

The Retail Game



Revenue Optimization

Statistical Data as Information

Fun with Optimization

The Situation

- Retailer with 2,000 units in inventory
- Full Retail Price \$60
- Discount Options 10%, 20% and 40% off
- Salvage Value of \$25 (over 58% off)
- 15 week selling season
- No restocking

Wk.	Inv. on hand	Price	Sales			StdErr	Revenue	
			Now	Ave.	Proj.		Now	Proj.
1	2000	60	29	29.0	435	.	1740	65225
2	1971	60	51	40.0	600	11.	4800	71000
3	1920							

Game number: 1
Salvage value \$25

Remaining Inventory

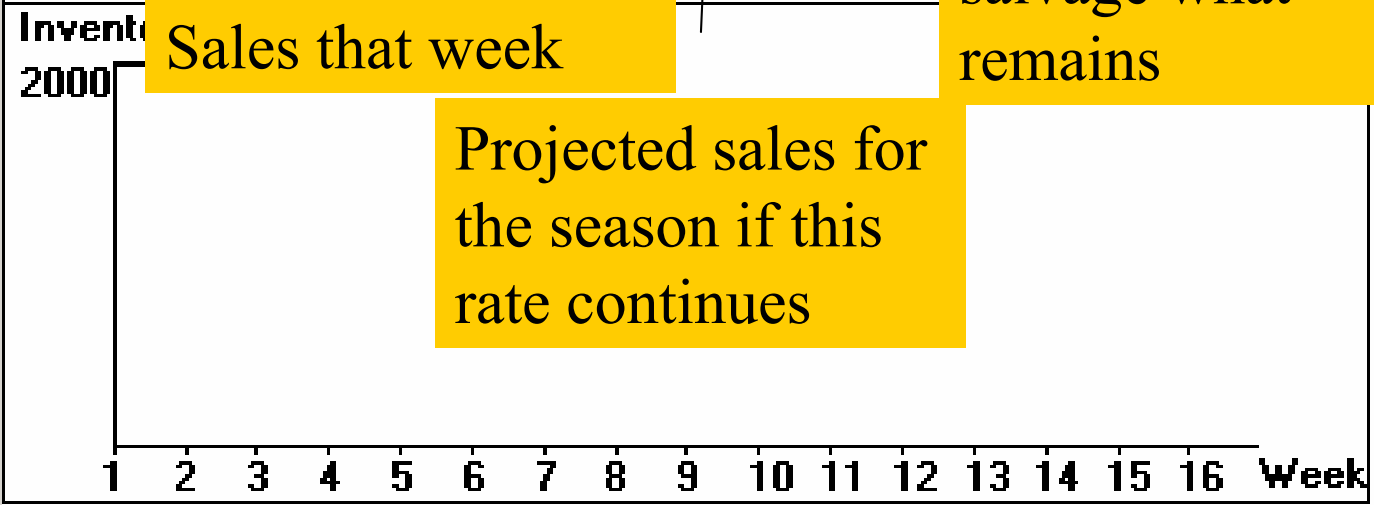
Average weekly sales at current price

Revenue to date

Projected revenues if you continue to sell at this price and at this rate and salvage what remains

Sales that week

Projected sales for the season if this rate continues



Price Markdowns

10% 20% 40%

Same Price

How to Play



Retail.xls

■ Histories of 15 items

Item	Week	Qty on hand	Price	Sales
1	1	2000	60	57
	2	1943	60	98
	3	1845	60	55
	4	1790	60	41
	5	1749	60	60
	6	1689	60	39
	7	1650	54	106
	8	1544	54	55
	9	1489	54	64
	10	1425	54	43
	11	1382	54	131
	12	1251	54	112
	13	1139	54	62
	14	1077	54	31
	15	1046	54	80
	16	966		

The Challenge

- Devise a discount strategy to earn maximum revenue
- What to do with the data in Retail.xls?



Competition



- Select Compete
- Enter the Random Seed:
- Your first weeks sales at \$60
- Implement your strategy for 5 items
- Record your total relative difference

Discussion



- What are the trade-offs?
- What information do you need?
- How to estimate that information?
- How to use the estimates?

Estimating Sales at Discount

Qty on

Est. Avg.

Item	Week	hand	Price	Sales	Average Price	Price	Sales
1	1	2000	\$ 60	75	Full Price	\$ 60	107.3
	2	1925	\$ 60	138	10%	\$ 54	113.0
	3	1787	\$ 60	109	20%	\$ 48	223.3
	4	1678	\$ 54	117	40%	\$ 36	223
	5	1561	\$ 54	141			
	6	1420	\$ 54	81			
	7	1339	\$ 48	304			
	8	1035	\$ 48	218			
	9	817	\$ 48	148			
	10	669	\$ 36	299			
	11	370	\$ 36	290			
	12	80	\$ 36	80			
	13	0					
	14	0					
	15	0					
	16	0					

Why lower sales?

Refining the Estimate

Qty on

Item	Week	hand	Price	Sales	Average Price	Est. Avg. Sales
1	1	2000	\$ 60	75	Full Price \$ 60	107.3
	2	1925	\$ 60	138	10% \$ 54	113.0
	3	1787	\$ 60	109	20% \$ 48	223.3
	4	1678	\$ 54	117	40% \$ 36	223
	5	1561	\$ 54	141		
	6	1420	\$ 54	81		
	7	1339	\$ 48	304		
	8	1035	\$ 48	218		
	9	817	\$ 48	148		
	10	669	\$ 36	299		
	11	370	\$ 36	290		
	12	80	\$ 36	80		
	13	0				
	14	0				
	15	0				
	16	0				

$$(290 + 299)/2 = 294.5$$

Sales, not demand!

Sales Rates

		Sales Rates			
Item	Full Price	10%	20%	40%	
1	58.33	76.00	-	-	
2	107.67	144.00	-	-	
3	59.33	82.33	-	-	
4	61.17	77.89	-	-	
5	92.50	113.67	-	-	
6	114.14	-	209.40	-	
7	67.43	-	119.63	-	
8	53.00	-	96.75	-	
9	73.71	-	131.88	-	
10	67.29	-	97.13	-	
11	100.44	-	-	264.25	
12	64.11	-	-	188.50	
13	65.56	-	-	196.67	
14	61.44	-	-	164.00	
15	62.33	-	-	175.00	
16	107.33	113.00	223.33	294.50	
Mean	75.99	101.15	146.35	213.82	
Std Dev	20.74	27.06	56.05	52.87	

Relative Sales Rates

Ratio of Sales Rates

Item	Full Price	10%	20%	40%
1	1	1.30	-	-
2	1	1.34	-	-
3	1	1.39	-	-
4	1	1.27	-	-
5	1	1.23	-	-
6	1	-	1.83	-
7	1	-	1.77	-
8	1	-	1.83	-
9	1	-	1.79	-
10	1	-	1.44	-
11	1	-	-	2.63
12	1	-	-	2.94
13	1	-	-	3.00
14	1	-	-	2.67
15	1	-	-	2.81
Mean	1	1.31	1.73	2.81
Std Dev	-	0.06	0.16	0.16

What to do with Info?



- Given initial sales data, how many weeks to sell at each discount?

Variables



- How many weeks to sell at each price
- What about the order?!
- How much to salvage?

Constraints



- Total Weeks \leq Weeks in the Season
- Total Sales and Salvage = Initial Inventory
- And?

A Model

Initial								
Weekly Rate		90						
	Full Price		10% off	20% off	40% off	Salvage		
Price	\$ 60.00	\$ 54.00	\$ 48.00	\$ 36.00	\$ 25.00			
Sales Lift	1	1.31	1.73	2.81				
						Salvage	Total	
Weeks at	1.00	7.12	6.88	-			15.00	
Sales	90.00	837.45	1,072.55	-		-	2,000.00	
						Salvage	Total	
						Revenue	Revenue	
Revenues	\$ 5,400	\$ 45,222	\$ 51,482	\$ -	\$ -	\$ -	\$ 102,105	

How Many Prices?



- Ignore period 1
- At most 2 prices (including salvage)
- Why?
- Two constraints - two basic variables

Two Constraints

■ P = Average Sales Rate at Full Price

■ $x[\text{price}]$ = Weeks we sell at price

■ S = Units we salvage

$$\max P^*(60x[60] + 54*1.31x[54] + 48*1.73x[48] + 36*2.81x[36]) + 25S$$

$$\text{s.t. } x[60] + x[54] + x[48] + x[36] \leq 15$$

$$\text{s.t. } P^*(x[60] + 1.31x[54] + 1.73x[48] + 2.81x[36]) + S = 2000$$

$$\text{s.t. } x[60] \geq 1 \text{ (This is a bound. Like } x[54] \geq 0)$$

non-negativity

Strategies



- Average Sales at Full Price
- $< 80/\text{wk}$ 20% off & Salvage
- 80-103/wk 10% off & 20% off
- 104-133/wk Full Price & 10% off
- $> 133/\text{wk}$ Full Price
- Where are the greatest errors?
- Why?

What about 40% off?

- Would it ever be wise to discount by 40%



Why Not 40% Off?

- Two Cases: Salvage and No Salvage
 - ▶ No Salvage means the lift in sales doesn't help
- We Salvage:
 - ▶ Weekly Revenue Rate at 20% discount
 - $\$48 * 1.73 * P = 83.04 * P$
 - ▶ Weekly Revenue Rate at 40% discount
 - $\$36 * 2.81 * P = 101.16 * P$

Something is wrong with this reasoning!

Lost Salvage Value

■ We Salvage:

- ▶ Revenue Rate over salvage at 20% discount
 - $(\$48 - \$25) * 1.73 * P = 39.79 * P$
- ▶ Revenue Rate over salvage at 40% discount
 - $(\$36 - \$25) * 2.81 * P = 30.91 * P$
- ▶ 40% is not competitive

Summary



- Use optimization to improve decision making even under uncertainty
- Even if it is only as good, it is automatic.
- Insights into strategy
 - ▶ Ignore period 1
 - One Price and salvage
 - Two prices and no salvage

A Parametric Model

- Rescale Sales volumes to units of Initial Weekly Sales (Rate)
- Initial Inventory becomes $2000/\text{Rate}$
- Rescale Revenues by dividing by Initial Weekly Sales
- Only RHS of Inventory Constraint depends on Rate
- Use Sensitivity Analysis

Parametric Model

■ R = Average Sales Rate at Full Price

■ $x[\text{price}]$ = Weeks we sell at price

■ S = Units we salvage/ R

$$\max 60x[60] + 54*1.31x[54] + 48*1.73x[48] + 36*2.81x[36] + 25S$$

$$\text{s.t. } x[60] + x[54] + x[48] + x[36] \leq 15$$

$$\text{s.t. } x[60] + 1.31x[54] + 1.73x[48] + 2.81x[36] + S = 2000/R$$

$$\text{s.t. } x[60] \geq 1$$

non-negativity

Parametric Study

Initial Weekly Rate		Initial Inv./Initial Weekly Rate				
1.00		2000				
	Full Price	10% off	20% off	40% off	Salvage	
Price	\$ 60.00	\$ 54.00	\$ 48.00	\$36.00	\$25.00	
Sales	1	1.31	1.73	2.81		
Weeks of sales at each price						Total
	1.00	0.62	13.38	-		15
Sales and Salvage in Units of Weeks at Full Price						Total
	1.00	0.81	23.19	-	-	25.00
Revenues/ Initial Weekly Rate						Total
	\$ 60	\$ 44	\$ 1,113	\$ -	\$ -	\$1,217

Using Sensitivity Analysis

■ Strategy:

- ▶ 1 week at full price
- ▶ 14 weeks at 20% off
- ▶ Salvage what's left

What increase in sales rate will decrease the Initial Inventory by 1974.73?

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$H\$7	Weeks :	15.00	39.87	15	1139.279735	14
\$H\$8	Sales :	2,000.00	25.00	2000	1E+30	1974.733554

The basis (strategy) remains the same until we decrease the RHS of the Inventory Constraint by 1974.73

When to Switch Strategies

- Current Inventory 2000/Current Rate
- Find New Rate that decreases this by 1974.73

$$2000/\text{Current Rate} - 2000/\text{New Rate} = 1974.73$$

$$\begin{aligned}\text{New Rate} &= 2000/(\text{2000/Current} - 1974.73) \\ &= 79.16\end{aligned}$$

Parametric Study

Initial Weekly Rate		Initial Inv./Initial Weekly Rate				
1.00		2000				
	Full Price	10% off	20% off	40% off	Salvage	
Price	\$ 60.00	\$ 54.00	\$ 48.00	\$36.00	\$25.00	
Sales	1	1.31	1.73	2.81		
Weeks of sales at each price						Total
	1.00	0.62	13.38	-		15
Sales and Salvage in Units of Weeks at Full Price						Total
	1.00	0.81	23.19	-	-	25.00
Revenues/ Initial Weekly Rate						Total
	\$ 60	\$ 44	\$ 1,113	\$ -	\$ -	\$1,217