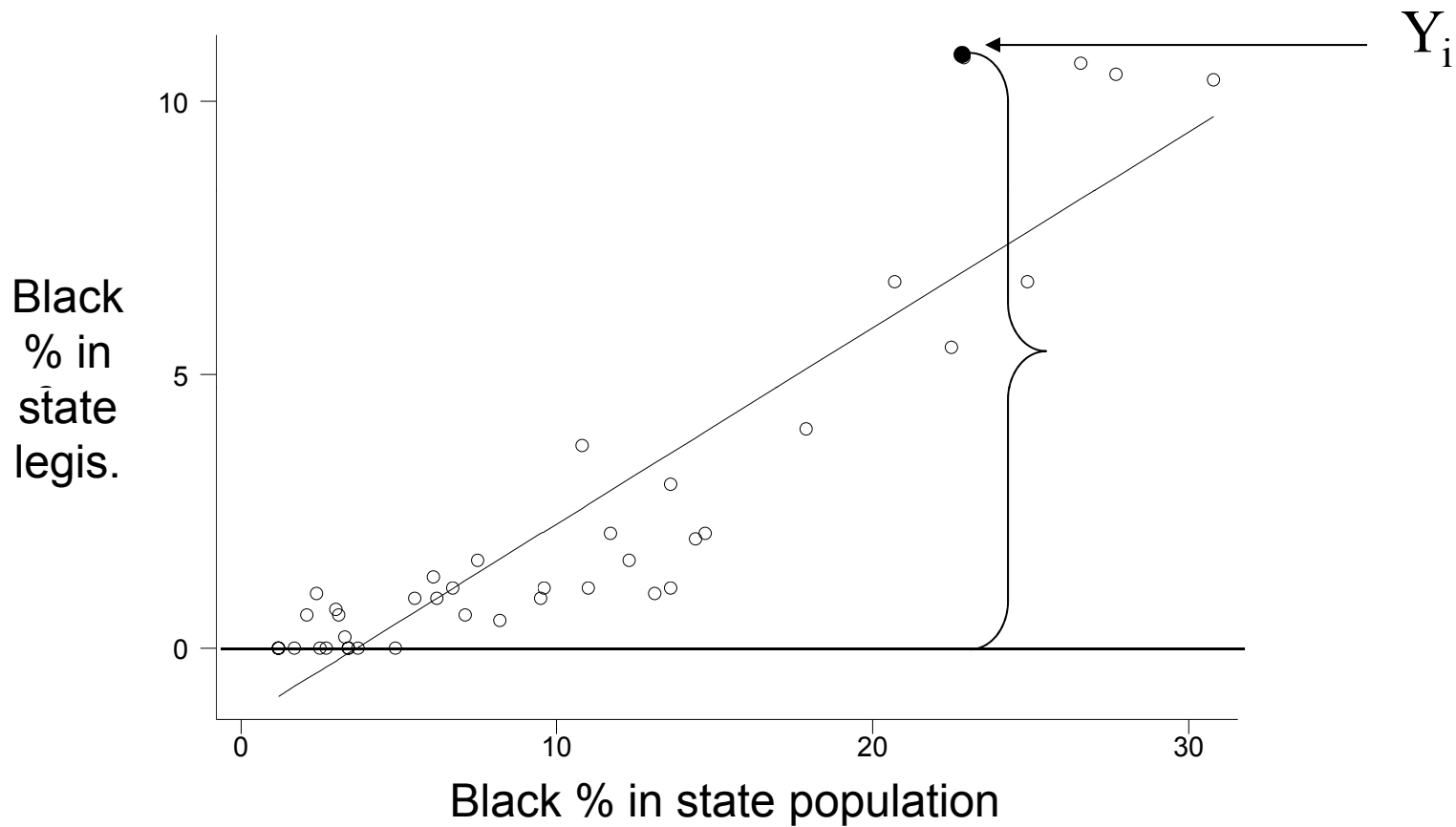
$$\hat{\beta}_1 = 0.359$$



$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$$

How did we get that line?

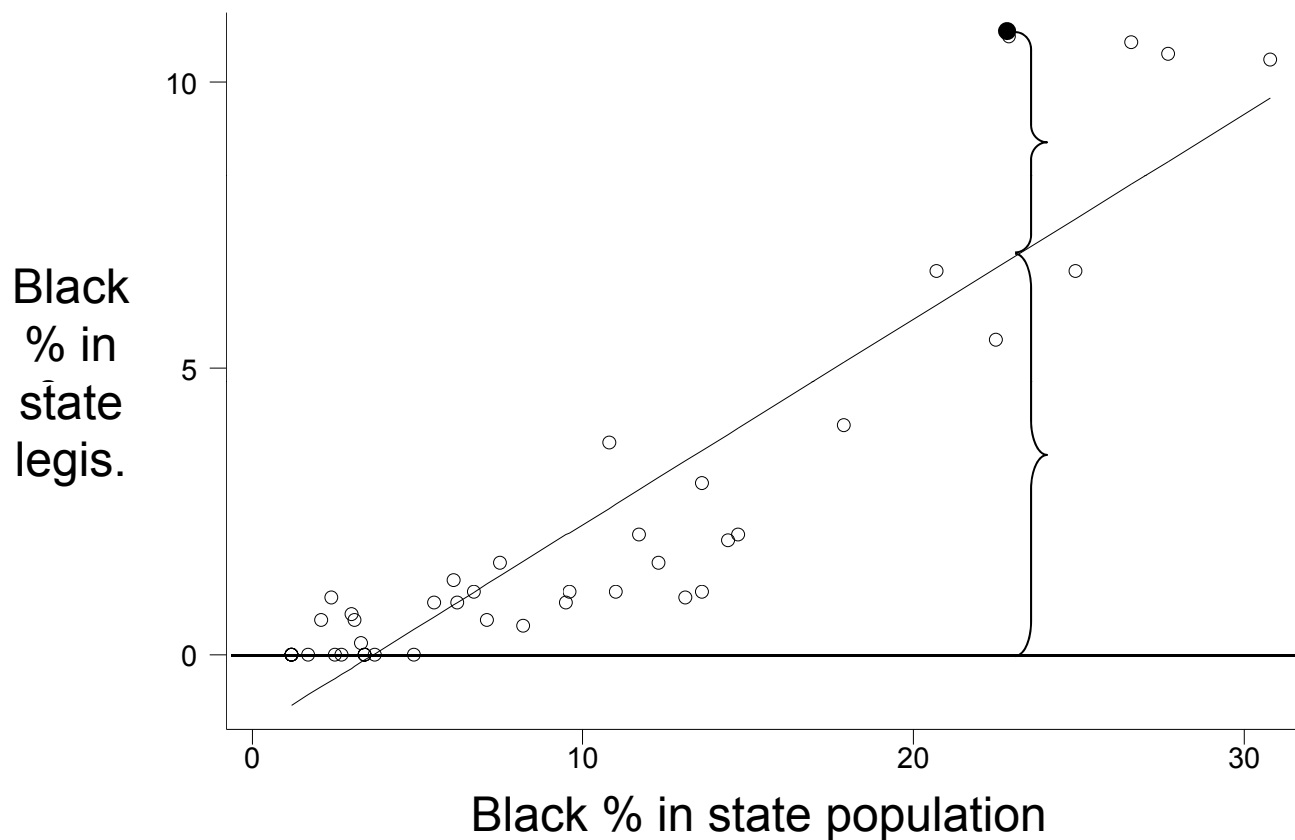
1. Pick a value of Y_i



$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$$

How did we get that line?

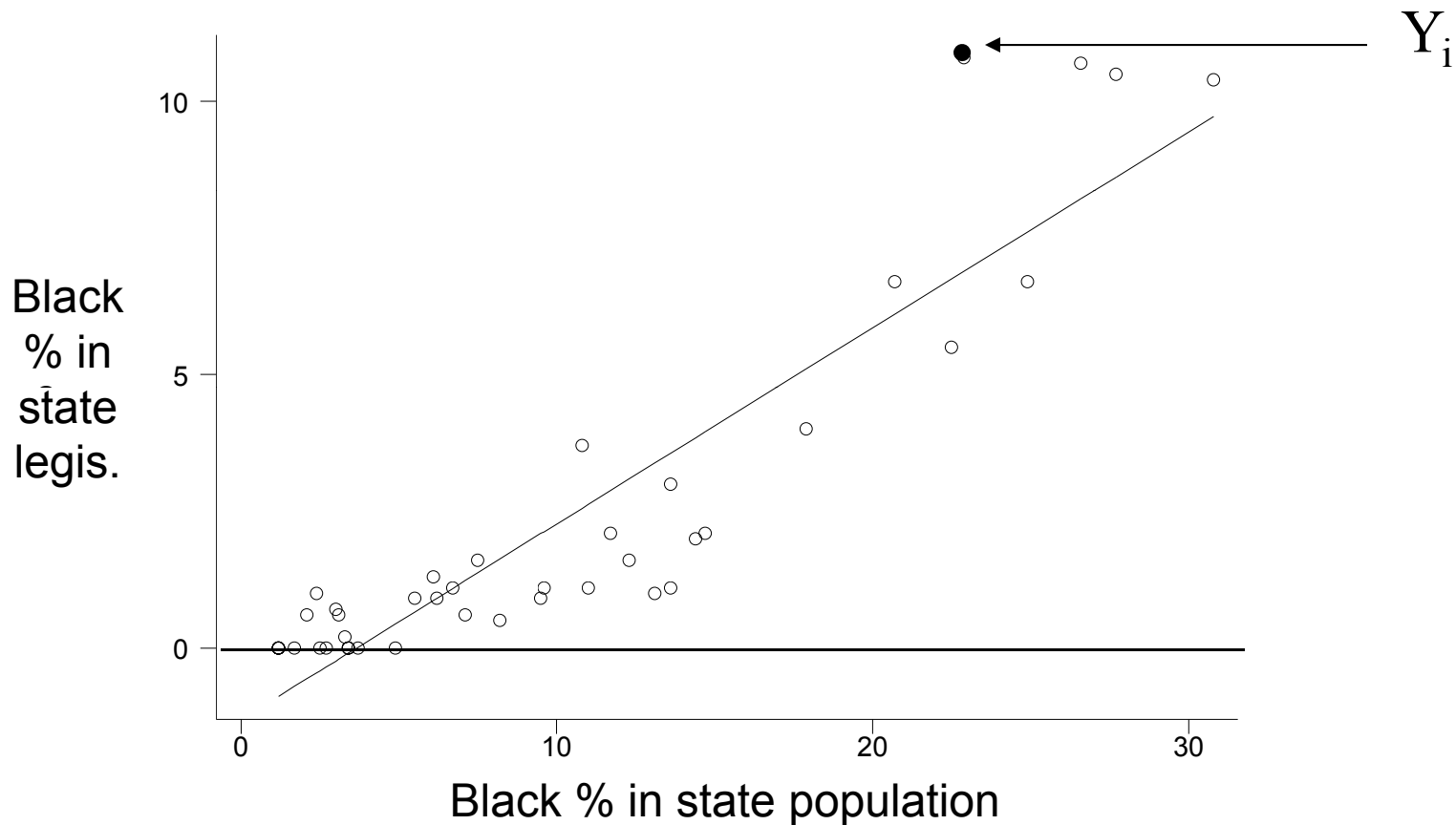
2. Decompose Y_i into two parts



$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

How did we get that line?

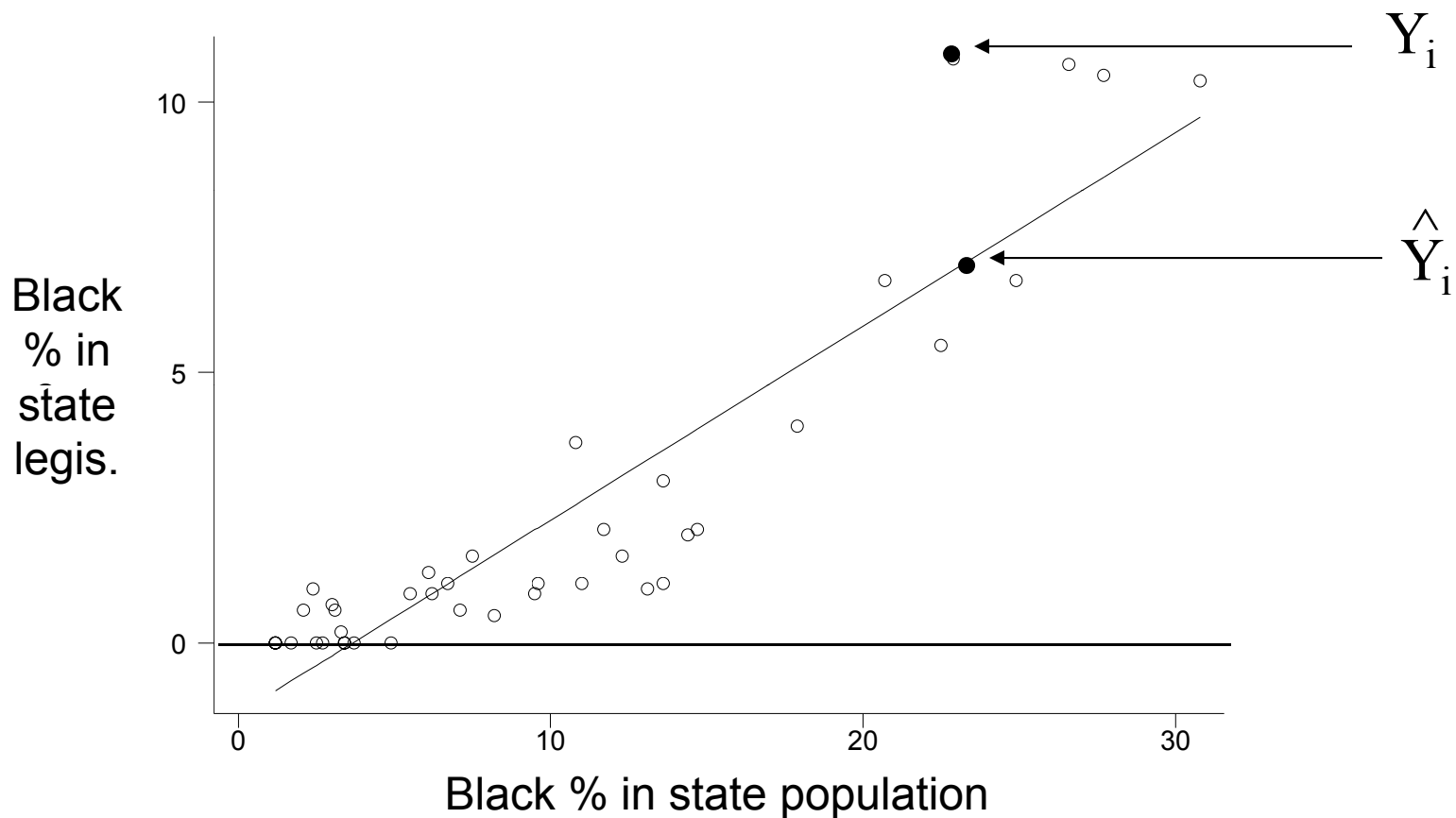
3. Label the points



$$Y_i = (\beta_0 + \beta_1 X_i) + \varepsilon_i$$

How did we get that line?

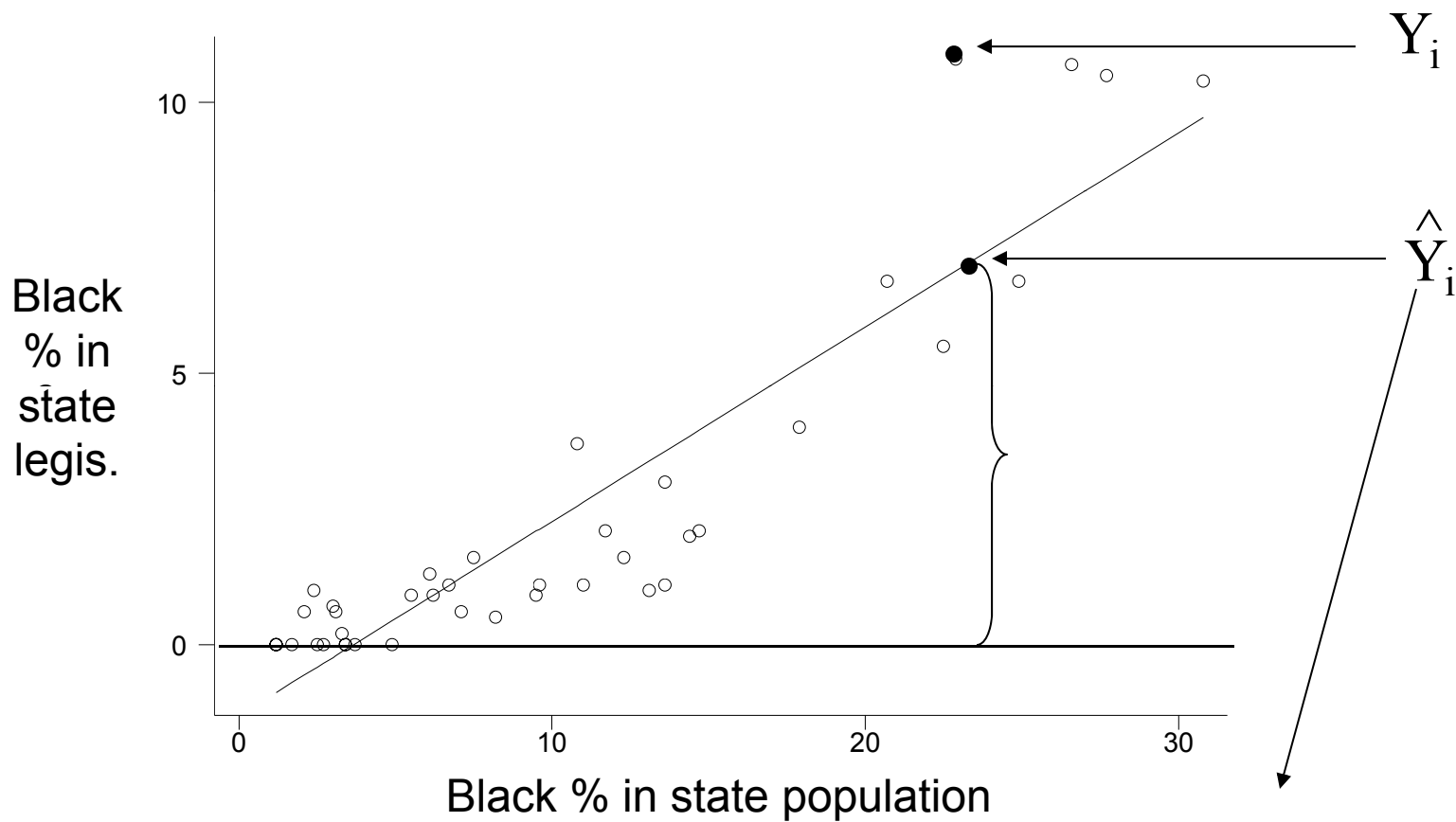
3. Label the points



$$Y_i = (\beta_0 + \beta_1 X_i) + \varepsilon_i$$

How did we get that line?

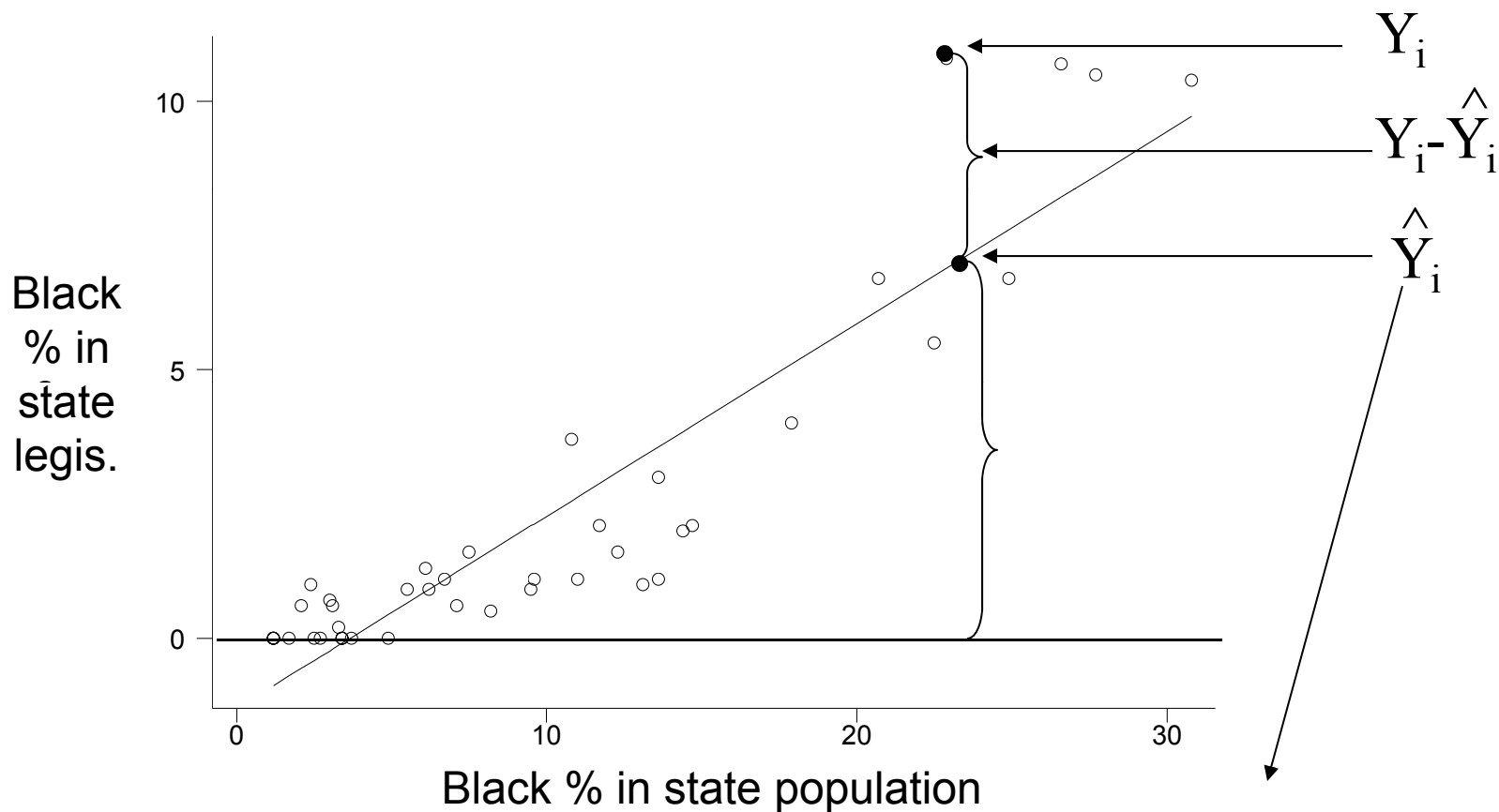
3. Label the points



$$Y_i = (\beta_0 + \beta_1 X_i) + \varepsilon_i$$

How did we get that line?

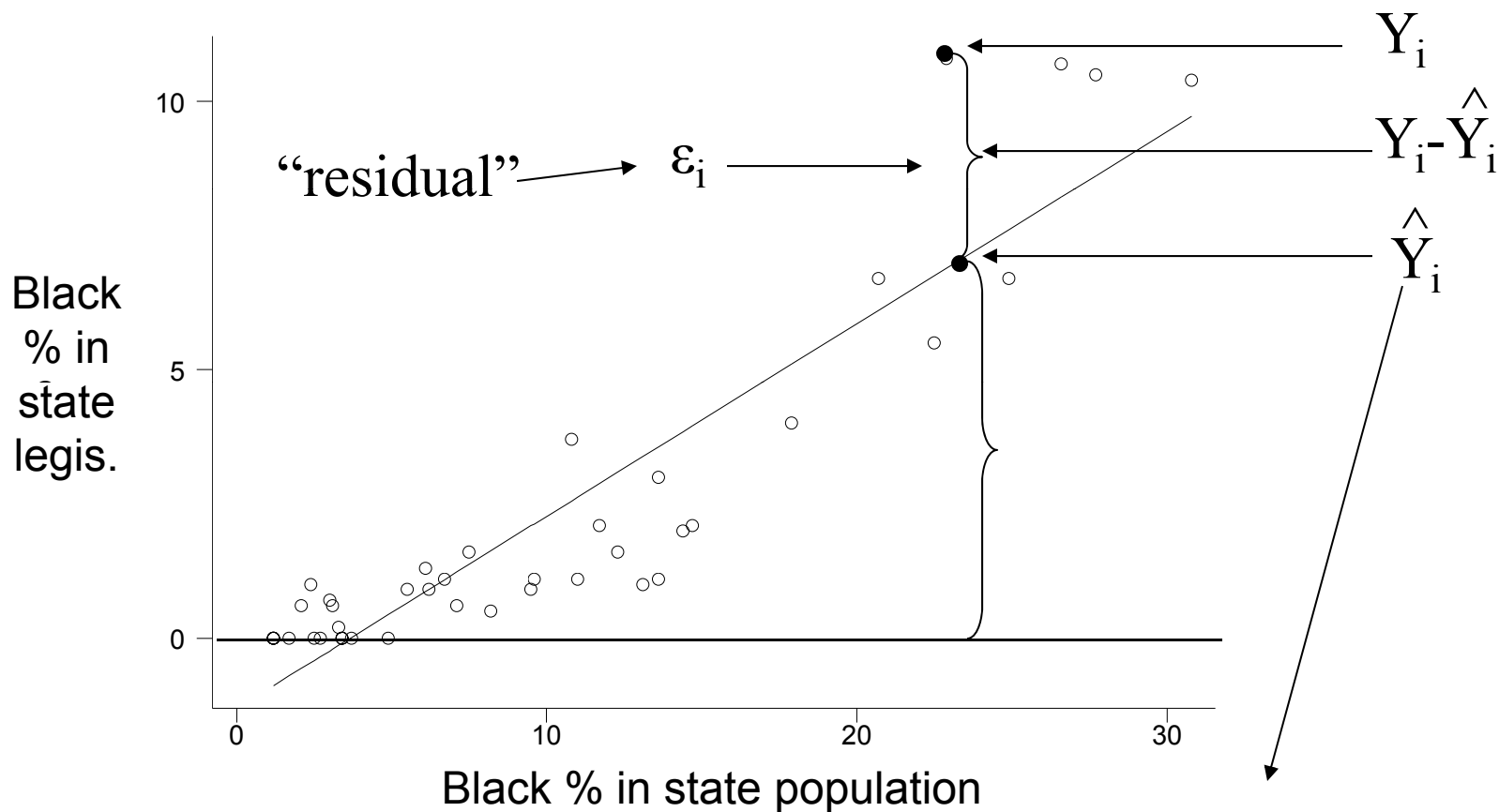
3. Label the points




$$Y_i = (\beta_0 + \beta_1 X_i) + \varepsilon_i$$

How did we get that line?

3. Label the points



$$Y_i = (\beta_0 + \beta_1 X_i) + \epsilon_i$$



What is ε_i ? (sometimes u_i)

- Wrong functional form
- Measurement error
- Stochastic component in Y
- Unmeasured influences on Y

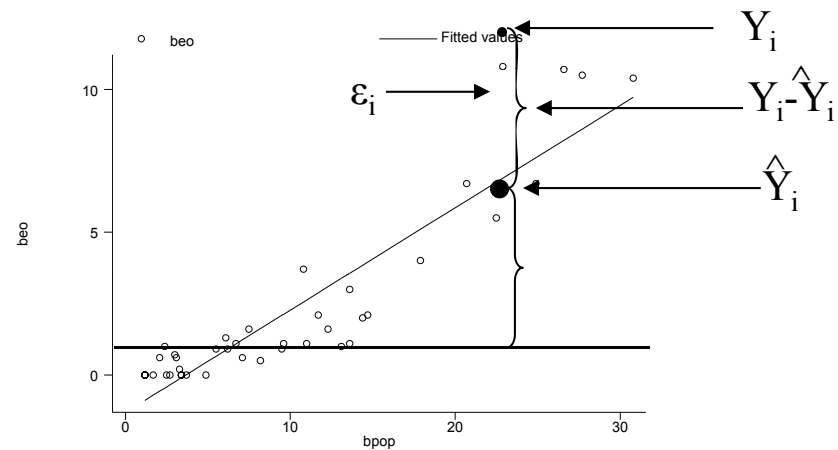
$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$$

The Method of Least Squares


Pick β_0 and β_1 to minimize $\sum_{i=1}^n \varepsilon_i^2$

$$\sum_{i=1}^n (Y_i - \hat{Y}_i)^2 \text{ or}$$

$$\sum_{i=1}^n (Y_i - \beta_0 - \beta_1 X_i)^2$$



$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$$



Solve for $\frac{\partial \sum_{i=1}^n (Y_i - \beta_0 - \beta_1 X_i)^2}{\partial \beta_1} = 0$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^n (\bar{Y} - Y_i)(\bar{X} - X_i)}{\sum_{i=1}^n (\bar{X} - X_i)^2} \quad \text{or}$$

$$\frac{\text{cov}(X, Y)}{\text{var}(X)}$$

Remember this for the problem set!



Regression commands in STATA

- `reg depvar expvars`
 - E.g., `reg y x`
 - E.g., `reg beo bpop`

- Making predictions from regression lines
 - `predict newvar`
 - `predict newvar, resid`
 - *newvar* will now equal ε_i

Black elected officials example

```
. reg beo bpop
```

Source	SS	df	MS	Number of obs = 41		
Model	351.26542	1	351.26542	F(1, 39)	=	202.56
Residual	67.6326195	39	1.73416973	Prob > F	=	0.0000
-----				R-squared	=	0.8385
-----				Adj R-squared	=	0.8344
Total	18.898039	40	10.472451	Root MSE	=	1.3169

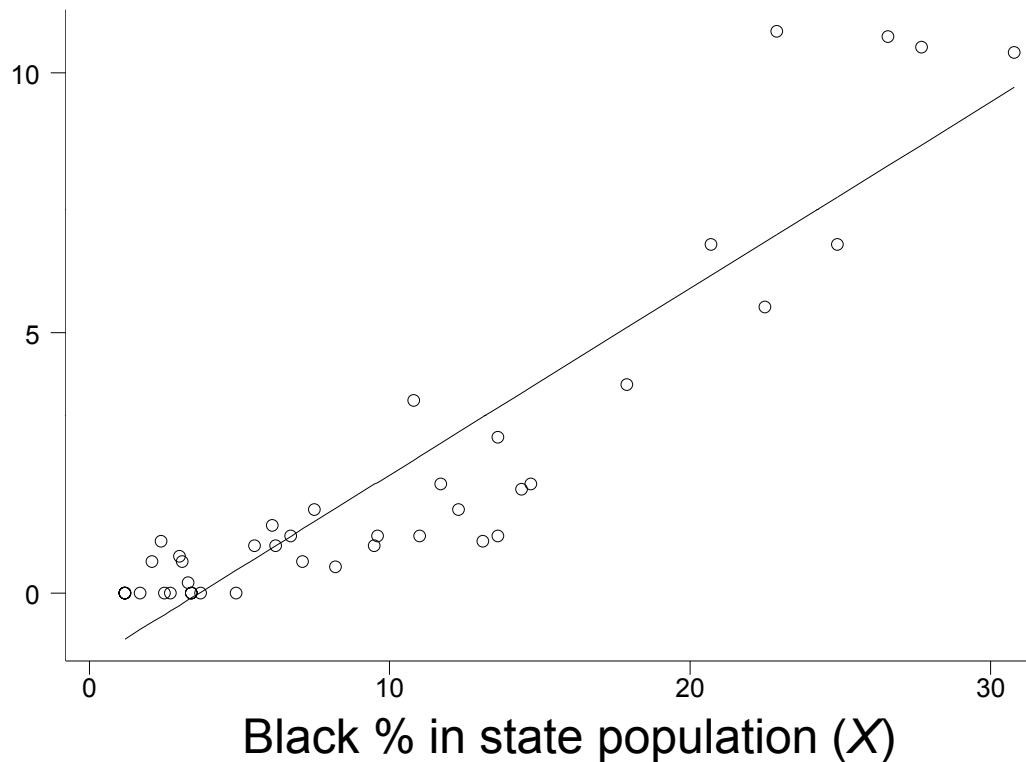
beo	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
bpop	.3586751	.0251876	14.23	0.000	.3075284	.4094219
_cons	-1.314892	.3277508	-4.01	0.000	-1.977831	-.6519535

Always include interpretation in your presentations and papers

Interpretation: a one percentage point increase in black population leads to a .36 percentage point increase in black composition in the legislature

The Linear Relationship between African American Population & Black Legislators

Black % in
state
legislatures
(Y)



$$\beta_0 = -1.31$$

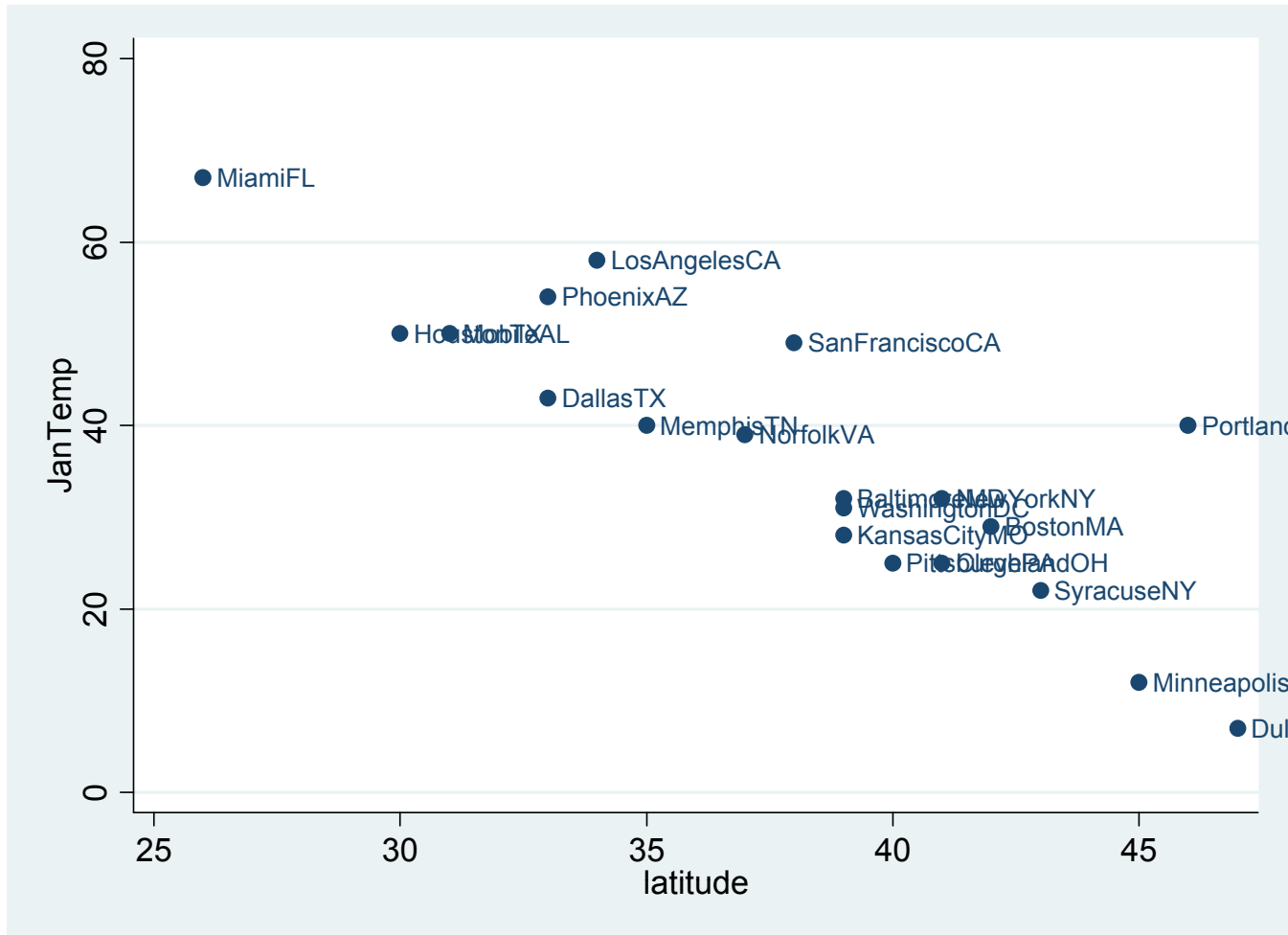
$$\beta_1 = 0.359$$

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$$



More regression examples

Temperature and Latitude



```
scatter JanTemp latitude, mlabel(city)
```

```
. reg jantemp latitude
```

Source	SS	df	MS	Number of obs =	20
Model	3250.72219	1	3250.72219	F(1, 18) =	49.34
Residual	1185.82781	18	65.8793228	Prob > F =	0.0000
Total	4436.55	19	233.502632	R-squared =	0.7327
				Adj R-squared =	0.7179
				Root MSE =	8.1166

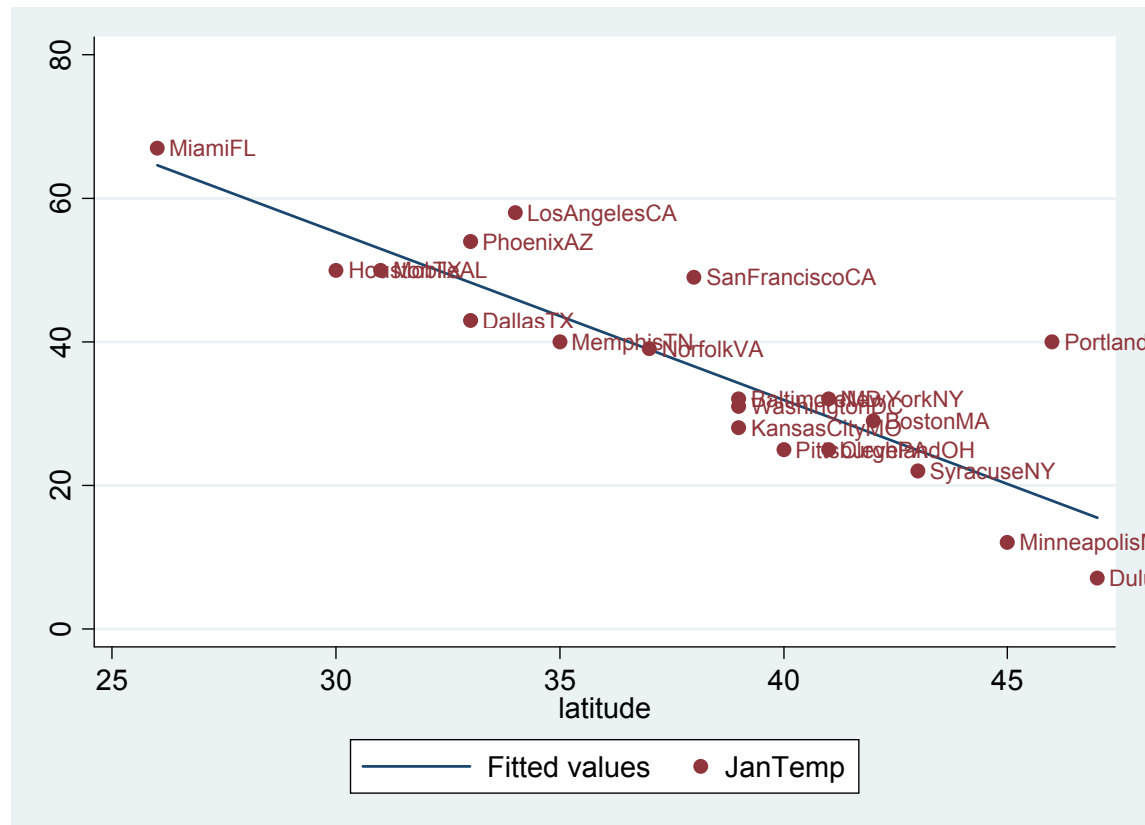
jantemp	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
latitude	-2.341428	.3333232	-7.02	0.000	-3.041714 -1.641142
_cons	125.5072	12.77915	9.82	0.000	98.65921 152.3552

Interpretation: a one point increase in latitude is associated with a 2.3 decrease in average temperature (in Fahrenheit).

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$$

How to add a regression line:

Stata command: `lfit`



```
scatter JanTemp latitude, mlabel(city) || lfit JanTemp latitude
```

or often better

```
scatter JanTemp latitude, mlabel(city) m(i) || lfit JanTemp latitude
```

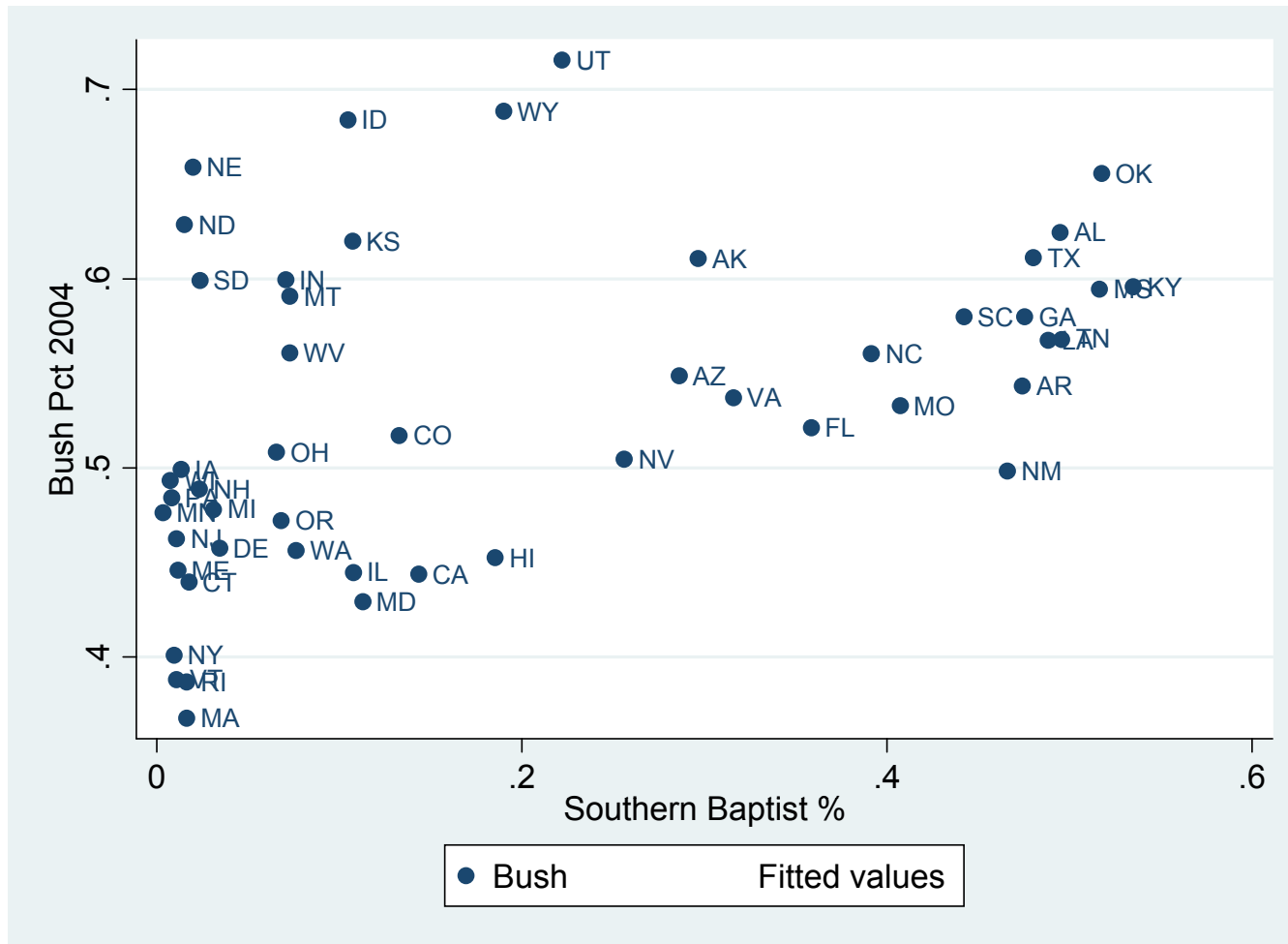



Presenting regression results

Brief aside

- First, show scatter plot
 - Label data points (if possible)
 - Include best-fit line
- Second, show regression table
 - Assess statistical significance with confidence interval or p-value
 - Assess robustness to control variables
(internal validity: nonrandom selection)

Bush vote and Southern Baptists



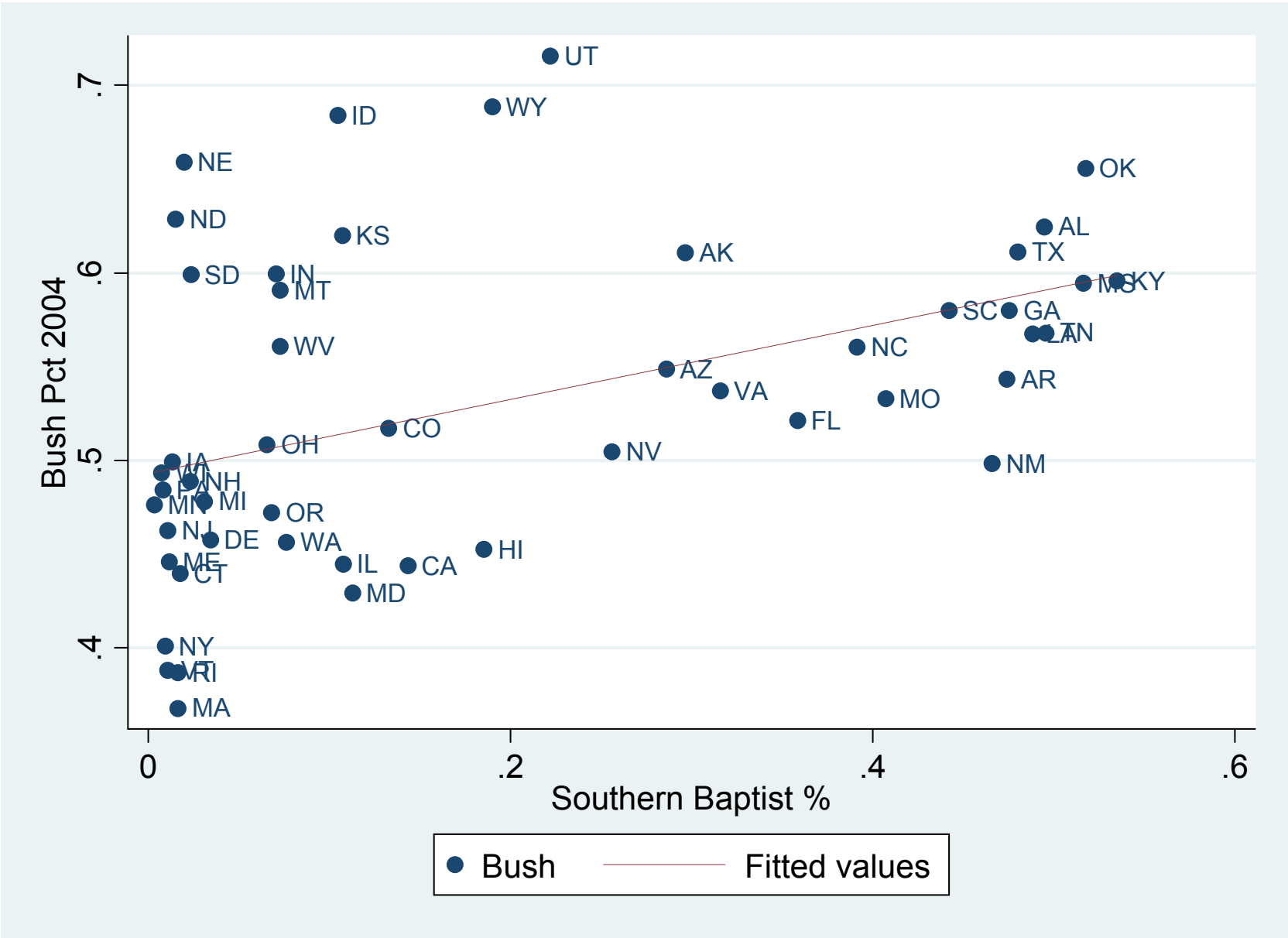
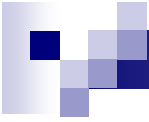
```
. reg bush sbc_mpct [aw=votes]
(sum of wgt is 1.2207e+08)
```

Source	SS	df	MS			
Model	.118925068	1	.118925068	Number of obs =	50	
Residual	.142084951	48	.002960103	F(1, 48) =	40.18	
Total	.261010018	49	.005326735	Prob > F =	0.0000	
				R-squared =	0.4556	
				Adj R-squared =	0.4443	
				Root MSE =	.05441	

bush	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
sbc_mpct	.261779	.0413001	6.34	0.000	.1787395	.3448185
_cons	.4563507	.0112155	40.69	0.000	.4338004	.4789011

Coefficient interpretation:

- A one percentage point increase in Baptist percentage is associated with a .26 percentage point increase in Bush vote share at the state level.



Interpreting confidence interval

```
. reg bush sbc_mpct [aw=votes]
(sum of wgt is 1.2207e+08)
```

Source	SS	df	MS	Number of obs =	50
Model	.118925068	1	.118925068	F(1, 48) =	40.18
Residual	.142084951	48	.002960103	Prob > F =	0.0000
Total	.261010018	49	.005326735	R-squared =	0.4556
				Adj R-squared =	0.4443
				Root MSE =	.05441

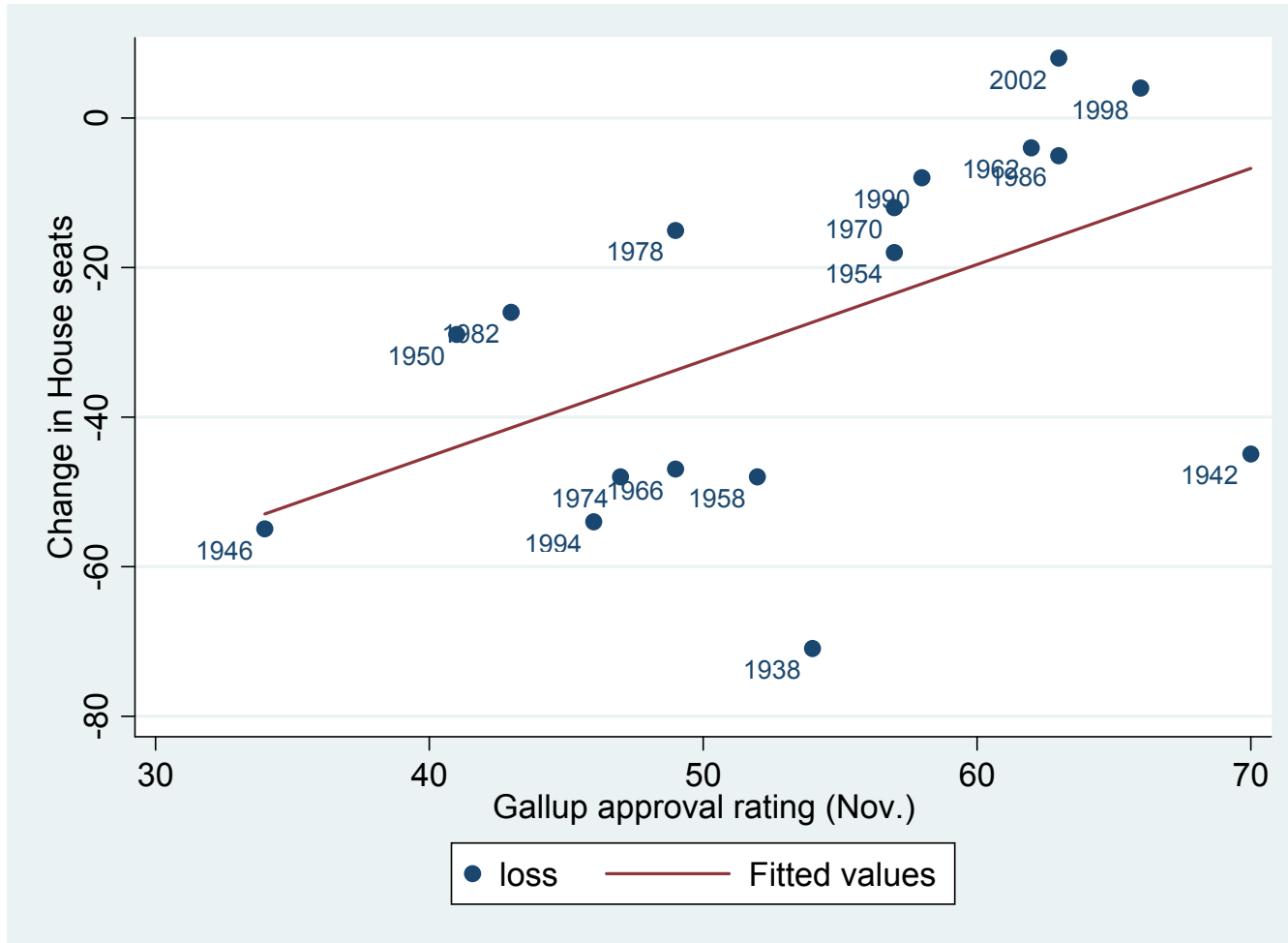
bush	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
sbc_mpct	.261779	.0413001	6.34	0.000	.1787395 .3448185
_cons	.4563507	.0112155	40.69	0.000	.4338004 .4789011

Coefficient interpretation:

- A 1 percentage point increase in Baptist percentage is associated with a .26 percentage point increase in Bush vote share at the state level.

Confidence interval interpretation

- The 95% confidence interval lies between .18 and .34.



```
. reg loss gallup
```

Source	SS	df	MS			
Model	2493.96962	1	2493.96962	Number of obs =	17	
Residual	6564.50097	15	437.633398	F(1, 15) =	5.70	
Total	9058.47059	16	566.154412	Prob > F =	0.0306	
				R-squared =	0.2753	
				Adj R-squared =	0.2270	
				Root MSE =	20.92	

Seats	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
gallup	1.283411	.53762	2.39	0.031	.1375011	2.429321
cons	-96.59926	29.25347	-3.30	0.005	-158.9516	-34.24697

Coefficient interpretation:

- A 1 percentage point increase in presidential approval is associated with an avg. of 1.28 more seats won by the president's party in the midterm.

Confidence interval interpretation

- The 95% confidence interval lies between .14 and 2.43.



Additional regression in bivariate relationship topics

- Residuals
- Comparing coefficients
- Functional form
- Goodness of fit (R^2 and SER)
- Correlation
- Discrete DV, discrete EV
- Using the appropriate graph/table



Residuals

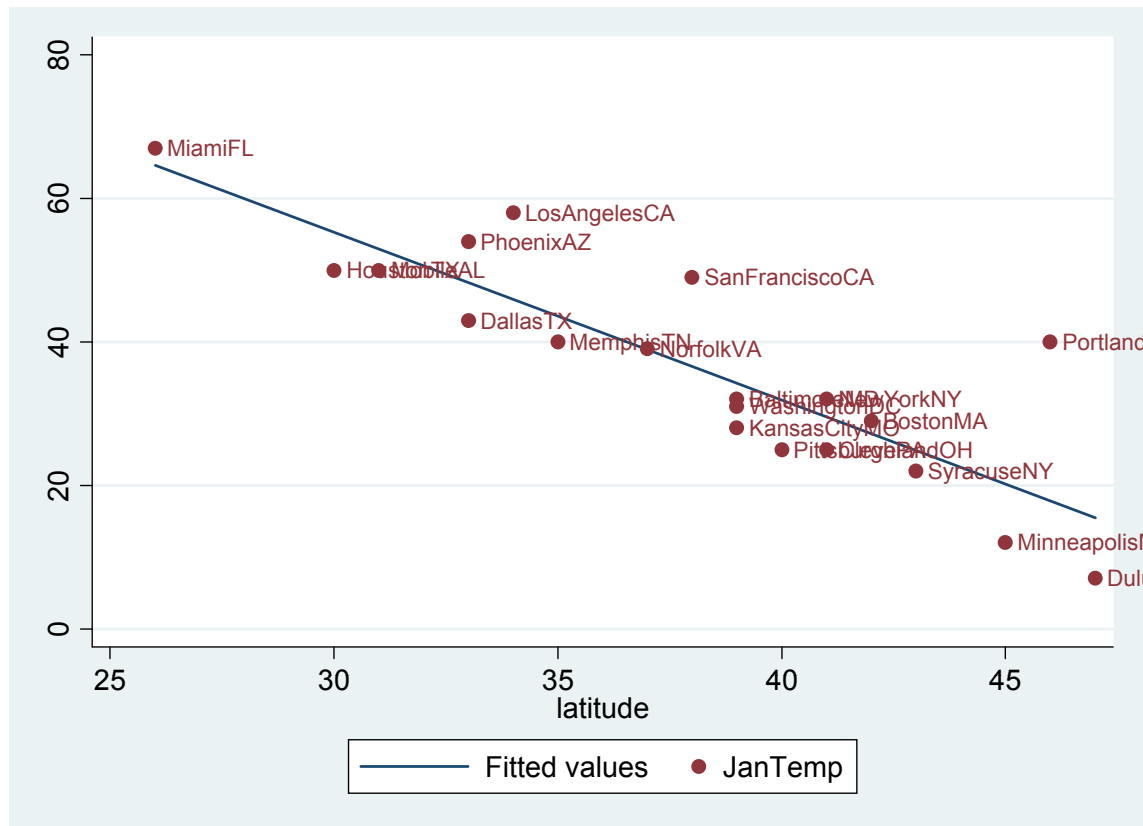


Residuals

$$e_i = Y_i - B_0 - B_1 X_i$$

One important numerical property of residuals

- The sum of the residuals is zero



Generating predictions and residuals

```
. reg jantemp latitude
```

Source	SS	df	MS			
Model	3250.72219	1	3250.72219	Number of obs =	20	
Residual	1185.82781	18	65.8793228	F(1, 18) =	49.34	
				Prob > F =	0.0000	
				R-squared =	0.7327	
				Adj R-squared =	0.7179	
				Root MSE =	8.1166	
Total	4436.55	19	233.502632			

jantemp	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
latitude	-2.341428	.3333232	-7.02	0.000	-3.041714	-1.641142
_cons	125.5072	12.77915	9.82	0.000	98.65921	152.3552

```
. predict py  
(option xb assumed; fitted values)
```

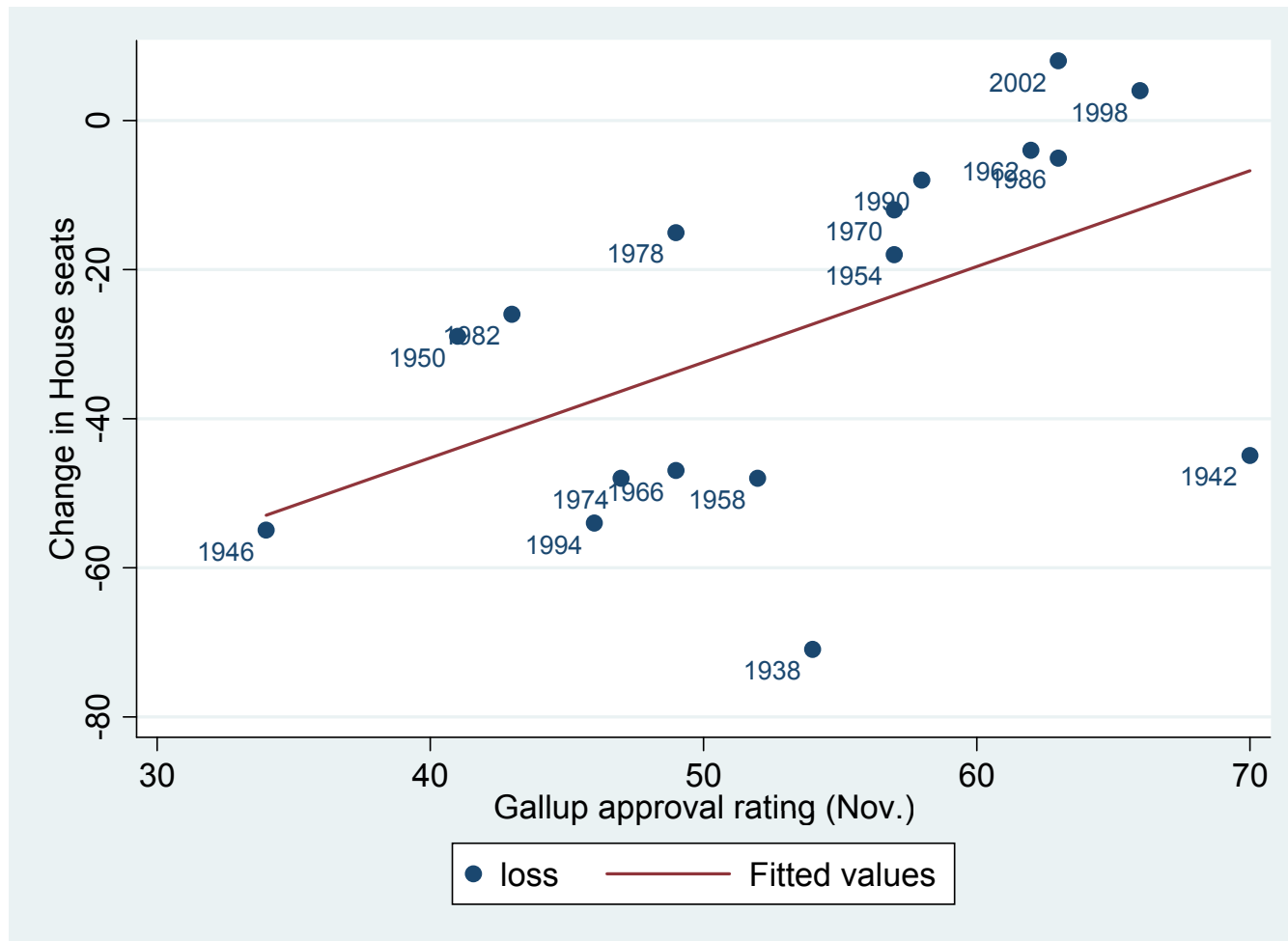
```
. predict ry, resid
```

gsort -ry

. list city jantemp py ry

	city	jantemp	py	ry
1.	PortlandOR	40	17.8015	22.1985
2.	SanFranciscoCA	49	36.53293	12.46707
3.	LosAngelesCA	58	45.89864	12.10136
4.	PhoenixAZ	54	48.24007	5.759929
5.	NewYorkNY	32	29.50864	2.491357
6.	MiamiFL	67	64.63007	2.36993
7.	BostonMA	29	27.16722	1.832785
8.	NorfolkVA	39	38.87436	.125643
9.	BaltimoreMD	32	34.1915	-2.1915
10.	SyracuseNY	22	24.82579	-2.825786
11.	MobileAL	50	52.92293	-2.922928
12.	WashingtonDC	31	34.1915	-3.1915
13.	MemphisTN	40	43.55721	-3.557214
14.	ClevelandOH	25	29.50864	-4.508643
15.	DallasTX	43	48.24007	-5.240071
16.	HoustonTX	50	55.26435	-5.264356
17.	KansasCityMO	28	34.1915	-6.1915
18.	PittsburghPA	25	31.85007	-6.850072
19.	MinneapolisMN	12	20.14293	-8.142929
20.	DuluthMN	7	15.46007	8.460073

Use residuals to diagnose potential problems



```
. reg loss gallup
```

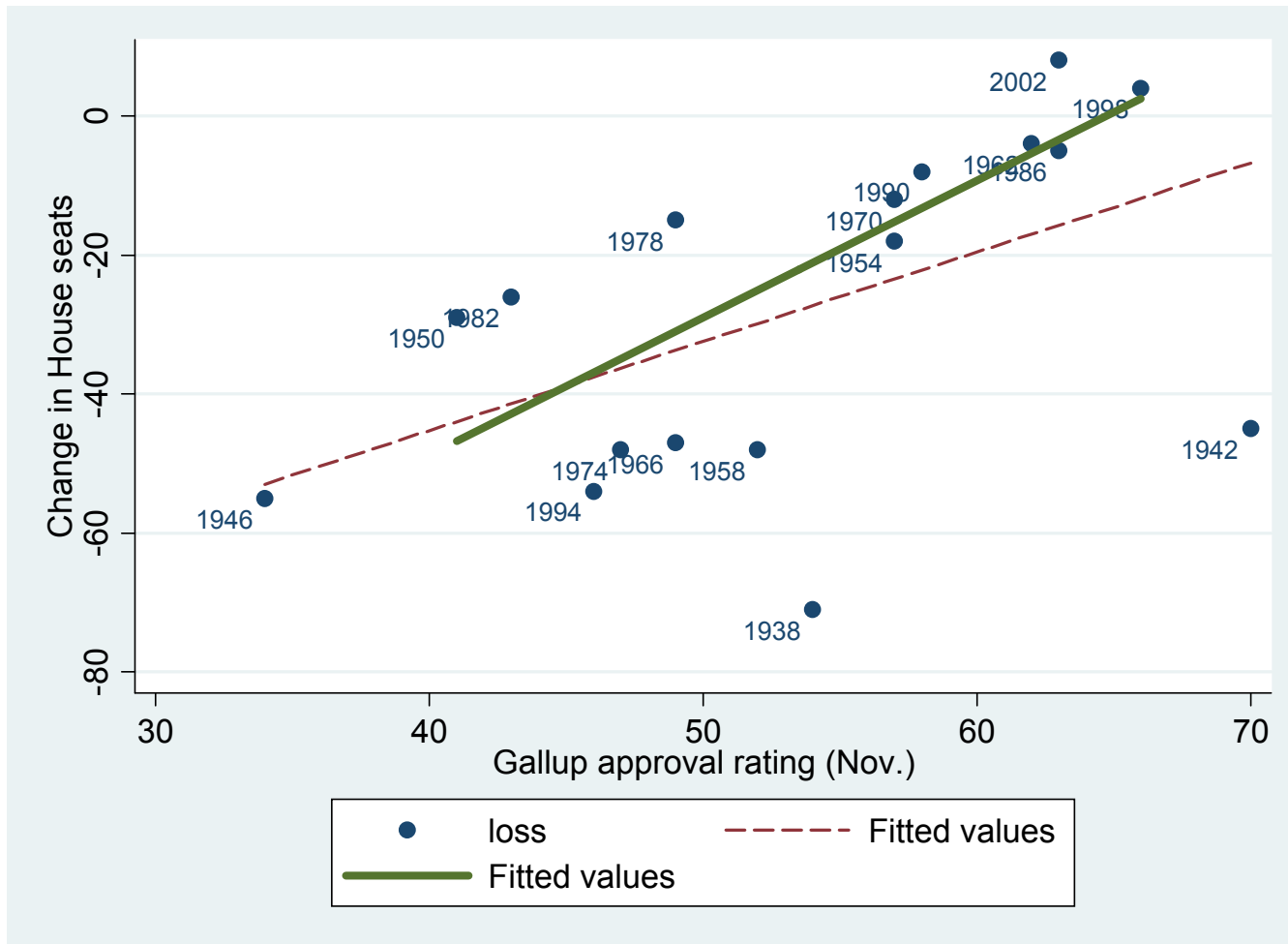
Source	SS	df	MS	Number of obs = 17		
Model	2493.96962	1	2493.96962	F(1, 15)	=	5.70
Residual	6564.50097	15	437.633398	Prob > F	=	0.0306
-----+-----				R-squared	=	0.2753
Total	9058.47059	16	566.154412	Adj R-squared	=	0.2270
-----+-----				Root MSE	=	20.92
Seats	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
gallup	1.283411	53762	2.39	0.031	1375011	2.429321
_cons	-96.59926	29.25347	-3.30	0.005	-158.9516	-34.24697

```
. reg loss gallup if year>1946
```

Source	SS	df	MS	Number of obs = 14		
Model	3332.58872	1	3332.58872	F(1, 12)	=	17.53
Residual	2280.83985	12	190.069988	Prob > F	=	0.0013
-----+-----				R-squared	=	0.5937
Total	5613.42857	13	431.802198	Adj R-squared	=	0.5598
-----+-----				Root MSE	=	13.787
seats	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
gallup	1.96812	.4700211	4.19	0.001	.9440315	2.992208
_cons	-127.4281	25.54753	-4.99	0.000	-183.0914	-71.76486



```
scatter loss gallup, mlabel(year) || lfit loss gallup || lfit loss gallup if year >1946
```





Comparing regression coefficients

- As a general rule:
 - Code all your variables to vary between 0 and 1
 - That is, minimum = 0, maximum = 1
 - Regression coefficients then represent the effect of shifting from the minimum to the maximum.
 - This allows you to more easily compare the relative importance of coefficients.



How to recode variables to 0-1 scale

- Party ID example: pid7
- Usually varies from
 - 1 (strong_R epublican)
 - to 8 (strong Democrat)
 - sometimes 0 needs to be recoded to missing (“.”).
- Stata code?
 - `replace pid7 = (pid7-1)/7`



Regression interpretation with 0-1 scale

■ Continue with pid7 example

- regress natlecon pid7 (both recoded to 0-1 scales)*
- pid7 coefficient: $b = -.46$ (CCES data from 2006)
- Interpretation?
 - Shifting from being a strong Republican to a strong Democrat corresponds with a .46 drop in evaluations of the national economy (on the one-point national economy scale)

*natlecon originally coded so that 1 = excellent, 4 = poor, 5 = not sure



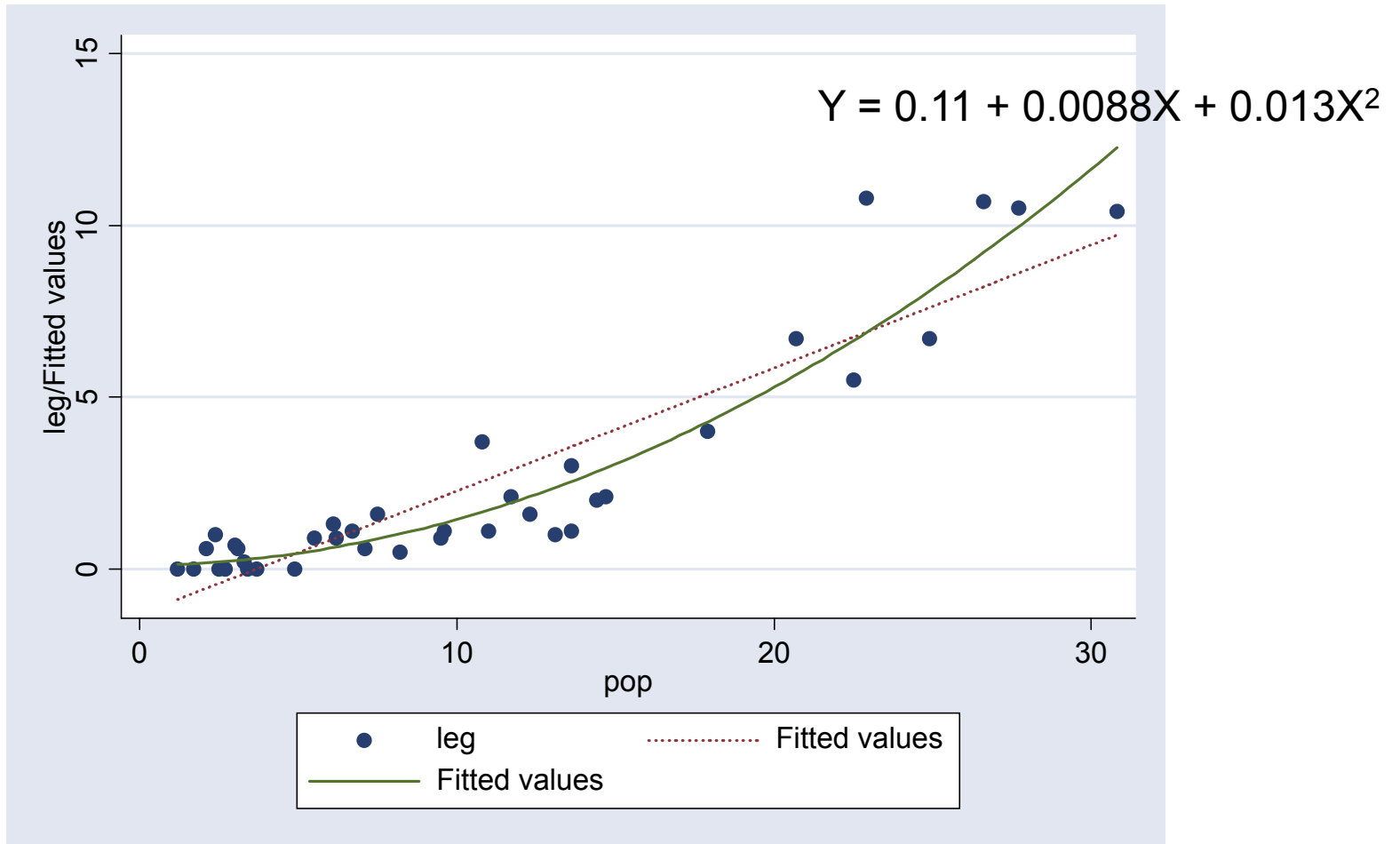
Functional Form



About the Functional Form

- Linear in the variables *vs.* linear in the parameters
 - $Y = a + bX + e$ (linear in both)
 - $Y = a + bX + cX^2 + e$ (linear in parms.)
 - $Y = a + X^b + e$ (linear in variables, not parms.)
- Regression must be linear in parameters

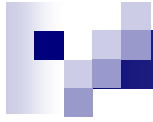
The Linear and Curvilinear Relationship between African American Population & Black Legislators



```
scatter beo pop || qfit beo pop
```

Log transformations (see Tufte, ch. 3)

$Y = a + bX + e$	$b = dY/dX$, or $b =$ the unit change in Y given a unit change in X	Typical case
$Y = a + b \ln X + e$	$b = dY/(dX/X)$, or $b =$ the unit change in Y given a % change in X	Log explanatory variable
$\ln Y = a + bX + e$	$b = (dY/Y)/dX$, or $b =$ the % change in Y given a unit change in X	Log dependent variable
$\ln Y = a + b \ln X + e$	$b = (dY/Y)/(dX/X)$, or $b =$ the % change in Y given a % change in X (elasticity)	Economic production

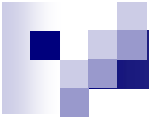


Goodness of regression fit

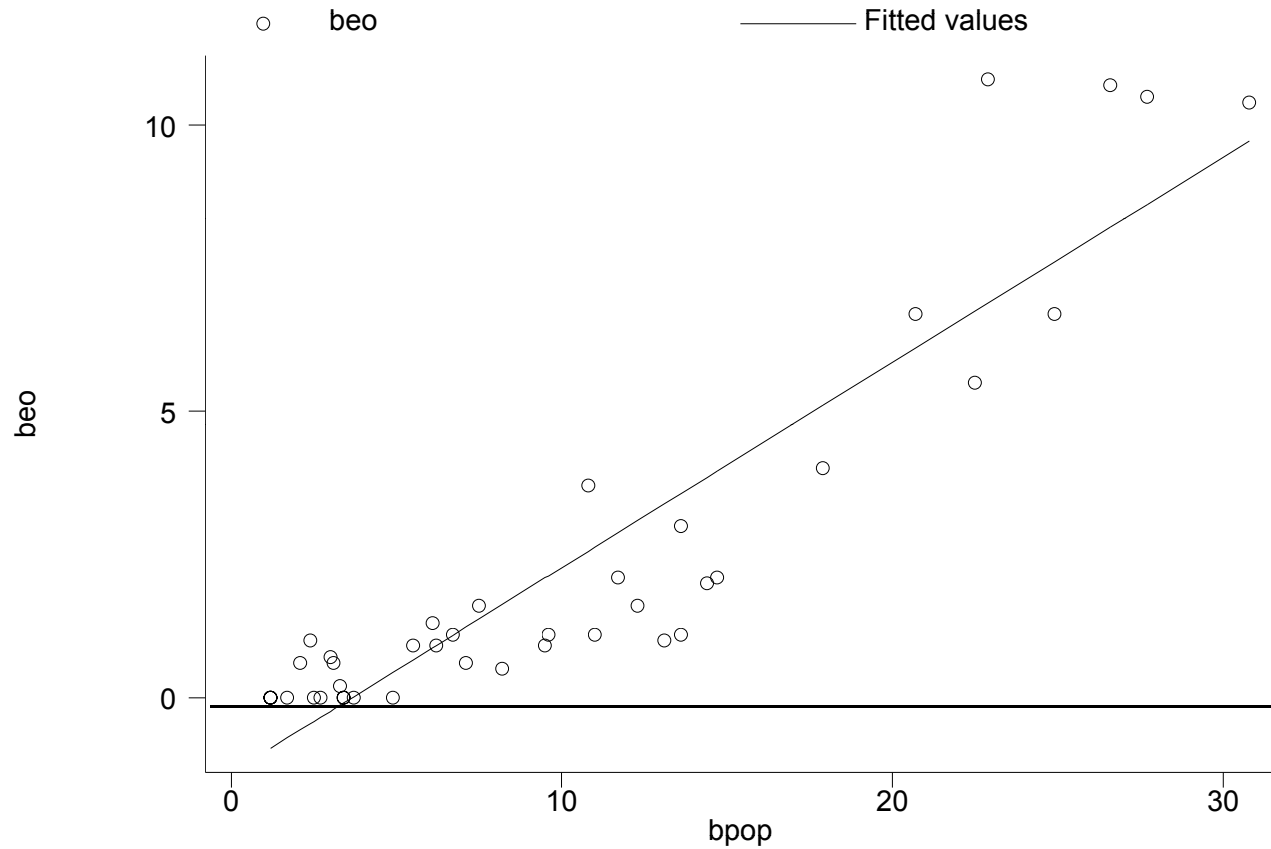


How “good” is the fitted line?

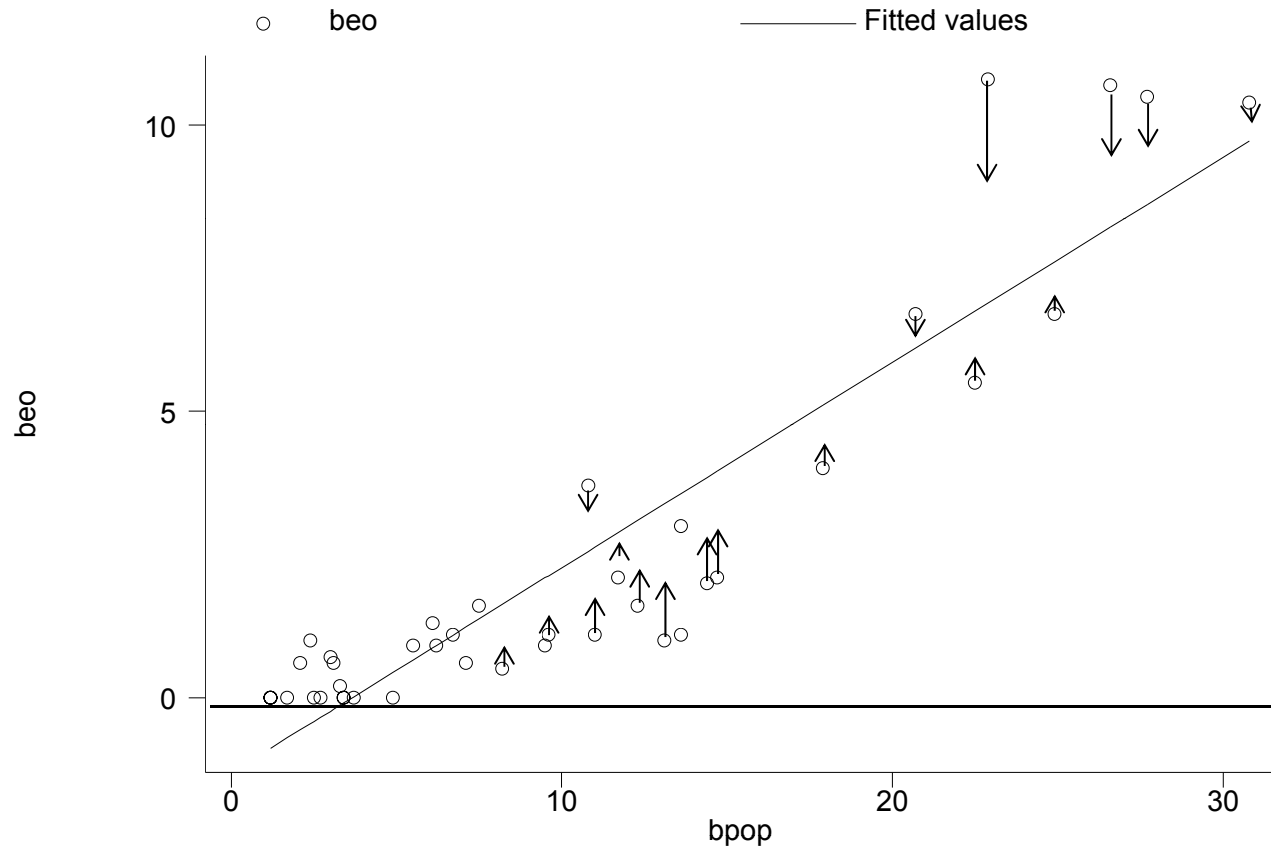
- Goodness-of-fit is often not relevant to research
- Goodness-of-fit receives too much emphasis
- Focus on
 - Substantive interpretation of coefficients (most important)
 - Statistical significance of coefficients (less important)
 - Confidence interval
 - Standard error of a coefficient
 - *t*-statistic: *coeff./s.e.*
- Nevertheless, you should know about
 - Standard Error of the Regression (SER)
 - Standard Error of the Estimate (SEE)
 - Also called Regrettably called Root Mean Squared Error (Root MSE) in Stata
 - R-squared (R^2)
 - Often not informative, use sparingly



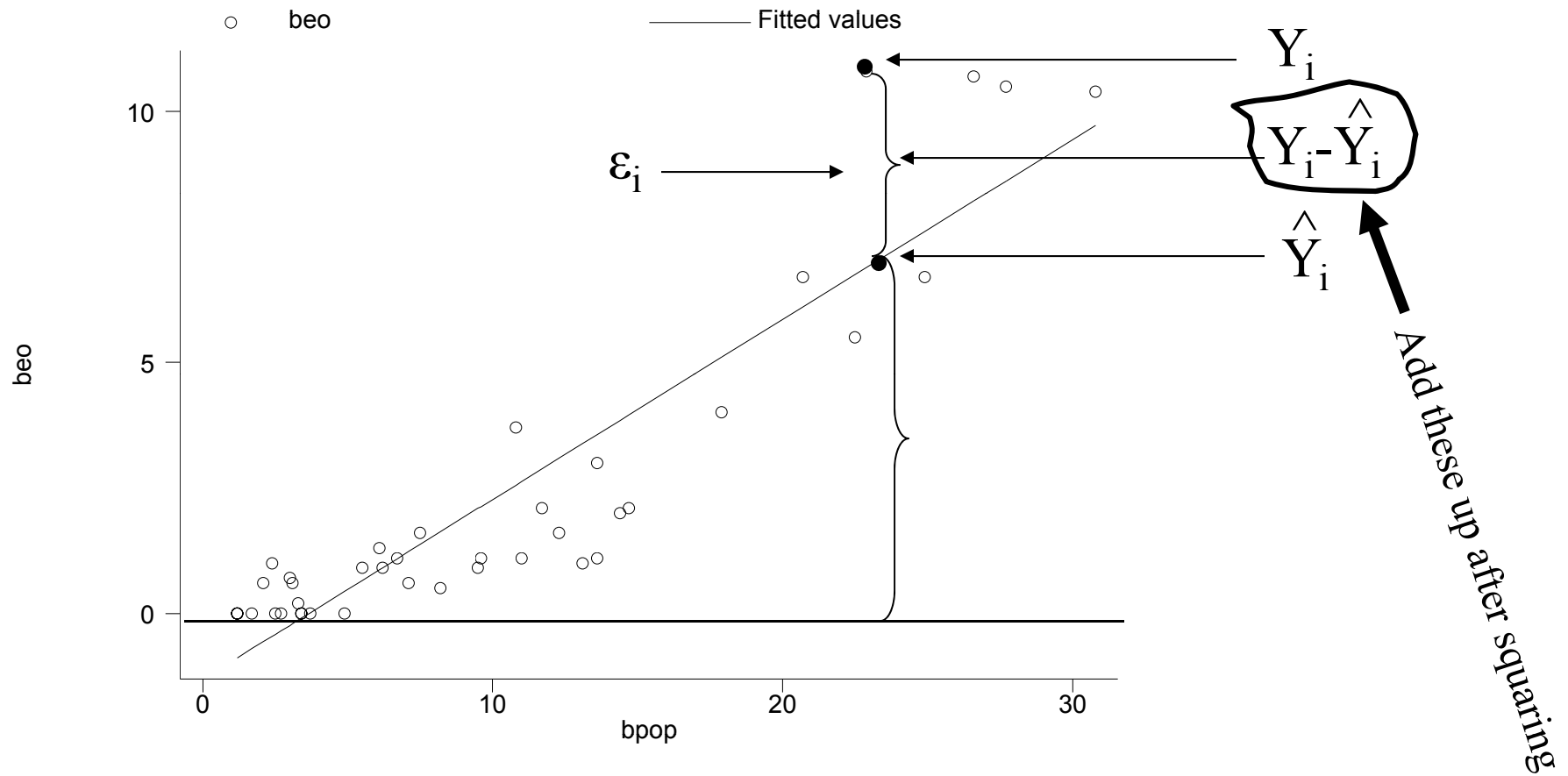
Standard Error of the Regression the idea



Standard Error of the Regression the idea



Standard Error of the Regression picture



Standard Error of the Regression (SER)

- or Standard Error of the Estimate
- or Root Mean Squared Error (Root MSE)

$$\sqrt{\frac{\sum_{i=1}^n (Y_i - \hat{Y}_i)^2}{d.f.}}$$

d.f. equals n minus the number of estimate coefficients (B s).
In bivariate regression case, $d.f. = n-2$.

SER interpretation called "Root MSE" in Stata

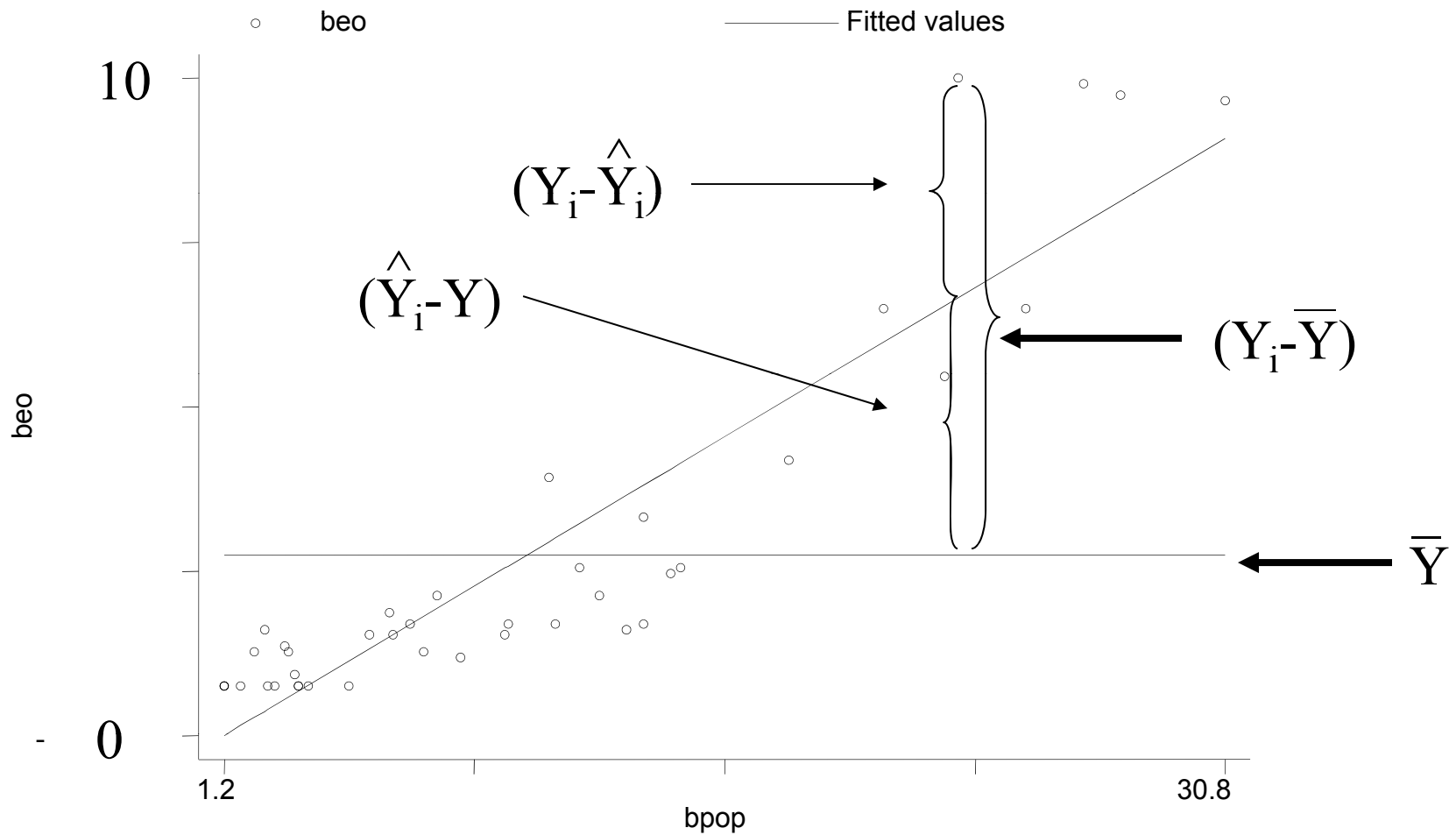
- On average, in-sample predictions will be off the mark by about one standard error of the regression

```
. reg beo bpop
```

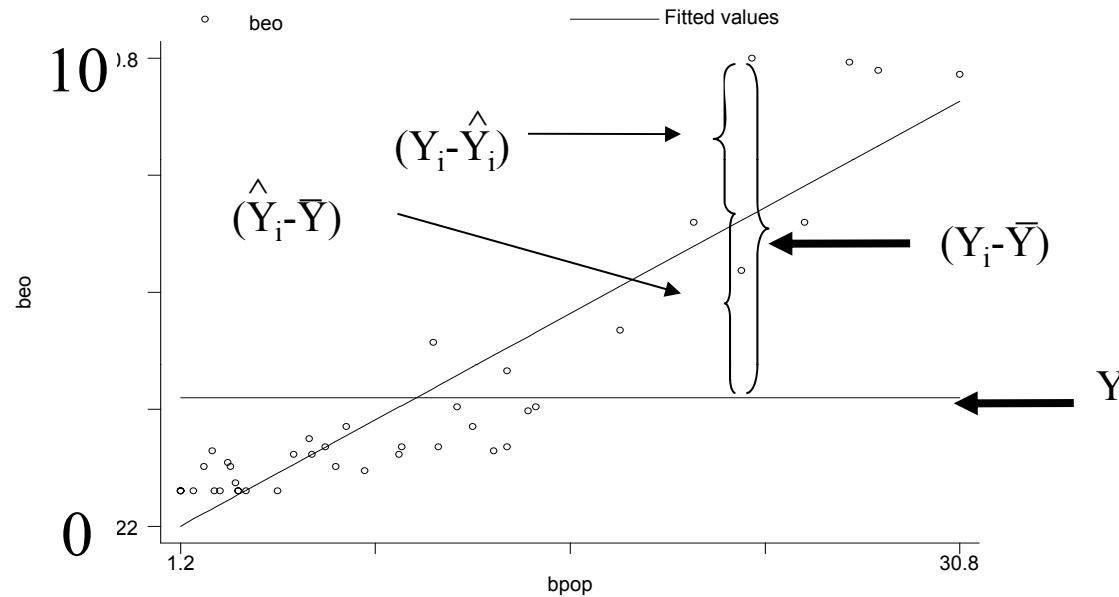
Source	SS	df	MS			
Model	351.26542	1	351.26542	Number of obs =	41	
Residual	67.6326195	39	1.73416973	F(1, 39) =	202.56	
Total	418.898039	40	10.472451	Prob > F =	0.0000	
				R-squared =	0.8385	
				Adj R-squared =	0.8344	
				Root MSE =	1.3169	

beo	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
bpop	.3586751	.0251876	14.23	0.000	.3075284	.4094219
_cons	-1.314892	.3277508	-4.01	0.000	-1.977831	-.6519535

R²: A less useful measure of fit



R²: A less useful measure of fit



$$\sum_{i=1}^n (Y_i - \bar{Y})^2 = \text{"total sum of squares"}$$

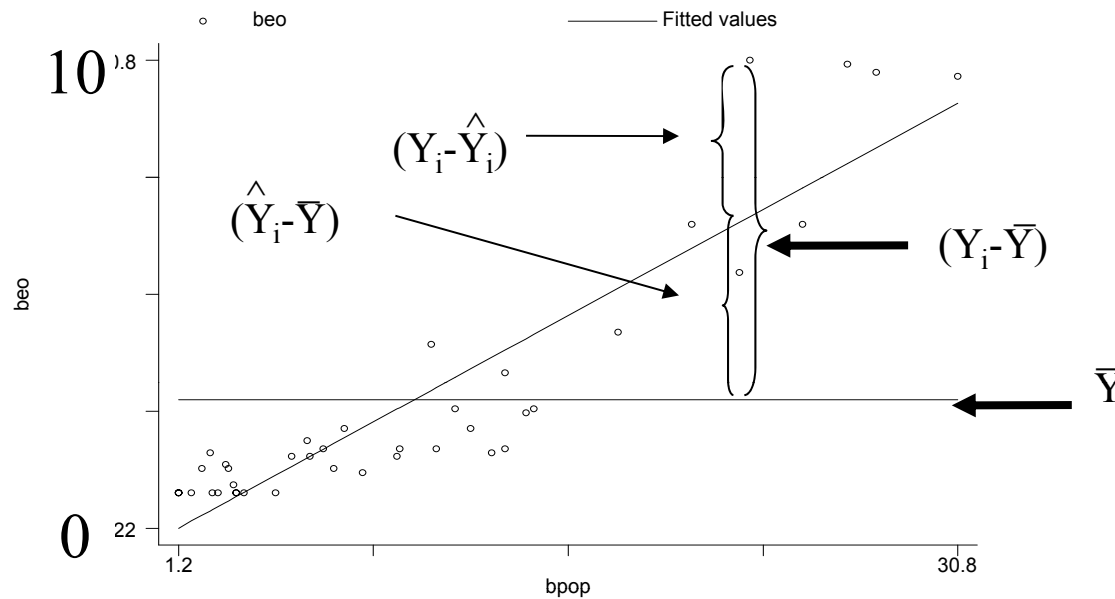
=

$$\sum_{i=1}^n (\hat{Y}_i - \bar{Y})^2 = \text{"regression sum of squares"}$$

+

$$\sum_{i=1}^n (Y_i - \hat{Y}_i)^2 = \text{"residual sum of squares"}$$

R-squared



$$r^2 = \frac{\sum_{i=1}^n (\hat{Y}_i - \bar{Y})^2}{\sum_{i=1}^n (Y_i - \bar{Y})^2} \quad \text{or}$$

pct. variance "explained"

Also called "coefficient of determination"

Interpreting SER (Root MSE) and R²

```
. reg bush sbc_mpct
```

Source	SS	df	MS			
Model	.069183833	1	.069183833	Number of obs =	50	
Residual	.280630922	48	.005846478	F(1, 48) =	11.83	
Total	.349814756	49	.007139077	Prob > F =	0.0012	
				R-squared =	0.1978	
				Adj R-squared =	0.1811	
				Root MSE =	.07646	

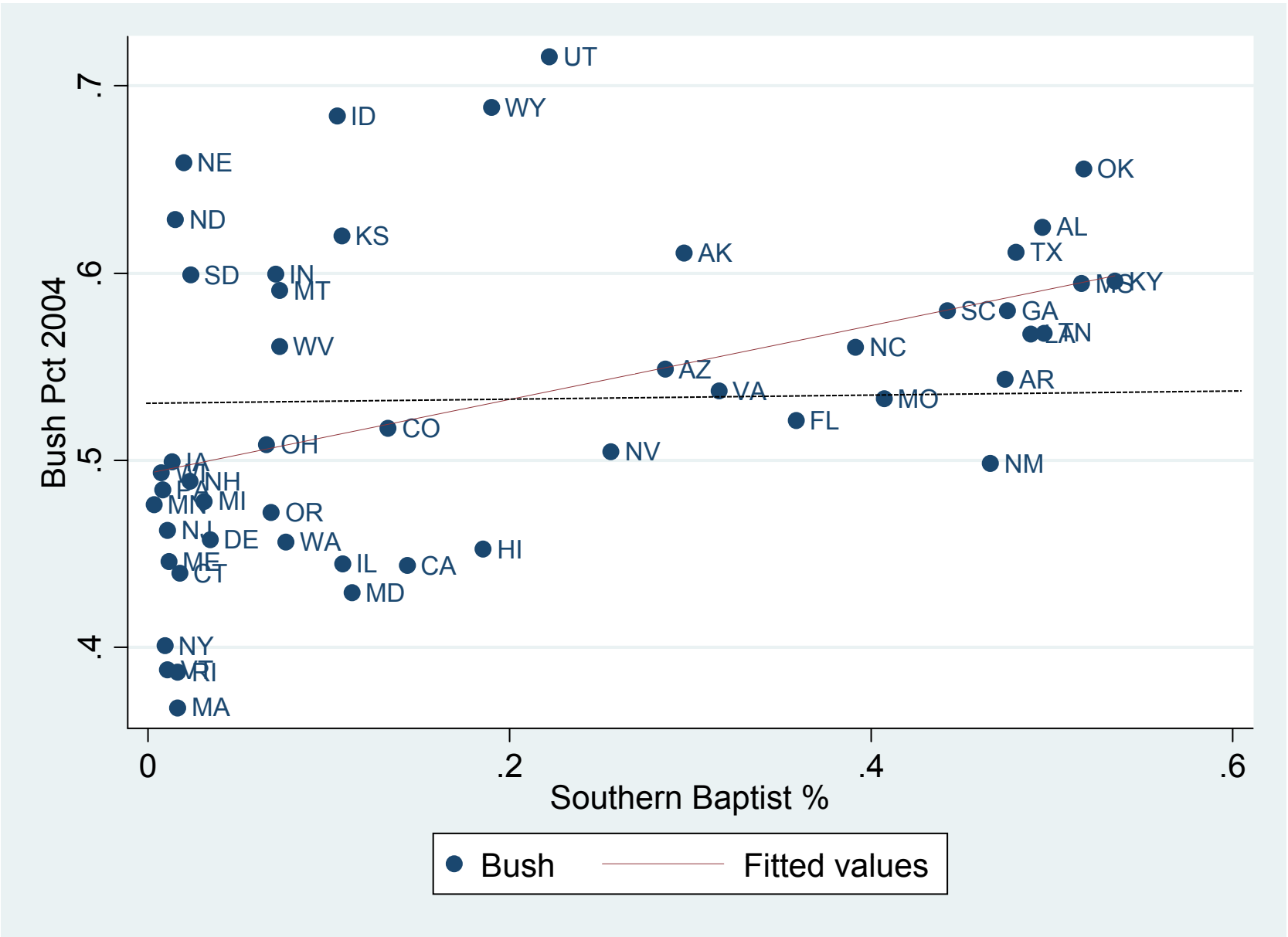
bush	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
sbc mpct	.196814	.0572138	3.44	0.001	.0817779	.3118501
_cons	.4931758	.0155007	31.82	0.000	.4620095	.524342

Interpreting SER (Root MSE):

- On average, in-sample predictions about Bush's vote share will be off the mark by about 7.6%

Interpreting R²

- Regression model explains about 19.8% of the variation in Bush vote.





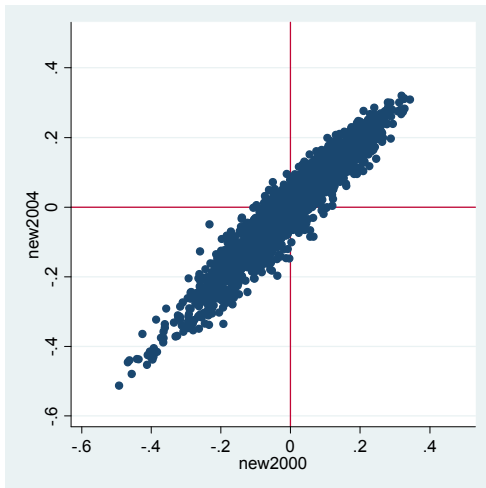
Correlation

Correlation

$$\text{Corr}(x, y) = \frac{\text{Cov}(x, y)}{\sigma_x \sigma_y} = r$$

$$\text{Corr}(\text{BushPct}_{00}, \text{BushPct}_{04}) = 0.96 =$$

$$\frac{0.014858}{\sqrt{0.01499} \times \sqrt{0.01605}} \approx .96$$




- Measures how closely data points fall along the line
- Varies between -1 and 1 (compare with Tufte p. 102)



Warning: Don't correlate often!

- Correlation only measures linear relationship
- Correlation is sensitive to variance
- Correlation usually doesn't measure a theoretically interesting quantity
- Same criticisms apply to R^2 , which is the squared correlation between predictions and data points.
- Instead, focus on regression coefficients (slopes)



Discrete DV, discrete EV

- Crosstabs
- χ^2
- Gamma, Beta, etc.



Example

- What is the relationship between abortion sentiments and vote choice?

- The abortion scale:

1. BY LAW, ABORTION SHOULD NEVER BE PERMITTED.
2. THE LAW SHOULD PERMIT ABORTION ONLY IN CASE OF RAPE, INCEST, OR WHEN THE WOMAN'S LIFE IS IN DANGER.
3. THE LAW SHOULD PERMIT ABORTION FOR REASONS OTHER THAN RAPE, INCEST, OR DANGER TO THE WOMAN'S LIFE, BUT ONLY AFTER THE NEED FOR THE ABORTION HAS BEEN CLEARLY ESTABLISHED.
4. BY LAW, A WOMAN SHOULD ALWAYS BE ABLE TO OBTAIN AN ABORTION AS A MATTER OF PERSONAL CHOICE.

Abortion and vote choice in 2006

. tab housevote abortopinion, col

Key
frequency
column percentage

us house candidate voting for	stmt most agrees w/ view on abortion law					Total
	Never	Rarely	Sometimes	Always	other (pl	
Democrat	446 13.60	1,749 20.21	1,903 36.90	8,759 57.93	770 34.30	13,627 39.55
Republican	1,900 57.93	4,381 50.62	1,639 31.78	2,006 13.27	758 33.76	10,684 31.01
other (please specify	157 4.79	384 4.44	228 4.42	671 4.44	190 8.46	1,630 4.73
i won't vote in this	65 1.98	201 2.32	117 2.27	299 1.98	52 2.32	734 2.13
haven't decided	712 21.71	1,939 22.41	1,270 24.63	3,386 22.39	475 21.16	7,782 22.58
Total	3,280 100.00	8,654 100.00	5,157 100.00	15,121 100.00	2,245 100.00	34,457 100.00



Use the appropriate graph/table

- Continuous DV, continuous EV
 - E.g., vote share by income growth
 - Use scatter plot
- Continuous DV, discrete and unordered EV
 - E.g., vote share by religion or by union membership
 - Box plot, dot plot
- Discrete DV, discrete EV
 - No graph: Use crosstabs (`tabulate`)



Two quick notes about comparing coefficients

- Recode/rescale independent variables to be in 0-1 interval
 - $\text{new_x} = [x - \min(x) + 1] / (\max(x) - \min(x) + 1)$
 - Interpretation: a move from the minimum to the maximum in the independent variable yields an average change of b in the d.v.

. reg beo bpop

Source	SS	df	MS	Number of obs =	41
Model	351.26542	1	351.26542	F(1, 39) =	202.56
Residual	67.6326195	39	1.73416973	Prob > F =	0.0000
				R-squared =	0.8385
				Adj R-squared =	0.8344
Total	418.898039	40	10.472451	Root MSE =	1.3169

beo	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
bpop	.3584751	.0251876	14.23	0.000	.3075284	.4094219
_cons	-1.314892	.3277508	-4.01	0.000	-1.977831	-.6519535

Variable	Obs	Mean	Std. Dev.	Min	Max
bpop	41	10.13171	8.266633	1.2	30.8

. gen bpop01=(bpop-1.2)/(30.8-1.2)

. reg beo bpop01

Source	SS	df	MS	Number of obs =	41
Model	351.265419	1	351.265419	F(1, 39) =	202.56
Residual	67.63262	39	1.73416974	Prob > F =	0.0000
				R-squared =	0.8385
				Adj R-squared =	0.8344
Total	418.898039	40	10.472451	Root MSE =	1.3169

beo	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
bpop01	10.61086	.7455536	14.23	0.000	9.10284	12.11889
_cons	-.8847219	.3048075	-2.90	0.006	-1.501253	-.2681905

- 
- Convert *all* variables, except dummy variables, to “unit deviates”:\

- $\text{new_x} = [x - \text{mean}(x)] / \text{sd}(x)$

- $\text{new_y} = [y - \text{mean}(y)] / \text{sd}(y)$ etc.

- Interpretation: a one standard deviation change in x yields, on average, a b standard deviation change in y .

- (For a dummy variable, a change from category 0 to category 1 yields, on average, a b standard deviation change in y .

```
. reg beo bpop
```

Source	SS	df	MS	Number of obs =	41
Model	351.26542	1	351.26542	F(1, 39) =	202.56
Residual	67.6326195	39	1.73416973	Prob > F =	0.0000
-----				R-squared =	0.8385
-----				Adj R-squared =	0.8344
Total	418.898039	40	10.472451	Root MSE =	1.3169

beo	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
bpop	.3584751	.0251876	14.23	0.000	.3075284	.4094219
_cons	-1.314892	.3277508	-4.01	0.000	-1.977831	-.6519535

```
. summ beo bpop
```

Variable	Obs	Mean	Std. Dev.	Min	Max
beo	41	2.317073	3.236117	0	10.8
bpop	41	10.13171	8.266633	1.2	30.8

```
. gen st_beo=(beo-2.317073)/3.236117  
(9 missing values generated)
```

```
. gen st_bpop=(bpop-10.13171)/8.266633  
(9 missing values generated)
```

```
. reg st_beo st_bpop
```

Source	SS	df	MS			
Model	33.5418469	1	33.5418469	Number of obs =	41	
Residual	6.45814509	39	.165593464	F(1, 39) =	202.56	
Total	39.9999919	40	.999999799	Prob > F =	0.0000	
				R-squared =	0.8385	
				Adj R-squared =	0.8344	
				Root MSE =	.40693	

st_beo	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
st_bpop	.9157217	.0643416	14.23	0.000	.7855786	1.045865
_cons	3.54e-07	.0635521	0.00	1.000	-.1285458	.1285465

```
. reg beo bpop,beta
```

Source	SS	df	MS			
Model	351.26542	1	351.26542	Number of obs =	41	
Residual	67.6326195	39	1.73416973	F(1, 39) =	202.56	
Total	418.898039	40	10.472451	Prob > F =	0.0000	
				R-squared =	0.8385	
				Adj R-squared =	0.8344	
				Root MSE =	1.3169	

beo	Coef.	Std. Err.	t	P> t	Beta
bpop	.3584751	.0251876	14.23	0.000	.9157218
_cons	-1.314892	.3277508	-4.01	0.000	

MIT OpenCourseWare
<http://ocw.mit.edu>

17.871 Political Science Laboratory
Spring 2012

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.