Problem Set 6

1. (To be solved in a group) Suppose we observe returns of N trading strategies over the same time period, x_t^n, $n = 1, ..., N$, $t = 1, ..., T$. We want to develop a test of the null hypothesis that all n strategies produce the same average return.

   (a) Assuming that returns are independent and identically distributed over time but potentially correlated contemporaneously, derive the estimator of the average return for the N strategies, $\widehat{\mu}_n$. Derive the asymptotic variance-covariance matrix $\widehat{\Omega}$ for the estimated mean vector.

   (b) Argue that the estimates $\widehat{\mu}_n$ have multivariate normal distribution asymptotically.

   (c) Define $\delta_k = \widehat{\mu}_k - \widehat{\mu}_1$, $k = 2, ..., N$. Argue that the asymptotic distribution of the vector $\delta = (\delta_2, ..., \delta_N)'$ is multivariate normal. What is the mean and var-cov matrix of this distribution under the null hypothesis? Show (i) how to derive the var-cov matrix of δ from $\widehat{\Omega}$; and (ii) how to estimate it directly from return data x_t^n.

   (d) Denote the var-cov matrix of the distribution of δ in (1c) by V. Argue that the test statistic
   $$W = \delta' V^{-1} \delta$$
   is distributed as $\chi^2(N - 1)$.

   (e) Using the cumulative distribution function for the χ^2 distribution,[1] construct a test of the null hypothesis with 5% size. Your test should rely on the statistic W in (1d).

2. (To be solved in a group) Conduct a Monte Carlo study to illustrate the performance of the test developed above. Assume that $N = 5$ and $T = 60$. Assume that the return series are normally distributed with mean 0.01 and standard deviation 0.05. All pairwise correlations between the return series are equal to 0.25. Thus, the variance-covariance matrix of returns has diagonal elements of 0.05^2 and off-diagonal elements of $0.05^2 \times 0.25$.

   (a) Simulate 100,000 independent samples of N return series of length T according to the above distribution. Perform the test developed in (1a-1e) for each simulated

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[1] $\chi^2(K)$ CDF is available in **MATLAB(R)** Function call is *cdf('chi2',x,K)*, where x is the argument, and K is the degrees of freedom parameter.

sample and compute the frequency of rejecting the null hypothesis. What is the size of the test according to your Monte Carlo study?

(b) Repeat the Monte Carlo study under a different distribution of returns. In particular, assume that the null hypothesis is false and the five average returns are

$$0.02, \ 0.015, \ 0.01, \ 0.015, \ 0.025$$

The variance-covariance matrix of returns is the same as in (2a), returns are still multi-variate normal. Estimate the probability of the test rejecting the null hypothesis when it's false. This probability is called *the power* of the test against different alternatives above.

(c) Assuming the return distribution in (2b), approximately how many observations T would it take for the above test to reject the null hypothesis of equal average returns with probability 99%? You should use your Monte Carlo set-up to support the answer this question.

3. (To be solved individually) In this problem we test for predictability of excess stock returns. Download the market data in file ReturnPred.xls, (this file is unavailable on MIT OpenCourseWare). The file contains historical data on S&P 500 returns, interest rates, dividend-price ratio for the S&P 500 index, and the term spread, defined as the difference in ten-year and three-month Treasury yields Consider the following predictive model:

$$r_{t+1}^e = a_0 + a_1(D/P)_t + a_2(\text{Term spread})_t + \varepsilon_{t+1}$$

where r_t^e denotes the monthly return on the S&P 500 value-weighted index in excess of the interest rate, approximated by the three-month Treasury Bill yield (the interest rate series is available at http://research.stlouisfed.org/fred2/series/TB3MS?cid=116).

(a) Using OLS, estimate the predictive model above. Compute standard errors for the coefficients a_1, a_2. Use the Newey-West procedure, defend your choice of the band width parameter.

(b) Test the null hypothesis that $a_1 = a_2 = 0$. Can you reject the null at 5% level? Can you reject the null at 1% level? Describe the details of the test.

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