

# Optimization Methods in Management Science

MIT 15.053, Spring 2013

PROBLEM SET 3, DUE: THURSDAY FEBRUARY 28TH, 2013

## Problem Set Rules:

1. Each student should hand in an individual problem set.
2. Discussing problem sets with other students is permitted. Copying from another person or solution set is *not* permitted.
3. Late assignments will *not* be accepted. No exceptions.
4. The Excel solutions should be posted on the website by the beginning of class on the day the problem set is due. Questions that require an Excel submission are marked with EXCEL SUBMISSION . For EXCEL SUBMISSION questions, only the Excel spreadsheet will be graded.

For this problem set, all questions are EXCEL SUBMISSION : there is no paper submission, only electronic submission. All answers should be given on the spreadsheet, in the sheet labeled “Answers”. There are clearly marked boxed for each answer.

## Problem 1

(24 points) EXCEL SUBMISSION See Problem 1 on the Excel Workbook.

This problem introduces the Phase 1 method of linear programming. What is the optimal solution for the Phase 1 problem as obtained by your pivoting? What is the optimal basic feasible solution for the original problem?

## Problem 2

(20 points) EXCEL SUBMISSION See Problem 2 on the Excel Workbook.

This problem is concerned with degeneracy. This problem is based on a cycling instance similar to the one in the tutorial on degeneracy. The instance can be found on the spreadsheet entitled “degeneracy example”. However, we use Bland’s rule for pivoting. Now if you use the simplex method, there will be no cycling. Try it yourself. What is the optimum basis?

## Problem 3

(28 points) EXCEL SUBMISSION See Problem 3 on the Excel Workbook.

In this problem, change the value of one cell of the tableau so a stated condition is satisfied. In most cases, you can check to see if your answer is correct by carrying out a pivot on the initial tableau.

**Example:** The bfs is not optimal, and  $x_3$  is the next variable to be pivoted into the basis.

**Answer:** change the value in cell I11 to a +1. (Any positive number would be acceptable as an answer.)

In most cases, more than one answer is acceptable. When any positive number is acceptable in one cell, please use the value +1 in your answer. This will make grading easier.

- (a) (4 points) The bfs is optimal, and there is an edge of the feasible region that is also optimal.
- (b) (4 points) The bfs is optimal, and there is a ray of the feasible region that is also optimal.
- (c) (4 points) The bfs is not optimal and  $x_5$  becomes non-basic after the next pivot.
- (d) (4 points) The bfs is not optimal and  $x_3$  becomes basic after the next pivot.
- (e) (4 points) The bfs is degenerate.

For the next two parts, assume that the “4” in cell P13 is changed to a “0”. The question asks what additional change is needed to achieve the stated property.

- (f) (4 points) The basis is not optimal, and the next pivot is degenerate, that is, the basic feasible solution remains the same.
- (g) (4 points) The basis is not optimal, and the next pivot is non-degenerate. That is, the basic feasible solution is different.

## Problem 4

(28 points) EXCEL SUBMISSION See Problem 4 on the Excel Workbook.

You are given a tableau. All parts refer to the basic solution for the above tableau. In each case, give the best answer; conditions should be as succinct and necessary as possible. For example, one possible answer to part (c) is:  $A < -2$  AND  $B \leq 0$  AND  $C = 0$ , but this is not the best answer: there is a more general condition on  $A, B$  and  $C$  to satisfy part (c). In each case express the answer in terms of equalities or inequalities involving one or more of the terms of  $A, B, C, D, E$ , and  $F$ .

- (a) (2 points) Under what conditions on  $F$  is the current basic solution feasible and not degenerate?
- (b) (2 points) Under what conditions on  $F$  is the current basic solution feasible and degenerate?
- (c) (4 points) Under what conditions on  $A, B$ , and  $C$  is the current basic solution optimal? (Assume it is feasible and not degenerate.)
- (d) (4 points) Assume that  $A, B, C$ , and  $F$  satisfy the conditions of parts (a) and (c). Write at least one additional condition for the problem to have multiple optimal solutions.
- (e) (4 points) Under what conditions on  $A, B$ , and  $C$  is the current basic solution optimal and the problem has a unique optimal solution? (Assume it is feasible and not degenerate.)
- (f) (4 points) Assume that  $A, B \leq 0$ . Under what conditions on  $C$  and  $E$  is  $x_7$  eligible to be the leaving variable in the simplex pivot while  $x_6$  is the entering variable?
- (g) (4 points) Under what conditions on  $A$  and  $D$  can the simplex algorithm immediately end with a proof that the objective value is unbounded from above?
- (h) (4 points) Assume that  $A, B, C$ , and  $F$  satisfy the conditions of parts (a) and (c). Under what conditions is there a ray of optimal solutions?

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