

15.401 Recitation

4: Forwards & Futures

Learning Objectives

- Review of Concepts
 - Forwards vs. futures
 - Spot-future parity
 - Hedging with F&F
- Examples
 - Mark to market
 - Oil!
 - Stock futures

Review: forwards vs. futures

□ Similarity:

- One party has the **obligation to buy** an asset at a fixed future date and at a price determined today; the counterparty has the **obligation to sell**.

□ Difference:

○ forwards

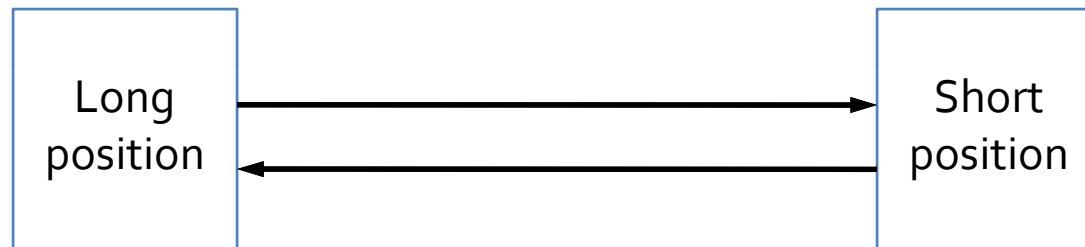
- custom made
- over the counter (OTC)
- no payments until maturity

○ futures

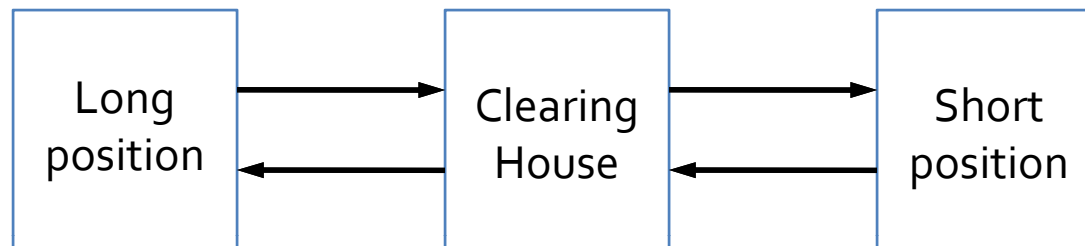
- standardized
- traded on exchange
- marked-to-market daily

Review: forwards vs. futures

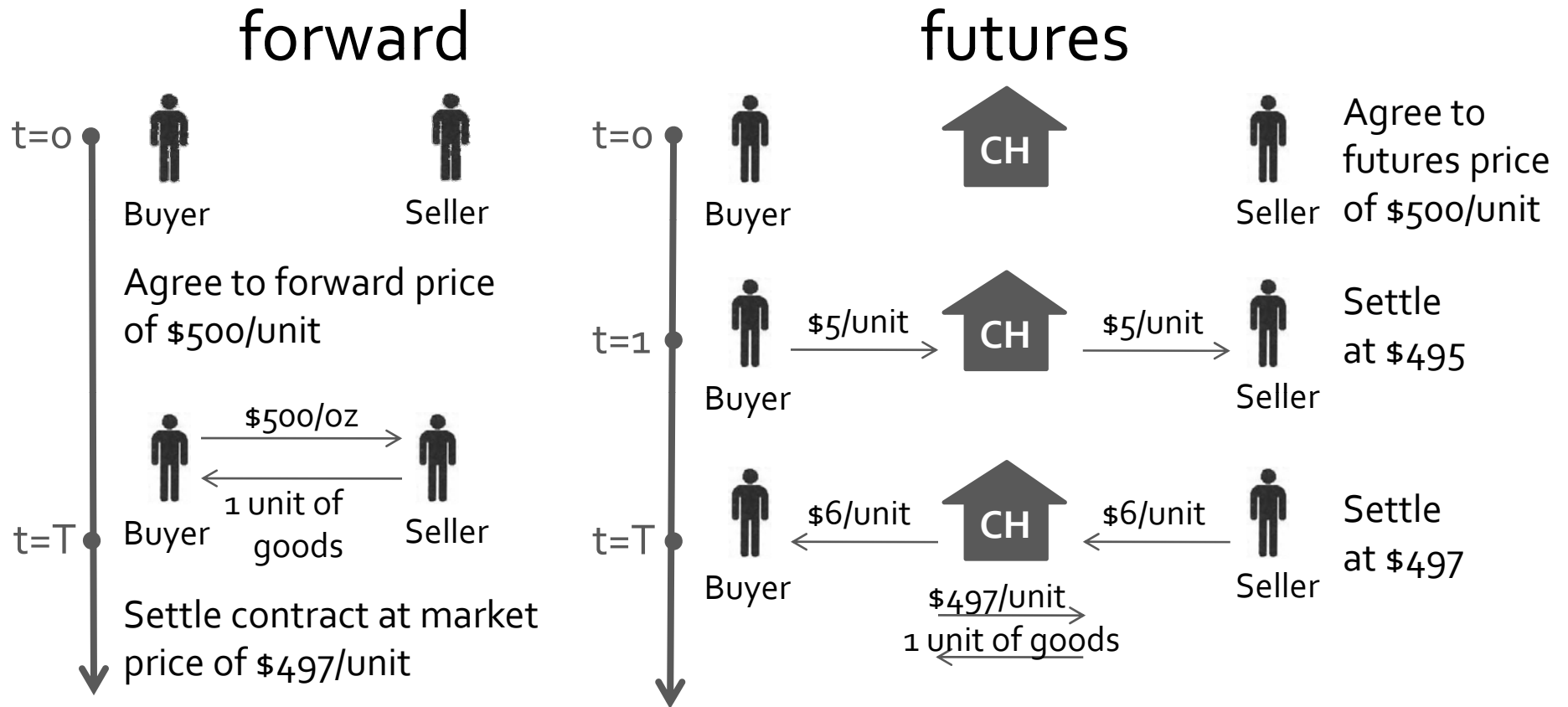
forwards



futures



Review: forwards vs. futures



In both cases, the seller earns \$3/unit.

Review: spot-future parity

- The **no-arbitrage** principle implies the following relationship between the spot price and futures price:

$$F_0 = P_0(1 + r_f - y + c)^T$$

- y is the convenience yield per period
 c is the storage cost.
 $(y - c)$ is the net convenience yield
 T is the time to maturity of the futures contract.
- If the spot-future parity does not hold, there is an **arbitrage opportunity**.

Review: spot-future parity

- ❑ If net convenience yield is positive (storage costs are low and convenience yields are high), futures price will be lower than the spot price adjusted for the time value of money. This is known as **backwardation**.
- ❑ If net convenience yield is negative, adjusted futures price is higher than the spot price. This is known as **contango**.

Review: hedging with F&F

- ❑ Forward contract can be used to hedge future inflow and outflow of commodities against price fluctuations:
 - Pro: contracts are custom-made to fit exact needs
 - Con: illiquid
- ❑ Futures contract can be used in a similar way:
 - Pro: liquid
 - Con: standardized contract

Review: hedging with F&F

- Suppose that you just discovered gold at a remote location in Yukon, but it will take 6 months to build a mine and start production. You can secure your revenue today by taking a short position in a gold forward contract:

	CF at t=0	CF at t=6
Long position in gold	0	S_6
Short position in forward	0	$F_0 - S_6$
Net Cash Flow	0	F_0

- Your future net cash flow, F_0 , is a fixed number at t=0.

Example 1: mark to market

- Assume the current futures price for silver for delivery 5 days from today is \$10.10 per ounce. Suppose that over the next 5 days, the futures price evolves as follows:

Day	0	1	2	3	4	5
Futures Price	\$10.10	10.2	10.25	10.18	10.18	10.21

- If you have a long position of 25,000 ounces at time 0, what are your cash flows for the next five days?

Example 1: mark to market

□ Answer:

Day	Profit/loss per ounce	Daily proceeds
1	$10.20 - 0.10 = 0.10$	\$2,500
2	$10.25 - 10.20 = 0.05$	1,250
3	$10.18 - 10.25 = -0.07$ $\times 25,000$	-1,750
4	$10.18 - 10.18 = 0.00$	0
5	$10.21 - 10.18 = 0.03$	750
		Sum = \$2,750

Example 2: oil!

- Oil is currently trading at \$50 per barrel. The 1-year risk-free interest rate is 3.8%, and the 1-year forward price of oil is \$50.40.
 - a. What is the net convenience yield of oil?
 - b. What would you do if the 1-year forward price is \$51 instead?

Example 2: oil!

□ Answer:

$$\text{a. } 50.4 = 50 \times (1 + 0.038 - \hat{y})^1 \Rightarrow \hat{y} = 3\%$$

Example 2: oil!

□ Answer:

b. there is an arbitrage opportunity:

	CF at t=0	CF at t=1
Sell 1 barrel forward	\$0.00	\$51.00
Buy 1 barrel now & hold	-\$50.00	\$1.50
Borrow \$50 today	\$50.00	-\$51.90
Net Cash Flow	\$0.00	\$0.60

Example 3: stock futures

- Stock price of Acme Inc. is trading at \$56, and is expected to pay \$10 of dividend in the next two years. The term structure of interest rates is flat at 3%. What is the 2-year forward price on company A's stock?

Example 3: stock futures

□ Answer: the no-arbitrage principle gives

$$F_2 = 56 \cdot 1.03^2 - 10 = \$49.41$$

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