# 15.S50 - Poker Theory and Analytics 

Pre-flop Analysis

## Pre-flop Analysis

- Motivator
- Range Definition
- Basic Assumptions
- Heads Up
- Other Positions

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## Motivator

- Why are we spending an entire day on pre-flop?
- In tournaments, most of your value will come from playing pre-flop close to optimally
- Most live players are way off in their decision-making
- Its easy to solve; fewer assumptions


## Scenario A



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## Scenario A

- No Limit Hold'em Tournament - 125/250 Blinds +25
- trifluvian (BB): 1100

$$
\mathrm{M}=2.59
$$

- Hero (BTN/SB): 3400

$$
\mathrm{M}=8
$$

- Pre Flop: (425) Hero is BTN/SB with 94 6
- Hero...


## Scenario A

- No Limit Hold'em Tournament - 125/250 Blinds +25
- trifluvian (BB): 1100
$\mathrm{M}=2.59$
- Hero (BTN/SB): 3400

M $=8$

- Pre Flop: (425) Hero is BTN/SB with 9 6 6
$\mathrm{EV}=($ Pot $*$ Fold\% $)+(1-\mathrm{Fold} \%) *($ Win $\% *$ WinAmt - Lose $\% *$ LoseAmt $)$
Assume call range of 22+, A2+, JT+ (27.60\%)
960 vs $22+, \mathrm{A} 2+, \mathrm{JT}+=$ ?


## Scenario A

Assume call range of 22+, A2+, JT+ (27.60\%)
960 vs $22+$, $\mathrm{A} 2+, \mathrm{JT}+=34.26 \%$

## Equity Calculator



| Game | Board Dead |
| :--- | :--- | :--- |
| Holdem | $\square$ |


| Sel | 960 | $34.26 \%$ |
| :--- | :--- | :--- |
| Sel | $22+$, ,A2s+,,KTs+,QTs+,JTs,A20+,KTo+,QTo, ,JTo | $65.74 \%$ |

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## Scenario A

- No Limit Hold'em Tournament - 125/250 Blinds +25
- trifluvian (BB): 1100
$\mathrm{M}=2.59$
- Hero (BTN/SB): 3400

M $=8$

- Pre Flop: (425) Hero is BTN/SB with 9 6 6

WinAmt $=4500-(3400-(125+25))=1250$
or
WinAmt $=125+25+25+250+(1100-(250+25))=1250$

LoseAmt $=1100-(125+25)=950$

## Scenario A - Solution

- No Limit Hold'em Tournament - 125/250 Blinds +25
- trifluvian (BB): 1100

$$
\begin{aligned}
& M=2.59 \\
& M=8
\end{aligned}
$$

- Hero (BTN/SB): 3400
- Pre Flop: (425) Hero is BTN/SB with 9 6 6
$\mathrm{EV}=($ Pot $*$ Fold \% $)+(1-$ Fold $\%) *($ Win $\% *$ WinAmt - Lose $\% *$ LoseAmt $)$
Assume call range of $22+$, A2+, JT+ (27.60\%)
960 vs $22+$, $\mathrm{A} 2+$, $\mathrm{JT}+=34.26 \%$
EV of Fold = 0
EV of Push $=(1-27.6 \%) * 425+(27.6 \%) *[34.26 \% * 1250-65.74 \%$ *950]
EV of Push $=307.7-54.2=253.5$


## Scenario A - Generalized

EV of Fold $=0$
EV of Push $=(1-\mathrm{F} \%) * 425+(\mathrm{F} \%)^{*}[\mathrm{~W} \% * 1250-(1-\mathrm{W} \%) * 950]$


## Scenario A - Generalized

EV of Fold $=0$
EV of Push $=(1-\mathrm{F} \%) * 425+(\mathrm{F} \%) *[\mathrm{~W} \% * 1250-(1-\mathrm{W} \%) * 950]$


## Scenario A - Generalized

EV of Fold $=0$
EV of Push $=(1-\mathrm{F} \%) * 425+(\mathrm{F} \%) *[\mathrm{~W} \% * 1250-(1-\mathrm{W} \%) * 950]$


## Scenario A

- No Limit Hold'em Tournament - 125/250 Blinds +25
- trifluvian (BB): 1100

$$
\mathrm{M}=2.59
$$

- Hero (BTN/SB): 3400 $\mathrm{M}=8$
- Pre Flop: (425) Hero is BTN/SB with 94 6
- Hero pushes all-in regardless of Villain playing style
- Fold costs about 175 chips in EV
- This is worse than calling all-in with $3 \times 4 \downarrow$ vs A K .
$-1250 * 35.42 \%-950 * 64.58 \%=-171$


## De-Motivator

- These push/fold decisions are not very intuitive, so we need to solve it out beforehand
- Variables that impact our decision are
- Our cards
- Our position
- Our stack
- Villain's call range
- Our goal is to develop quick rules that work
"for all call ranges" or "for any two cards" etc.


## Pre-flop Analysis

- Motivator
- Range Definition
- Basic Assumptions
- Heads Up
- Other Positions

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## Range of Hands

－A range in poker is a set of hands
－Suitedness can be represented by an o or $s$ next to the hand
－Here is PokerTracker＇s representation of 33＋，A4s＋，KTs＋， A8o＋

| AA | AKs | AQs | As | ATs | A9s | A8s | A7s | A6s | A5s | A4s | A3s | A2s |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AKo | KK | KQs | Kls | KTs | K9s | K8s | K7s | K6s | K5s | K4s | K3s | K2s |
| AQo | KQo | QQ | QJs | QTs | Q9s | Q8s | Q7s | Q6s | Q5s | Q4s | Q3s | Q2s |
| Ao | KJo | QJo | J | JTs | 99s | 18 s | 775 | J6s | J5s | J4s | J3s | 12s |
| ATo | KTo | QTo | JTo | TT | T9s | T8s | T7s | T6s | T5s | T4s | T3s | T2s |
| A90 | K9。 | Q9。 | 190 | T9。 | 99 | 985 | 975 | 96s | 95s | 94s | 93s | 925 |
| A8o | K80 | Q80 | 180 | T80 | 980 | 88 | 875 | 86s | 85s | 84s | 83s | 825 |
| A70 | K70 | Q70 | 170 | 770 | 970 | 870 | 77 | 76s | 75s | 74s | 73s | 725 |
| A60 | K60 | Q60 | J60 | T60 | 960 | 860 | 760 | 66 | 65s | 64s | 63s | 62 s |
| A50 | K50 | Q50 | 150 | T50 | 950 | 850 | 750 | 650 | 55 | 54s | 53s | 52s |
| A40 | K4o | Q4o | 140 | T40 | 940 | 840 | 740 | 640 | 540 | 44 | 43s | 42 s |
| A30 | K3o | Q30 | 130 | T30 | 930 | 830 | 730 | 630 | 530 | 430 | 33 | 32 s |
| A20 | K20 | Q20 | 120 | T20 | 920 | 820 | 720 | 620 | 520 | 420 | 320 | 22 |

## Range of Hands

- A range in poker is a set of hands
- Ranges are denoted by the lowest or highest cards in a series
- For example, AA KK QQ would be written $\mathrm{QQ}^{+}$
- Another example is AA KK AK AQ KQ would be written $\mathrm{KK}+, \mathrm{AQ}^{+}, \mathrm{KQ}+$
- Less commonly, the best hand in a series is written with a -
- For example, 554433225453524342 could be written 55-, 54-, 43-


## Range of Hands

- We will use ranges for
- Analyzing Opponents
- Estimating equity against likely opponent cards
- e.g. if we are holding AT, we are $63 \%$ vs ATC
- Determining Our Plays
- Developing range-based rules for our plays
- e.g. we will push with AT+, $88+$ and fold otherwise


## Range of Hands

- We can also map percentiles to ranges based on hand value preflop
- Most commonly we use Sklansky-Karlson rankings, based on likelihood of being ahead preflop in a 2-player hand
- Top 10 hands are AA, KK, AKs, QQ, AKo, JJ, AQs, TT, AQo, 99


## David Sklansky

- David Sklansky
- Poker Theorist
- Three WSOP Bracelets
- One WPT Title
- Studied at Wharton

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- Authored 13 books published by TwoPlusTwo Publishing
- Active member of TwoPlusTwo Poker Community


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## David Sklansky

Sklansky, D. The Theory of Poker: A Professional Poker Player
OBuy at Amazon) Teaches You How To Think Like One. Two Plus Two Pub, 1999.
 Big With Expert Play. Two Plus Two Pub, 2004.
©ByatAmzzon Sklansky, D. and M. Malmuth. Hold'em Poker: For Advanced Players. Two Plus Two Pub, 1999.

ØByatAmazon Sklansky, D. Poker, Gaming \& Life. Two Plus Two Pub, 2009.

## David Sklansky

## Fundamental Theorem of Poker


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Every time you play a hand differently from the way you would have played it if you could see all your opponents' cards, they gain; and every time you play your hand the same way you would have played it if you could see all their cards, they lose.

## Sklansky-Karlson Rankings

- David Sklansky - Oct 04 ‘03 "Important No Limit Math Problem"
- One and two dollar blinds. You are in the small blind. Everyone folds. Your hand is A8o. Your opponent in the two dollar big blind has more money than you.
- You have only two options. Fold, or turn your cards face up and move in. He sees your A8 before he acts. You should not move in if your bankroll is above x dollars. What's x ?
$-\mathrm{x} \sim 70$

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## Sklansky-Karlson Rankings

- David Sklansky - Oct 04 ' 03 "Important No Limit Math Problem"
- $1 / 2$ Blinds, Hero is SB with A8o
- Hero bets $x$ all in
- BB will call at $\mathrm{EV}>0$
- For simplicity, let's say BB calls at Win $\% \geq 50 \%$
- Minor differences in $x$ will be caused by Villain's calling behavior but it will generally not impact hand rankings


## Sklansky-Karlson Rankings

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- $1 / 2$ Blinds, Hero is SB with A8o
- Hero bets $x$ all in
- BB will call at $\mathrm{EV}>0$
- For simplicity, let's say BB calls at Win $\% \geq 50 \%$
- BB calls with A8+, 22+, or 13.12\%
- Hero wins $33 \%$ vs this range


## Sklansky-Karlson Rankings

- BB calls with A8+, $22+$, or $13.12 \%$
- Hero wins $33 \%$ vs this range



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- $1 / 2$ Blinds, Hero is SB with A8o
- Hero bets $x$ all in
- BB will call at $\mathrm{EV}>0$
- For simplicity, let's say BB calls at Win $\% \geq 50 \%$
- BB calls with A8+, 22+, or 13.12\%
- Hero wins $33 \%$ vs this range
- Hero EV $=87 \%$ * $3+13 \%$ * $[33 \% * x+67 \% *-(x-1)]$
- $\mathrm{EV}=0$ at $x \approx 62$
- So at a stack of 62 or less, open-face all-in with A8o is +EV


## Sklansky-Karlson Rankings

- David Sklansky - Oct 24 ‘03
- What is the rating $x$ for all hands?
- Function of
- Number of hands you are marginally better then
- Chance of winning when you are behind
- Victor Chubukov (Karlson) - Oct 24 ‘03
- AA
$\infty$
- KK 1290
- You can check KK with
$-0=.9955 * 3+.0045 *(.1805 * x-.8195 *(x-1))$ at $x \approx 1040$

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## Sklansky-Karlson Rankings

- David Sklansky - Oct 24 ‘03
- What is the rating $x$ for all hands?
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- Victor Chubukov (Karlson) - Oct 24 ‘03
- AA
$\infty$
- KK 1290
- You can check KK with
$-0=.9955 * 3+.0045 *\left(.1805^{*} \mathrm{x}-.8195 *(x-1)\right)$ at $x \approx 1040$


## Sklansky-Karlson Rankings

- For example, the top $1 \%$ of hands is AA
- The top $5 \%$ of hands is TT+, AQs+, AQo +
- The top $30 \%$ of hands is $22+$, A2s+, K4s+, Q9s+, JTs, A2o+, K8o+, QJo


## Range of Hands

- Simplified ranges you can memorize
- TT,$+ \mathrm{AQ}+=5 \%$
- $55+$, AT $+=10 \%$
$-22+, \mathbf{A 2 +}, \mathrm{KQ}=20 \%$
$-22+$, A2+, Broadway = 30\%
- Pairs and cards adding to $16=50 \%$
- Any two cards = 100\%


## Range of Hands

| AA | AKs | AQs | AJs | ATs | A9s | A8s | A7s | A6s | A5s | A4s | A3s | A2s |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AKo | KK | KQs | KJs | KTs | K9s | K8s | K7s | K6s | K5s | K4s | K3s | K2s |
| AQo | KQo | QQ | QJs | QTs | Q9s | Q8s | Q7s | Q6s | Q5s | Q4s | Q3s | Q2s |
| Alo | KJo | QJo | J | Js | J9s | J8s | J7s | J6s | J5s | J4s | J3s | J 2 s |
| ATo | KTo | QTo | Jо | TT | T9s | T8s | T7s | T6s | T5s | T4s | T3s | T2s |
| A9o | K90 | Q9o | J90 | T90 | 99 | 98s | 97 s | 965 | 95s | 94s | 93 s | 92 s |
| A8o | K80 | Q80 | J80 | T80 | 980 | 88 | 87s | 865 | 85s | 84s | 83 s | 82 s |
| A70 | K7o | Q7o | J70 | T7o | 970 | 87o | 77 | 765 | 75s | 74 s | 73s | 72 s |
| A60 | K60 | Q60 | J60 | T60 | 960 | 860 | 760 | 66 | $65 s$ | 64s | 63 s | 62 s |
| A50 | K50 | Q50 | J50 | T5o | 950 | 850 | 750 | 650 | 55 | 54 s | 53 s | 52 s |
| A4o | K4o | Q40 | J40 | T4o | 940 | 840 | 740 | 640 | 540 | 44 | 43 s | 42 s |
| A30 | K3o | Q3o | J3o | T3o | 930 | 830 | 730 | 630 | 530 | 430 | 33 | 32 s |
| A20 | K2o | Q2o | J20 | T20 | 920 | 820 | 720 | 620 | 520 | 420 | 320 | 22 |

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## Range of Hands

$A+E V$ range for a decision is profitable on average
An optimal range the most profitable set of hands in dollar terms
For example, vs. AQ, the range $55+$, AT+ is profitable at $53 \%$ to win. But it's not optimal because AT and AJ lose to AQ

An optimal range is $22+\mathrm{AQ}^{+}$, which wins $60 \%$ of the time
If a range is optimal, then every hand in that range is +EV

## Pre-flop Analysis

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- Other Positions

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## Basic Assumptions

- Hero has effective $\mathrm{M}<10$
- Villain calling range is some top $\mathrm{x} \%$ of hands and everyone generally agrees on the order of hand rankings
- ICM doesn't matter, just cEV
- $\mathrm{M}<10$ is basically push or fold
- Why not raise? A bet of 3BB (i.e. 2 M ) lets you fold to a re-raise if it's as much as 6 M (i.e. giving you pot odds of $8 / 20$ or $40 \%$ ). You can fold 99- A5- K9- on a very small margin. Less than 4 M remaining gives you pot odds of 4 / 12 or $30 \%$ which is basically always a call. So maximizing fold equity is most important


## Preflop Strategy

Goal - Develop optimal push/call range for Ms up to 10 in blind vs blind

1. Build table of range vs range equities
2. Derive two-factor model to estimate range vs range equities
3. Develop EV model for semi-bluffs
4. For each M, find Nash Equilibrium if one exists
5. For unstable equilibriums, find reasonable ranges

## \&DFXCDMOJIMRSTI MYMRSTII



| $\mathrm{TT}+, \mathrm{AQs}+, \mathrm{AQ} 0+$ | $59.72 \%$ |
| :--- | ---: |
| $55+, \mathrm{A} 8 \mathrm{~s}+, \mathrm{ATo}+$ | $40.28 \%$ |

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## Range Table

Villain's Range

|  | 4.68\% | 9.95\% | 14.78\% | 19.91\% | 29.71\% | 39.67\% | 50.00\% | 74.66\% | 100.00\% |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4.68\% | 50\% | 60\% | 63\% | 66\% | 67\% | 69\% | 70\% | 72\% | 73\% |  |
| 9.95\% | 40\% | 50\% | 55\% | 56\% | 60\% | 62\% | 63\% | 66\% | 68\% |  |
| 14.78\% | 37\% | 45\% | 50\% | 52\% | 56\% | 58\% | 60\% | 63\% | 65\% | Hero's Favor |
| 19.91\% | 35\% | 44\% | 48\% | 50\% | 54\% | 57\% | 58\% | 61\% | 63\% | Neutral |
| 29.71\% | 33\% | 40\% | 44\% | 46\% | 50\% | 53\% | 55\% | 59\% | 61\% |  |
| 39.67\% | 31\% | 38\% | 42\% | 43\% | 47\% | 50\% | 52\% | 57\% | 60\% | Villain's Favor |
| 50.00\% | 30\% | 37\% | 40\% | 42\% | 45\% | 48\% | 50\% | 55\% | 58\% | Villain's Favor |
| 74.66\% | 28\% | $34 \%$ | 37\% | 39\% | 41\% | 43\% | 45\% | 50\% | 55\% |  |
| 100.00\% | 27\% | $32 \%$ | 35\% | 37\% | 39\% | 40\% | 42\% | 45\% | 50\% |  |

Hero Win \% at Showdown

## Hero's Range

## Preflop Strategy

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## Range vs Range Model

Hero's Range $=$ Top 50\%


## Range vs Range Model

Hero's Range $=$ Top 50\%


## Range vs Range Model

Hero's Range $=$ Top 50\%


[^0]
## Range vs Range Model

Hero's Range $=$ Top 30\%


[^1]
## Range vs Range Model

Hero's Range $=$ Top $10 \%$


[^2]
## Range vs Range Model

Hero's Range $=$ Top 5\%


[^3]
## Range vs Range Model

Hero's Range $=$ Top 3\%


[^4]
## Range vs Range Model

Hero's Range = Top 1\%


[^5]
## Range vs Range Model

## Villain's Range = Top 50\%



## Range vs Range Model

Villain's Range = Top 50\%


[^6]
## Range vs Range Model

Villain's Range $=$ Top 30\%


[^7]
## Range vs Range Model

Villain's Range $=$ Top 2\%


[^8]
## Range vs Range Model

Takeaway - range vs range equity relationship probably logarithmic, but not good in top 5\%

## Range vs Range Model

| Villain's Range |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Hero's Range |  | 4.688\%9.95\% |  | 4.78\%/ $19.91 \%$ |  | 9.71\% 33.67\% |  | 50.00\% | 74.6\%\% | $\begin{array}{r\|} \hline 100.00 \% \\ \hline 73 \% \\ \hline \end{array}$ | Hero's Favor |
|  | 4.68\% |  |  |  |  |  |  |  |  |  |  |
|  | 9.95\% | $40 \%$ | 50\% | 55\% | 56\% | 60\% | ${ }^{62 \%}$ | 63\% | 66\% |  |  |
|  | 19.78\% | ${ }^{37 \%}$ | ${ }^{45 \%}$ | 50\% | 52\% | 55\% | ${ }_{58 \%}^{58 \%}$ | ${ }^{60 \%}$ | ${ }^{63 \%}$ | ${ }^{65 \%}$ |  |
|  | ${ }^{19.92 \%}$ | ${ }^{33 \%}$ | ${ }^{44 \%}$ | ${ }_{4}^{48 \%}$ | 50\% | 54\% | ${ }_{\text {57\% }}^{53}$ | ${ }^{58 \%}$ | ${ }_{\text {69\% }}^{618}$ | ${ }_{618}^{63 \%}$ | Neutral |
|  | -33.67\% | ${ }^{318}$ | ${ }^{38 \%}$ | ${ }^{42 \%}$ | ${ }^{43 \%}$ | ${ }^{47 \%}$ | 50\% | 52\% | ${ }^{577 \%}$ | -60\% | Villain's Favor |
|  | 50.06\% | ${ }^{388}$ | ${ }_{\text {34\% }}$ | 30\%\% | ${ }^{\text {32\%\% }}$ | ${ }_{418}$ | 43\% | ${ }^{505 \%}$ | ${ }^{55 \%}$ |  |  |
|  | 100.0\% | ${ }^{278}$ | ${ }^{32 \%}$ | 35\% | ${ }^{37 \%}$ | 39\% | 40\% | ${ }^{428 \%}$ | 45\% | 50\% |  |

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## Multiple Regression

|  | $4,68 \%$ | $9.95 \%$ | $14.78 \%$ | $19.91 \%$ | $29.71 \%$ | $39.67 \%$ | $50.00 \%$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | $\mathbf{7 4 . 6 5 \%}|100.00 \%|$


| SUMMARY OUTPUT |  |  |
| :--- | ---: | :---: |
| Regression Statistics |  |  |
| Multiple R | 0.99 |  |
| R Square | 0.98 |  |
| Adjusted R Square | 0.98 |  |
| Standard Error | 0.01 |  |
| Observations | 81 |  |
|  |  |  |
|  | Coefficients |  |
| $\ln$ nercept | 0.5 |  |
| $\ln (\mathrm{H})$ | $(0.085)$ |  |
| $\ln (\mathrm{V})$ | 0.085 |  |

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## Multiple Regression

| SUMMARY OUTPUT |  |  |
| :--- | ---: | :---: |
| Regression Statistics |  |  |
| Multiple R | 0.99 |  |
| R Square | 0.98 |  |
| Adjusted R Square | 0.98 |  |
| Standard Error | 0.01 |  |
| Observations | 81 |  |
| Coefficients |  |  |
|  | 0.5 |  |
| Intercept | $(0.085)$ |  |
| $\ln (H)$ | 0.085 |  |
| $\ln (V)$ |  |  |

Equity Model

| $\begin{aligned} \text { HeroWin } \%=50 \% & +.085 * \ln (\text { VillainRange }) \\ & -.085 * \ln (\text { HeroRange })+\varepsilon \end{aligned}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |

## Error Analysis



## Range vs Range Model

Two-factor logarithmic model is good estimate for range vs range preflop equities

Error is large at ranges less than 5\%, but tolerable otherwise

$$
\begin{aligned}
\text { HeroWin } \%=50 \% & +.085 * \ln (\text { VillainRange }) \\
& -.085 * \ln (\text { HeroRange })+\varepsilon
\end{aligned}
$$

## Preflop Strategy

Goal - Develop optimal push/call range for Ms up to 10 in blind vs blind

1. Build table of range vs range equities
2. Derive two-factor model to estimate range vs range equities
3. Develop EV model for semi-bluffs
4. For each M, find Nash Equilibrium if one exists
5. For unstable equilibriums, find reasonable ranges

## EV Equation (Semi-Bluffs)

$E V=$ FoldEquity + ShowdownValue<br>FoldEquity $=$ Blinds $*(1-$ VillainCall\% $)$

## EV Equation (Semi-Bluffs)

$E V=$ FoldEquity + ShowdownValue
FoldEquity $=1 *(1-$ VillainCall\% $)$
ShowdownValue $=$
VillainCall\% * [SDWinAmt * HeroWin\% - SDLoseAmt * HeroLose\%]
SDWinAmt $=$ Stack $+\frac{2}{3}$
SDLoseAmt $=$ Stack
HeroWin $\%=50 \%+.085 * \ln ($ VillainCall\% $)-.085 * \ln ($ HeroPush $\%)$

## EV Equation (Semi-Bluffs)

$E V=$ FoldEquity + ShowdownValue
FoldEquity $=1 *(1-$ VillainCall\% $)$
ShowdownValue $=$
VillainCall\% * [SDWinAmt * HeroWin\% - SDLoseAmt * HeroLose\%]
SDWinAmt $=$ Stack $+\frac{2}{3}$
SDLoseAmt $=$ Stack

HeroWin $\%=50 \%+.085 * \ln ($ VillainCall\%) $-.085 * \ln ($ HeroPush\% $)$

## EV Equation (Semi-Bluffs)

$$
\begin{aligned}
& \text { EV }=(1-\text { VillainCall\% })+\text { VillainCall } \% *\left[\left(\operatorname{Stack}+\frac{2}{3}\right) * \text { HeroWin } \%-\text { Stack } * \text { HeroLose } \%\right] \\
& \text { HeroWin } \%=50 \%+.085 * \ln (\text { VillainCall } \%)-.085 * \ln (\text { HeroPush } \%)
\end{aligned}
$$

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## Preflop Strategy

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## EV when $M=20$

SB's EV (in terms of M)

|  |  | 5\% |
| :---: | :---: | :---: |
|  | 5\% | 0.05 |
| BB | 25.0\% | 0.11 |
| Picks | 50.0\% | 0.23 |
|  | 75.0\% | 0.36 |
|  | 100.0\% | 0.52 |

$\square$ BB's Favor $\square$ Neutral
$\square$ SB's Favor
SB loses .49M/hand

## EV when $M=20$

SB's EV (in terms

| BB Wants <br> to <br> Minimize <br> EV |  | 5\% | 25.0\% | 50.0\% | 75.0\% | 100.0\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 5\% | 0.05 | 0.17 | 0.29 | 0.39 | 0.47 |
|  | 25.0\% | 0.11 | 0.21 | 0.13 | -0.06 | -0.32 |
|  | 50.0\% | 0.23 | 0.46 | 0.33 | -0.01 | -0.49 |
|  | 75.0\% | 0.36 | 0.81 | 0.76 | 0.38 | -0.22 |
|  | 100.0\% | 0.52 | 1.24 | 1.32 | 0.97 | 0.33 |

$\square$ BB's Favor $\quad \square$ Neutral $\square$ SB's Favor

## EV when $M=20$

SB Picks

|  |  | 5\% | 10.0\% | 15.0\% | 20.0\% | 25.0\% | 30.0\% | 35.0\% | 40.0\% | 45.0\% | 50.0\% | 55.0\% | 60.0\% | 65.0\% | 70.0\% | 75.0\% | 80.0\% | 85.0\% | 90.0\% | 95.0\% | 100.0\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 5\% | 0.05 | 0.09 | 0.12 | 0.15 | 0.17 | 0.20 | 0.22 | 0.25 | 0.27 | 0.29 | 0.31 | 0.33 | 0.35 | 0.37 | 0.39 | 0.40 | 0.42 | 0.44 | 0.45 | 0.47 |
|  | 10.0\% | 0.06 | 0.09 | 0.12 | 0.14 | 0.16 | 0.17 | 0.18 | 0.19 | 0.19 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.19 | 0.19 | 0.18 | 0.17 | 0.16 |
|  | 15.0\% | 0.07 | 0.11 | 0.14 | 0.15 | 0.16 | 0.17 | 0.17 | 0.16 | 0.16 | 0.15 | 0.14 | 0.12 | 0.11 | 0.09 | 0.07 | 0.05 | 0.03 | 0.00 | -0.02 | -0.05 |
|  | 20.0\% | 0.09 | 0.13 | 0.16 | 0.17 | 0.18 | 0.18 | 0.17 | 0.16 | 0.15 | 0.13 | 0.11 | 0.08 | 0.05 | 0.02 | -0.01 | -0.05 | -0.08 | -0.12 | -0.17 | -0.21 |
|  | 25.0\% | 0.11 | 0.16 | 0.19 | 0.20 | 0.21 | 0.20 | 0.19 | 0.18 | 0.15 | 0.13 | 0.10 | 0.06 | 0.02 | -0.02 | -0.06 | -0.11 | -0.16 | -0.21 | -0.27 | -0.32 |
|  | 30.0\% | 0.13 | 0.19 | 0.22 | 0.24 | 0.25 | 0.24 | 0.23 | 0.20 | 0.18 | 0.14 | 0.11 | 0.06 | 0.02 | -0.03 | -0.09 | -0.15 | -0.21 | -0.27 | -0.34 | -0.41 |
|  | 35.0\% | 0.15 | 0.22 | 0.26 | 0.28 | 0.29 | 0.28 | 0.27 | 0.24 | 0.21 | 0.17 | 0.13 | 0.08 | 0.03 | -0.03 | -0.09 | -0.16 | -0.23 | -0.30 | -0.38 | -0.46 |
| DP | 40.0\% | 0.18 | 0.26 | 0.31 | 0.33 | 0.34 | 0.34 | 0.32 | 0.29 | 0.26 | 0.22 | 0.17 | 0.12 | 0.06 | -0.01 | -0.08 | -0.15 | -0.23 | -0.31 | -0.40 | -0.49 |
| $D D$ | 45.0\% | 0.20 | 0.30 | 0.35 | 0.38 | 0.40 | 0.39 | 0.38 | 0.35 | 0.32 | 0.27 | 0.22 | 0.16 | 0.10 | 0.03 | -0.05 | -0.13 | -0.22 | -0.31 | -0.40 | -0.50 |
|  | 50.0\% | 0.23 | 0.34 | 0.40 | 0.44 | 0.46 | 0.46 | 0.44 | 0.42 | 0.38 | 0.33 | 0.28 | 0.22 | 0.15 | 0.07 | -0.01 | -0.09 | -0.19 | -0.28 | -0.38 | -0.49 |
| D1C | 55.0\% | 0.25 | 0.38 | 0.45 | 0.50 | 0.52 | 0.52 | 0.51 | 0.49 | 0.45 | 0.40 | 0.35 | 0.28 | 0.21 | 0.13 | 0.05 | -0.04 | -0.14 | -0.24 | -0.35 | -0.46 |
| P1CKN | 60.0\% | 0.28 | 0.42 | 0.51 | 0.56 | 0.59 | 0.60 | 0.59 | 0.56 | 0.53 | 0.48 | 0.43 | 0.36 | 0.29 | 0.20 | 0.11 | 0.02 | -0.08 | -0.19 | -0.30 | -0.42 |
|  | 65.0\% | 0.31 | 0.46 | 0.56 | 0.62 | 0.66 | 0.67 | 0.67 | 0.65 | 0.61 | 0.57 | 0.51 | 0.44 | 0.37 | 0.28 | 0.19 | 0.09 | -0.01 | -0.13 | -0.24 | -0.37 |
|  | 70.0\% | 0.34 | 0.51 | 0.62 | 0.69 | 0.73 | 0.75 | 0.75 | 0.74 | 0.70 | 0.66 | 0.60 | 0.54 | 0.46 | 0.37 | 0.28 | 0.18 | 0.07 | -0.05 | -0.17 | -0.30 |
|  | 75.0\% | 0.36 | 0.55 | 0.68 | 0.76 | 0.81 | 0.84 | 0.84 | 0.83 | 0.80 | 0.76 | 0.70 | 0.64 | 0.56 | 0.47 | 0.38 | 0.27 | 0.16 | 0.04 | -0.09 | -0.22 |
|  | 80.0\% | 0.39 | 0.60 | 0.74 | 0.83 | 0.89 | 0.93 | 0.94 | 0.93 | 0.90 | 0.86 | 0.81 | 0.74 | 0.66 | 0.58 | 0.48 | 0.37 | 0.26 | 0.14 | 0.01 | -0.13 |
|  | 85.0\% | 0.42 | 0.65 | 0.80 | 0.91 | 0.98 | 1.02 | 1.03 | 1.03 | 1.01 | 0.97 | 0.92 | 0.85 | 0.78 | 0.69 | 0.59 | 0.48 | 0.37 | 0.24 | 0.11 | -0.03 |
|  | 90.0\% | 0.45 | 0.70 | 0.87 | 0.98 | 1.06 | 1.11 | 1.13 | 1.13 | 1.12 | 1.08 | 1.03 | 0.97 | 0.90 | 0.81 | 0.71 | 0.60 | 0.49 | 0.36 | 0.23 | 0.08 |
|  | 95.0\% | 0.49 | 0.75 | 0.93 | 1.06 | 1.15 | 1.21 | 1.24 | 1.24 | 1.23 | 1.20 | 1.16 | 1.09 | 1.02 | 0.93 | 0.84 | 0.73 | 0.61 | 0.48 | 0.35 | 0.20 |
|  | 100.0\% | 0.52 | 0.80 | 1.00 | 1.14 | 1.24 | 1.31 | 1.34 | 1.36 | 1.35 | 1.32 | 1.28 | 1.22 | 1.15 | 1.07 | 0.97 | 0.86 | 0.74 | 0.62 | 0.48 | 0.33 |

- BB's Favor

Neutral
SB's Favor

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## EV when $M=10$

## Hero's Push Range

## Villain's Call <br> Range

|  | 5\% | 10.0\% | 15.0\% | 20.0\% | 25.0\% | 30.0\% | 35.0\% | 40.0\% | 45.0\% | 50.0\% | 55.0\% | 60.0\% | 65.0\% | 70.0\% | 75.0\% | 80.0\% | 85.0\% | 90.0\% | 95.0\% | 100.0\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5\% | 0.05 | 0.09 | 0.13 | 0.17 | 0.21 | 0.24 | 0.28 | 0.32 | 0.35 | 0.39 | 0.42 | 0.45 | 0.49 | 0.52 | 0.55 | 0.59 | 0.62 | 0.65 | 0.68 | 0.71 |
| 10.0\% | 0.05 | 0.09 | 0.13 | 0.16 | 0.19 | 0.22 | 0.25 | 0.28 | 0.31 | 0.33 | 0.35 | 0.38 | 0.40 | 0.42 | 0.44 | 0.46 | 0.48 | 0.50 | 0.52 | 0.54 |
| 15.0\% | 0.06 | 0.10 | 0.14 | 0.17 | 0.19 | 0.22 | 0.24 | 0.26 | 0.28 | 0.30 | 0.31 | 0.33 | 0.34 | 0.36 | 0.37 | 0.38 | 0.39 | 0.40 | 0.41 | 0.42 |
| 20.0\% | 0.07 | 0.11 | 0.14 | 0.17 | 0.20 | 0.22 | 0.24 | 0.25 | 0.27 | 0.28 | 0.29 | 0.30 | 0.30 | 0.31 | 0.31 | 0.32 | 0.32 | 0.32 | 0.32 | 0.32 |
| 25.0\% | 0.08 | 0.12 | 0.16 | 0.19 | 0.21 | 0.23 | 0.24 | 0.25 | 0.26 | 0.27 | 0.27 | 0.28 | 0.28 | 0.28 | 0.28 | 0.27 | 0.27 | 0.26 | 0.25 | 0.25 |
| 30.0\% | 0.09 | 0.14 | 0.17 | 0.20 | 0.22 | 0.24 | 0.25 | 0.26 | 0.27 | 0.27 | 0.27 | 0.27 | 0.26 | 0.26 | 0.25 | 0.24 | 0.23 | 0.22 | 0.20 | 0.19 |
| 35.0\% | 0.10 | 0.15 | 0.19 | 0.22 | 0.24 | 0.26 | 0.27 | 0.27 | 0.28 | 0.28 | 0.27 | 0.27 | 0.26 | 0.25 | 0.24 | 0.22 | 0.20 | 0.19 | 0.16 | 0.14 |
| 40.0\% | 0.11 | 0.17 | 0.21 | 0.24 | 0.26 | 0.28 | 0.29 | 0.29 | 0.29 | 0.29 | 0.28 | 0.27 | 0.26 | 0.25 | 0.23 | 0.21 | 0.19 | 0.16 | 0.14 | 0.11 |
| 45.0\% | 0.12 | 0.18 | 0.23 | 0.26 | 0.29 | 0.30 | 0.31 | 0.32 | 0.32 | 0.31 | 0.30 | 0.29 | 0.27 | 0.25 | 0.23 | 0.21 | 0.18 | 0.15 | 0.12 | 0.09 |
| 50.0\% | 0.13 | 0.20 | 0.25 | 0.29 | 0.31 | 0.33 | 0.34 | 0.34 | 0.34 | 0.33 | 0.32 | 0.31 | 0.29 | 0.27 | 0.24 | 0.21 | 0.18 | 0.15 | 0.12 | 0.08 |
| 55.0\% | 0.14 | 0.22 | 0.28 | 0.32 | 0.34 | 0.36 | 0.37 | 0.37 | 0.37 | 0.36 | 0.35 | 0.33 | 0.31 | 0.29 | 0.26 | 0.23 | 0.19 | 0.16 | 0.12 | 0.08 |
| 60.0\% | 0.16 | 0.24 | 0.30 | 0.34 | 0.37 | 0.39 | 0.40 | 0.41 | 0.40 | 0.39 | 0.38 | 0.36 | 0.34 | 0.31 | 0.28 | 0.25 | 0.21 | 0.17 | 0.13 | 0.08 |
| 65.0\% | 0.17 | 0.26 | 0.33 | 0.37 | 0.41 | 0.43 | 0.44 | 0.44 | 0.44 | 0.43 | 0.41 | 0.39 | 0.37 | 0.34 | 0.31 | 0.27 | 0.23 | 0.19 | 0.14 | 0.09 |
| 70.0\% | 0.18 | 0.28 | 0.35 | 0.40 | 0.44 | 0.46 | 0.47 | 0.48 | 0.48 | 0.47 | 0.45 | 0.43 | 0.40 | 0.37 | 0.34 | 0.30 | 0.26 | 0.21 | 0.16 | 0.11 |
| 75.0\% | 0.20 | 0.31 | 0.38 | 0.44 | 0.47 | 0.50 | 0.51 | 0.52 | 0.52 | 0.51 | 0.49 | 0.47 | 0.44 | 0.41 | 0.38 | 0.33 | 0.29 | 0.24 | 0.19 | 0.13 |
| 80.0\% | 0.21 | 0.33 | 0.41 | 0.47 | 0.51 | 0.54 | 0.56 | 0.56 | 0.56 | 0.55 | 0.54 | 0.51 | 0.49 | 0.45 | 0.42 | 0.37 | 0.33 | 0.28 | 0.22 | 0.16 |
| 85.0\% | 0.23 | 0.35 | 0.44 | 0.50 | 0.55 | 0.58 | 0.60 | 0.61 | 0.61 | 0.60 | 0.58 | 0.56 | 0.53 | 0.50 | 0.46 | 0.42 | 0.37 | 0.32 | 0.26 | 0.20 |
| 90.0\% | 0.24 | 0.38 | 0.47 | 0.54 | 0.59 | 0.62 | 0.64 | 0.66 | 0.66 | 0.65 | 0.63 | 0.61 | 0.58 | 0.55 | 0.51 | 0.46 | 0.41 | 0.36 | 0.30 | 0.24 |
| 95.0\% | 0.26 | 0.40 | 0.50 | 0.58 | 0.63 | 0.67 | 0.69 | 0.70 | 0.71 | 0.70 | 0.69 | 0.66 | 0.64 | 0.60 | 0.56 | 0.51 | 0.46 | 0.41 | 0.35 | 0.28 |
| 100.0\% | 0.27 | 0.42 | 0.53 | 0.61 | 0.67 | 0.71 | 0.74 | 0.76 | 0.76 | 0.75 | 0.74 | 0.72 | 0.69 | 0.66 | 0.62 | 0.57 | 0.52 | 0.46 | 0.40 | 0.33 |

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## $E V$ when $M=1$

## Hero's Push Range

## Villain's Call <br> Range

|  | 5\% | 10.0\% | 15.0\% | 20.0\% | 25.0\% | 30.0\% | 35.0\% | 40.0\% | 45.0\% | 50.0\% | 55.0\% | 60.0\% | 65.0\% | 70.0\% | 75.0\% | 80.0\% | 85.0\% | 90.0\% | 95.0\% | 100.0\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5\% | 0.05 | 0.10 | 0.14 | 0.19 | 0.24 | 0.28 | 0.33 | 0.38 | 0.42 | 0.47 | 0.52 | 0.56 | 0.61 | 0.66 | 0.70 | 0.75 | 0.80 | 0.84 | 0.89 | 0.93 |
| 10.0\% | 0.05 | 0.09 | 0.14 | 0.18 | 0.23 | 0.27 | 0.32 | 0.36 | 0.41 | 0.45 | 0.49 | 0.54 | 0.58 | 0.62 | 0.67 | 0.71 | 0.75 | 0.80 | 0.84 | 0.88 |
| 15.0\% | 0.05 | 0.09 | 0.14 | 0.18 | 0.22 | 0.26 | 0.31 | 0.35 | 0.39 | 0.43 | 0.47 | 0.51 | 0.55 | 0.59 | 0.64 | 0.68 | 0.72 | 0.76 | 0.80 | 0.84 |
| 20.0\% | 0.05 | 0.09 | 0.13 | 0.17 | 0.21 | 0.25 | 0.29 | 0.33 | 0.37 | 0.41 | 0.45 | 0.49 | 0.53 | 0.57 | 0.61 | 0.64 | 0.68 | 0.72 | 0.76 | 0.80 |
| 25.0\% | 0.05 | 0.09 | 0.13 | 0.17 | 0.21 | 0.25 | 0.29 | 0.32 | 0.36 | 0.40 | 0.43 | 0.47 | 0.51 | 0.54 | 0.58 | 0.62 | 0.65 | 0.69 | 0.72 | 0.76 |
| 30.0\% | 0.05 | 0.09 | 0.13 | 0.17 | 0.20 | 0.24 | 0.28 | 0.31 | 0.35 | 0.38 | 0.42 | 0.45 | 0.49 | 0.52 | 0.55 | 0.59 | 0.62 | 0.66 | 0.69 | 0.72 |
| 35.0\% | 0.05 | 0.09 | 0.12 | 0.16 | 0.20 | 0.23 | 0.27 | 0.30 | 0.34 | 0.37 | 0.40 | 0.44 | 0.47 | 0.50 | 0.53 | 0.56 | 0.59 | 0.62 | 0.66 | 0.69 |
| 40.0\% | 0.05 | 0.09 | 0.12 | 0.16 | 0.19 | 0.23 | 0.26 | 0.29 | 0.33 | 0.36 | 0.39 | 0.42 | 0.45 | 0.48 | 0.51 | 0.54 | 0.57 | 0.60 | 0.62 | 0.65 |
| 45.0\% | 0.05 | 0.08 | 0.12 | 0.16 | 0.19 | 0.22 | 0.25 | 0.28 | 0.32 | 0.34 | 0.37 | 0.40 | 0.43 | 0.46 | 0.49 | 0.51 | 0.54 | 0.57 | 0.60 | 0.62 |
| 50.0\% | 0.05 | 0.08 | 0.12 | 0.15 | 0.19 | 0.22 | 0.25 | 0.28 | 0.31 | 0.33 | 0.36 | 0.39 | 0.41 | 0.44 | 0.47 | 0.49 | 0.52 | 0.54 | 0.57 | 0.59 |
| 55.0\% | 0.05 | 0.08 | 0.12 | 0.15 | 0.18 | 0.21 | 0.24 | 0.27 | 0.30 | 0.32 | 0.35 | 0.37 | 0.40 | 0.42 | 0.45 | 0.47 | 0.49 | 0.52 | 0.54 | 0.56 |
| 60.0\% | 0.05 | 0.08 | 0.12 | 0.15 | 0.18 | 0.21 | 0.23 | 0.26 | 0.29 | 0.31 | 0.34 | 0.36 | 0.38 | 0.41 | 0.43 | 0.45 | 0.47 | 0.49 | 0.51 | 0.53 |
| 65.0\% | 0.05 | 0.08 | 0.12 | 0.15 | 0.18 | 0.20 | 0.23 | 0.25 | 0.28 | 0.30 | 0.32 | 0.35 | 0.37 | 0.39 | 0.41 | 0.43 | 0.45 | 0.47 | 0.49 | 0.51 |
| 70.0\% | 0.05 | 0.08 | 0.12 | 0.15 | 0.17 | 0.20 | 0.22 | 0.25 | 0.27 | 0.29 | 0.31 | 0.33 | 0.35 | 0.37 | 0.39 | 0.41 | 0.43 | 0.45 | 0.46 | 0.48 |
| 75.0\% | 0.05 | 0.08 | 0.11 | 0.14 | 0.17 | 0.20 | 0.22 | 0.24 | 0.26 | 0.28 | 0.30 | 0.32 | 0.34 | 0.36 | 0.38 | 0.39 | 0.41 | 0.42 | 0.44 | 0.45 |
| 80.0\% | 0.05 | 0.08 | 0.11 | 0.14 | 0.17 | 0.19 | 0.21 | 0.24 | 0.26 | 0.27 | 0.29 | 0.31 | 0.33 | 0.34 | 0.36 | 0.37 | 0.39 | 0.40 | 0.41 | 0.43 |
| 85.0\% | 0.05 | 0.08 | 0.11 | 0.14 | 0.17 | 0.19 | 0.21 | 0.23 | 0.25 | 0.27 | 0.28 | 0.30 | 0.31 | 0.33 | 0.34 | 0.36 | 0.37 | 0.38 | 0.39 | 0.40 |
| 90.0\% | 0.05 | 0.08 | 0.11 | 0.14 | 0.16 | 0.18 | 0.21 | 0.22 | 0.24 | 0.26 | 0.27 | 0.29 | 0.30 | 0.31 | 0.33 | 0.34 | 0.35 | 0.36 | 0.37 | 0.38 |
| 95.0\% | 0.05 | 0.08 | 0.11 | 0.14 | 0.16 | 0.18 | 0.20 | 0.22 | 0.23 | 0.25 | 0.26 | 0.28 | 0.29 | 0.30 | 0.31 | 0.32 | 0.33 | 0.34 | 0.35 | 0.36 |
| 100.0\% | 0.05 | 0.08 | 0.11 | 0.14 | 0.16 | 0.18 | 0.20 | 0.21 | 0.23 | 0.24 | 0.26 | 0.27 | 0.28 | 0.29 | 0.30 | 0.31 | 0.31 | 0.32 | 0.33 | 0.33 |

- Villain's Favor

Neutral
Hero's Favor

Massachusetts Institute of Technology

## EV when M = 1 (Equilibrium)

Hero's Push Range


## Hero Picks

## EV when M = 2 (Equilibrium)



## EV when M = 3 (Unstable Nash)

Hero's Push Range

Villain's Call Range

|  | 55.0\% | 60.0\% | 65.0\% | 70.0\% | 75.0\% | 80.0\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 55.0\% | 0.35 | 0.36 | 0.38 | 0.39 | 0.40 | 0.42 |
| 60.0\% | 0.35 | 0.36 | 0.37 | 0.38 | 0.40 | 0.40 |
| 65.0\% | 0.34 | 0.36 | 0.37 | 0.38 | 0.39 | 0.39 |
| 70.0\% | 0.34 | 0.36 | 0.37 | 0.37 | 0.38 | 0.39 |
| 75.0\% | 0.35 | 0.35 | 0.36 | 0.37 | 0.38 | 0.38 |
| 80.0\% | 0.35 | 0.36 | 0.36 | 0.3 | 9\%7 | 0.37 |
| 85.0\% | 0.35 | 0.36 | 0.36 | 0.37 | 0.37 | . 37 |
| 90.0\% | 0.35 | 0.36 | 0.36 | 0.37 | 0.37 | . 37 |
| 95.0\% | 0.36 | 0.36 | 0.37 | 0.37 | 0.37 | $\checkmark 3.36$ |
| 100.0\% | 0.36 | 0.37 | 0.37 | $\leftarrow 0.37$ | 0.37 | 0.36 |

## When Hero is SB (and first to act)

Goal: Determine optimal pushing ranges for likely scenarios
Methodology: For each column, identify max EV for the following scenarios
Average Case (No info about Villain's call range)
Worst Case (Villain has best response)
Tight Villain (Assumes 15\% Calling Range) [33+,A4+,KT+]
/ RRVHVillain (Assumes 80 \% Calling Range) [22+, T2+, 93+, etc]

## EV when $\mathrm{M}=1$ to 10

1M

|  | $5 \%$ | $10 \%$ | $15 \%$ | $20 \%$ | $25 \%$ | $30 \%$ | $35 \%$ | $40 \%$ | $45 \%$ | $50 \%$ | $55 \%$ | $60 \%$ | $65 \%$ | $70 \%$ | $75 \%$ | $80 \%$ | $85 \%$ | $90 \%$ | $95 \%$ | $100 \%$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Average | 0.05 | 0.09 | 0.12 | 0.16 | 0.19 | 0.22 | 0.25 | 0.28 | 0.31 | 0.34 | 0.37 | 0.39 | 0.42 | 0.45 | 0.47 | 0.50 | 0.52 | 0.55 | 0.57 | 0.60 |
| Worst | 0.05 | 0.08 | 0.11 | 0.14 | 0.16 | 0.18 | 0.20 | 0.21 | 0.23 | 0.24 | 0.26 | 0.27 | 0.28 | 0.29 | 0.30 | 0.31 | 0.31 | 0.32 | 0.33 | 0.33 |
| Tight(15\%) | 0.05 | 0.09 | 0.14 | 0.18 | 0.22 | 0.26 | 0.31 | 0.35 | 0.39 | 0.43 | 0.47 | 0.51 | 0.55 | 0.59 | 0.64 | 0.68 | 0.72 | 0.76 | 0.80 | 0.84 |
| Loose $(80 \%)$ | 0.05 | 0.08 | 0.11 | 0.14 | 0.17 | 0.19 | 0.21 | 0.24 | 0.26 | 0.27 | 0.29 | 0.31 | 0.33 | 0.34 | 0.36 | 0.37 | 0.39 | 0.40 | 0.41 | 0.43 |

## EV when $\mathrm{M}=1$ to 10

1M

|  | 5\% | 10\% | 15\% | 20\% | 25\% | 30\% | 35\% | 40\% | 45\% | 50\% | 55\% | 60\% | 65\% | 70\% | 75\% | 80\% | 85\% | 90\% | 95\% | 100\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Average | 0.05 | 0.09 | 0.12 | 0.16 | 0.19 | 0.22 | 0.25 | 0.28 | 0.31 | 0.34 | 0.37 | 0.39 | 0.42 | 0.45 | 0.47 | 0.50 | 0.52 | 0.55 | 0.57 | 0.60 |
| Worst | 0.05 | 0.08 | 0.11 | 0.14 | 0.16 | 0.18 | 0.20 | 0.21 | 0.23 | 0.24 | 0.26 | 0.27 | 0.28 | 0.29 | 0.30 | 0.31 | 0.31 | 0.32 |  | 0.33 |
| Tight(15\%) | 0.05 | 0.09 | 0.14 | 0.18 | 0.22 | 0.26 | 0.31 | 0.35 | 0.39 | 0.43 | 0.47 | 0.51 | 0.55 | 0.59 | 0.64 | 0.68 | 0.72 |  |  | 0.84 |
| Losese 880 | 0.05 | 0.08 | 0.11 | 0.14 | 0.17 | 0.19 | 0.21 | 0.24 | 0.26 | 0.27 | 0.29 | 0.31 | 0.33 | 0.34 | 0.36 | 0.37 | 0.39 | 40 | 0.41 | 0.43 |


|  | 5\% | 10\% | 15\% | 20\% | 25\% | 30\% | 35\% | 40\% | 45\% | 50\% | 55\% | 60\% | 65\% | 70\% | 75\% | 80\% |  | 90\% | 95\% | 10\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Average | 0.07 | 0.12 | 0.16 | 0.20 | 0.23 | 0.26 | 0.29 | 0.31 | 0.33 | 0.36 | 0.38 | 0.40 | 0.41 | 0.43 | 0.45 |  | 0.48 | 0.49 | 0.50 | 0.51 |
| Worst | 0.05 | 0.09 | 0.13 | 0.17 | 0.21 | 0.24 | 0.27 | 0.29 | 0.31 | 0.33 | 0.34 | 0.35 | 0.36 | 0.37 | 0.37 |  | 0.36 | 0.35 | 0.34 | 0.33 |
| Tight (15\%) | 0.05 | 0.09 | 0.14 | 0.18 | 0.21 | 0.25 | 0.29 | 0.33 | 0.36 | 0.40 | 0.44 | 0.47 | 0.51 | 0.54 |  | 0.61 | 0.64 | 0.68 | 0.71 | 0.74 |
| Loose (80\%) | 0.08 | 0.14 | 0.18 | 0.21 | 0.24 | 0.27 | 0.29 | 0.31 | 0.32 | 0.34 | 0.35 | 0.36 | 0.36 | 0.37 |  | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 |

5M

|  | 5\% | 10\% | 15\% | 20\% | 25\% | 30\% | 35\% | 40\% | 45\% | 50\% | 55\% | 60\% | 65\% |  | 75\% | 80\% | 85\% | 90\% | 95\% | 100\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Average | 0.09 | 0.15 | 0.20 | 0.23 | 0.27 | 0.30 | 0.32 | 0.34 | 0.36 | 0.38 | 0.39 | 0.40 |  | 42 | 0.42 | 0.43 | 0.43 | 0.43 | 0.43 | 0.43 |
| Worst | 0.05 | 0.09 | 0.13 | 0.17 | 0.21 | 0.24 | 0.27 | 0.29 | 0.31 | 0.33 | 0.34 |  |  | 0.36 | 0.36 | 0.36 | 0.35 | 0.34 | 0.33 | 0.31 |
| Tight (15\%) | 0.05 | 0.10 | 0.14 | 0.17 | 0.21 | 0.24 | 0.28 | 0.31 | 0.34 | 0.37 | 0.40 |  |  | 0.49 | 0.52 | 0.54 | 0.57 | 0.60 | 0.62 | 0.65 |
| Loose (88\%) | 0.12 | 0.19 | 0.25 | 0.29 | 0.32 | 0.35 | 0.37 | 0.38 | 0.39 | 0.40 | 0.40 | . 40 | 0.40 | 0.39 | 0.38 | 0.37 | 0.36 | 0.35 | 0.33 | 0.31 |


|  | 5\% | 10\% | 15\% | 20\% | 25\% | 30\% | 35\% | 40\% | 45\% | 50\% | N\% | 60\% | 65\% | 70\% | 75\% | 80\% | 85\% | 90\% | 95\% | 10\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Average | 0.14 | 0.23 | 0.29 | 0.33 | 0.37 | 0.39 | 0.41 | 0.42 | 0.42 |  | 0.42 | 0.41 | 0.40 | 0.38 | 0.36 | 0.34 | 0.32 | 0.29 | 0.26 | 0.23 |
| Worst | 0.05 | 0.09 | 0.13 | 0.16 | 0.19 | 0.22 | 0.24 | 0.25 | 0.26 | 140] | 0.27 | 0.27 | 0.26 | 0.25 | 0.23 | 0.21 | 0.18 | 0.15 | 0.12 | 0.08 |
| Tight(15\%) | 0.06 | 0.10 | 0.14 | 0.17 | 0.19 | 0.22 | 0.24 | 0.26 | 0.28 | 0.30 | 0.31 | 0.33 | 0.34 | 0.36 | 0.37 | 0.38 | 0.39 | 0.40 | 0.41 | 0.42 |
| Loose(80\%) | 0.21 | 0.33 | 0.41 | 0.47 | 0.51 | 0.54 | 0.56 | 0.56 | 0.56 | 0.55 | 0.54 | 0.51 | 0.49 | 0.45 | 0.42 | 0.37 | 0.33 | 0.28 | 0.22 | 0.16 |

## SB Rules of Thumb

If you think Villain is tight, top $100 \%$ is always optimal
In other scenarios, top $100 \%$ is optimal for $\mathrm{M}=1$, then optimal slowly drifts to top $50 \%$ for 10 M

The biggest mistake would be pushing less often than $50 \%$ of hands

## When Hero is BB (second to act)

Goal: Determine optimal calling ranges given SB's behavior
SB will always have an edge for $\mathrm{M}<10$ because of fold equity
We want to develop a practical rule for our calling range
We want to make as few assumptions as possible

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## EV when M = 5

## Villain's Push Range

|  | 5\% | 10.0\% | 15.0\% | 20.0\% | 25.0\% | 30.0\% | 35.0\% | 40.0\% | 45.0\% | 50.0\% | 55.0\% | 60.0\% | 65.0\% | 70.0\% | 75.0\% | 80.0\% | 85.0\% | 90.0\% | 95.0\% | 100.0\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5\% | 0.05 | 0.09 | 0.14 | 0.18 | 0.22 | 0.27 | 0.31 | 0.35 | 0.39 | 0.43 | 0.47 | 0.51 | 0.56 | 0.60 | 0.64 | 0.68 | 0.72 | 0.76 | 0.80 | 0.84 |
| 10.0\% | 0.05 | 0.09 | 0.13 | 0.17 | 0.21 | 0.25 | 0.29 | 0.32 | 0.36 | 0.40 | 0.43 | 0.47 | 0.50 | 0.53 | 0.57 | 0.60 | 0.63 | 0.67 | 0.70 | 0.73 |
| 15.0\% | 0.05 | 0.10 | 0.14 | 0.17 | 0.21 | 0.24 | 0.28 | 0.31 | 0.34 | 0.37 | 0.40 | 0.43 | 0.46 | 0.49 | 0.52 | 0.54 | 0.57 | 0.60 | 0.62 | 0.65 |
| 20.0\% | 0.06 | 0.10 | 0.14 | 0.17 | 0.21 | 0.24 | 0.27 | 0.30 | 0.33 | 0.35 | 0.38 | 0.40 | 0.43 | 0.45 | 0.48 | 0.50 | 0.52 | 0.54 | 0.56 | 0.58 |
| 25.0\% | 0.06 | 0.10 | 0.14 | 0.18 | 0.21 | 0.24 | 0.27 | 0.29 | 0.32 | 0.34 | 0.36 | 0.38 | 0.41 | 0.43 | 0.44 | 0.46 | 0.48 | 0.50 | 0.51 | 0.53 |
| 30.0\% | 0.06 | 0.11 | 0.15 | 0.18 | 0.21 | 0.24 | 0.27 | 0.29 | 0.31 | 0.33 | 0.35 | 0.37 | 0.39 | 0.40 | 0.42 | 0.43 | 0.45 | 0.46 | 0.47 | 0.48 |
| 35.0\% | 0.07 | 0.12 | 0.15 | 0.19 | 0.22 | 0.24 | 0.27 | 0.29 | 0.31 | 0.33 | 0.35 | 0.36 | 0.38 | 0.39 | 0.40 | 0.41 | 0.42 | 0.43 | 0.44 | 0.44 |
| 40.0\% | 0.07 | 0.12 | 0.16 | 0.20 | 0.22 | 0.25 | 0.27 | 0.29 | 0.31 | 0.33 | 0.34 | 0.35 | 0.37 | 0.38 | 0.38 | 0.39 | 0.40 | 0.40 | 0.41 | 0.41 |
| 45.0\% | 0.08 | 0.13 | 0.17 | 0.20 | 0.23 | 0.26 | 0.28 | 0.30 | 0.32 | 0.33 | 0.34 | 0.35 | 0.36 | 0.37 | 0.37 | 0.38 | 0.38 | 0.38 | 0.39 | 0.39 |
| 50.0\% | 0.08 | 0.14 | 0.18 | 0.21 | 0.24 | 0.27 | 0.29 | 0.31 | 0.32 | 0.33 | 0.34 | 0.35 | 0.36 | 0.36 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 0.36 |
| 55.0\% | 0.09 | 0.15 | 0.19 | 0.22 | 0.25 | 0.28 | 0.30 | 0.31 | 0.33 | 0.34 | 0.35 | 0.35 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.35 | 0.35 |
| 60.0\% | 0.10 | 0.15 | 0.20 | 0.24 | 0.27 | 0.29 | 0.31 | 0.33 | 0.34 | 0.35 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.35 | 0.35 | 0.34 | 0.33 |
| 65.0\% | 0.10 | 0.16 | 0.21 | 0.25 | 0.28 | 0.30 | 0.32 | 0.34 | 0.35 | 0.36 | 0.36 | 0.37 | 0.37 | 0.37 | 0.36 | 0.36 | 0.35 | 0.34 | 0.33 | 0.32 |
| 70.0\% | 0.11 | 0.17 | 0.22 | 0.26 | 0.29 | 0.32 | 0.34 | 0.35 | 0.36 | 0.37 | 0.37 | 0.38 | 0.38 | 0.37 | 0.37 | 0.36 | 0.35 | 0.34 | 0.33 | 0.31 |
| 75.0\% | 0.11 | 0.18 | 0.23 | 0.27 | 0.31 | 0.33 | 0.35 | 0.37 | 0.38 | 0.38 | 0.39 | 0.39 | 0.39 | 0.38 | 0.38 | 0.37 | 0.36 | 0.34 | 0.33 | 0.31 |
| 80.0\% | 0.12 | 0.19 | 0.25 | 0.29 | 0.32 | 0.35 | 0.37 | 0.38 | 0.39 | 0.40 | 0.40 | 0.40 | 0.40 | 0.39 | 0.38 | 0.37 | 0.36 | 0.35 | 0.33 | 0.31 |
| 85.0\% | 0.13 | 0.20 | 0.26 | 0.30 | 0.34 | 0.36 | 0.38 | 0.40 | 0.41 | 0.41 | 0.42 | 0.42 | 0.41 | 0.40 | 0.39 | 0.38 | 0.37 | 0.35 | 0.33 | 0.31 |
| 90.0\% | 0.13 | 0.21 | 0.27 | 0.32 | 0.35 | 0.38 | 0.40 | 0.42 | 0.43 | 0.43 | 0.43 | 0.43 | 0.43 | 0.42 | 0.41 | 0.39 | 0.38 | 0.36 | 0.34 | 0.32 |
| 95.0\% | 0.14 | 0.22 | 0.29 | 0.33 | 0.37 | 0.40 | 0.42 | 0.43 | 0.44 | 0.45 | 0.45 | 0.45 | 0.44 | 0.43 | 0.42 | 0.41 | 0.39 | 0.37 | 0.35 | 0.32 |
| 100.0\% | 0.15 | 0.23 | 0.30 | 0.35 | 0.39 | 0.42 | 0.44 | 0.45 | 0.46 | 0.47 | 0.47 | 0.47 | 0.46 | 0.45 | 0.44 | 0.42 | 0.40 | 0.38 | 0.36 | 0.33 |

Hero's Favor
Neutral
Villain's Favor

## EV when M = 5 (steeper gradient)

## Villain's Push Range

Hero's
Call
Range

|  | 5\% | 10.0\% | 15.0\% | 20.0\% | 25.0\% | 30.0\% | 35.0\% | 40.0\% | 45.0\% | 50.0\% | 55.0\% | 60.0\% | 65.0\% | 70.0\% | 75.0\% | 80.0\% | 85.0\% | 90.0\% | 95.0\% | 100.0\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5\% | 0.05 | 0.09 | 0.14 | 0.18 | 0.22 | 0.27 | 0.31 | 0.35 | 0.39 | 0.43 | 0.47 | 0.51 | 0.56 | 0.60 | 0.64 | 0.68 | 0.72 | 0.76 | 0.80 | 0.84 |
| 10.0\% | 0.05 | 0.09 | 0.13 | 0.17 | 0.21 | 0.25 | 0.29 | 0.32 | 0.36 | 0.40 | 0.43 | 0.47 | 0.50 | 0.53 | 0.57 | 0.60 | 0.63 | 0.67 | 0.70 | 0.73 |
| 15.0\% | 0.05 | 0.10 | 0.14 | 0.17 | 0.21 | 0.24 | 0.28 | 0.31 | 0.34 | 0.37 | 0.40 | 0.43 | 0.46 | 0.49 | 0.52 | 0.54 | 0.57 | 0.60 | 0.62 | 0.65 |
| 20.0\% | 0.06 | 0.10 | 0.14 | 0.17 | 0.21 | 0.24 | 0.27 | 0.30 | 0.33 | 0.35 | 0.38 | 0.40 | 0.43 | 0.45 | 0.48 | 0.50 | 0.52 | 0.54 | 0.56 | 0.58 |
| 25.0\% | 0.06 | 0.10 | 0.14 | 0.18 | 0.21 | 0.24 | 0.27 | 0.29 | 0.32 | 0.34 | 0.36 | 0.38 | 0.41 | 0.43 | 0.44 | 0.46 | 0.48 | 0.50 | 0.51 | 0.53 |
| 30.0\% | 0.06 | 0.11 | 0.15 | 0.18 | 0.21 | 0.24 | 0.27 | 0.29 | 0.31 | 0.33 | 0.35 | 0.37 | 0.39 | 0.40 | 0.42 | 0.43 | 0.45 | 0.46 | 0.47 | 0.48 |
| 35.0\% | 0.07 | 0.12 | 0.15 | 0.19 | 0.22 | 0.24 | 0.27 | 0.29 | 0.31 | 0.33 | 0.35 | 0.36 | 0.38 | 0.39 | 0.40 | 0.41 | 0.42 | 0.43 | 0.44 | 0.44 |
| 40.0\% | 0.07 | 0.12 | 0.16 | 0.20 | 0.22 | 0.25 | 0.27 | 0.29 | 0.31 | 0.33 | 0.34 | 0.35 | 0.37 | 0.38 | 0.38 | 0.39 | 0.40 | 0.40 | 0.41 | 0.41 |
| 45.0\% | 0.08 | 0.13 | 0.17 | 0.20 | 0.23 | 0.26 | 0.28 | 0.30 | 0.32 | 0.33 | 0.34 | 0.35 | 0.36 | 0.37 | 0.37 | 0.38 | 0.38 | 0.38 | 0.39 | 0.39 |
| 50.0\% | 0.08 | 0.14 | 0.18 | 0.21 | 0.24 | 0.27 | 0.29 | 0.31 | 0.32 | 0.33 | 0.34 | 0.35 | 0.36 | 0.36 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 0.36 |
| 55.0\% | 0.09 | 0.15 | 0.19 | 0.22 | 0.25 | 0.28 | 0.30 | 0.31 | 0.33 | 0.34 | 0.35 | 0.35 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.35 | 0.35 |
| 60.0\% | 0.10 | 0.15 | 0.20 | 0.24 | 0.27 | 0.29 | 0.31 | 0.33 | 0.34 | 0.35 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.35 | 0.35 | 0.34 | 0.33 |
| 65.0\% | 0.10 | 0.16 | 0.21 | 0.25 | 0.28 | 0.30 | 0.32 | 0.34 | 0.35 | 0.36 | 0.36 | 0.37 | 0.37 | 0.37 | 0.36 | 0.36 | 0.35 | 0.34 | 0.33 | 0.32 |
| 70.0\% | 0.11 | 0.17 | 0.22 | 0.26 | 0.29 | 0.32 | 0.34 | 0.35 | 0.36 | 0.37 | 0.37 | 0.38 | 0.38 | 0.37 | 0.37 | 0.36 | 0.35 | 0.34 | 0.33 | 0.31 |
| 75.0\% | 0.11 | 0.18 | 0.23 | 0.27 | 0.31 | 0.33 | 0.35 | 0.37 | 0.38 | 0.38 | 0.39 | 0.39 | 0.39 | 0.38 | 0.38 | 0.37 | 0.36 | 0.34 | 0.33 | 0.31 |
| 80.0\% | 0.12 | 0.19 | 0.25 | 0.29 | 0.32 | 0.35 | 0.37 | 0.38 | 0.39 | 0.40 | 0.40 | 0.40 | 0.40 | 0.39 | 0.38 | 0.37 | 0.36 | 0.35 | 0.33 | 0.31 |
| 85.0\% | 0.13 | 0.20 | 0.26 | 0.30 | 0.34 | 0.36 | 0.38 | 0.40 | 0.41 | 0.41 | 0.42 | 0.42 | 0.41 | 0.40 | 0.39 | 0.38 | 0.37 | 0.35 | 0.33 | 0.31 |
| 90.0\% | 0.13 | 0.21 | 0.27 | 0.32 | 0.35 | 0.38 | 0.40 | 0.42 | 0.43 | 0.43 | 0.43 | 0.43 | 0.43 | 0.42 | 0.41 | 0.39 | 0.38 | 0.36 | 0.34 | 0.32 |
| 95.0\% | 0.14 | 0.22 | 0.29 | 0.33 | 0.37 | 0.40 | 0.42 | 0.43 | 0.44 | 0.45 | 0.45 | 0.45 | 0.44 | 0.43 | 0.42 | 0.41 | 0.39 | 0.37 | 0.35 | 0.32 |
| 100.0\% | 0.15 | 0.23 | 0.30 | 0.35 | 0.39 | 0.42 | 0.44 | 0.45 | 0.46 | 0.47 | 0.47 | 0.47 | 0.46 | 0.45 | 0.44 | 0.42 | 0.40 | 0.38 | 0.36 | 0.33 |

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## EV when $\mathrm{M}=1$

## Villain's Push Range

|  |  | 5\% | 10\% | 15\% | 20\% | 25\% | 30\% | 35\% | 40\% | 45\% | 50\% | 55\% | 60\% | 65\% | 70\% | 75\% | 80\% | 85\% | 90\% | 95\% | 100\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 5\% | 0.05 | 0.10 | 0.14 | 0.19 | 0.24 | 0.28 | 0.33 | 0.38 | 0.42 | 0.47 | 0.52 | 0.56 | 0.61 | 0.66 | 0.70 | 0.75 | 0.80 | 0.84 | 0.89 | 0.93 |
|  | 10\% | 0.05 | 0.09 | 0.14 | 0.18 | 0.23 | 0.27 | 0.32 | 0.36 | 0.41 | 0.45 | 0.49 | 0.54 | 0.58 | 0.62 | 0.67 | 0.71 | 0.75 | 0.80 | 0.84 | 0.88 |
|  | 15\% | 0.05 | 0.09 | 0.14 | 0.18 | 0.22 | 0.26 | 0.31 | 0.35 | 0.39 | 0.43 | 0.47 | 0.51 | 0.55 | 0.59 | 0.64 | 0.68 | 0.72 | 0.76 | 0.80 | 0.84 |
|  | 20\% | 0.05 | 0.09 | 0.13 | 0.17 | 0.21 | 0.25 | 0.29 | 0.33 | 0.37 | 0.41 | 0.45 | 0.49 | 0.53 | 0.57 | 0.61 | 0.64 | 0.68 | 0.72 | 0.76 | 0.80 |
|  | 25\% | 0.05 | 0.09 | 0.13 | 0.17 | 0.21 | 0.25 | 0.29 | 0.32 | 0.36 | 0.40 | 0.43 | 0.47 | 0.51 | 0.54 | 0.58 | 0.62 | 0.65 | 0.69 | 0.72 | 0.76 |
| T | 30\% | 0.05 | 0.09 | 0.13 | 0.17 | 0.20 | 0.24 | 0.28 | 0.31 | 0.35 | 0.38 | 0.42 | 0.45 | 0.49 | 0.52 | 0.55 | 0.59 | 0.62 | 0.66 | 0.69 | 0.72 |
| Hero | 35\% | 0.05 | 0.09 | 0.12 | 0.16 | 0.20 | 0.23 | 0.27 | 0.30 | 0.34 | 0.37 | 0.40 | 0.44 | 0.47 | 0.50 | 0.53 | 0.56 | 0.59 | 0.62 | 0.66 | 0.69 |
|  | 40\% | 0.05 | 0.09 | 0.12 | 0.16 | 0.19 | 0.23 | 0.26 | 0.29 | 0.33 | 0.36 | 0.39 | 0.42 | 0.45 | 0.48 | 0.51 | 0.54 | 0.57 | 0.60 | 0.62 | 0.65 |
|  | 45\% | 0.05 | 0.08 | 0.12 | 0.16 | 0.19 | 0.22 | 0.25 | 0.28 | 0.32 | 0.34 | 0.37 | 0.40 | 0.43 | 0.46 | 0.49 | 0.51 | 0.54 | 0.57 | 0.60 | 0.62 |
| - | 50\% | 0.05 | 0.08 | 0.12 | 0.15 | 0.19 | 0.22 | 0.25 | 0.28 | 0.31 | 0.33 | 0.36 | 0.39 | 0.41 | 0.44 | 0.47 | 0.49 | 0.52 | 0.54 | 0.57 | 0.59 |
|  | 55\% | 0.05 | 0.08 | 0.12 | 0.15 | 0.18 | 0.21 | 0.24 | 0.27 | 0.30 | 0.32 | 0.35 | 0.37 | 0.40 | 0.42 | 0.45 | 0.47 | 0.49 | 0.52 | 0.54 | 0.56 |
|  | 60\% | 0.05 | 0.08 | 0.12 | 0.15 | 0.18 | 0.21 | 0.23 | 0.26 | 0.29 | 0.31 | 0.34 | 0.36 | 0.38 | 0.41 | 0.43 | 0.45 | 0.47 | 0.49 | 0.51 | 0.53 |
| 91 O | 65\% | 0.05 | 0.08 | 0.12 | 0.15 | 0.18 | 0.20 | 0.23 | 0.25 | 0.28 | 0.30 | 0.32 | 0.35 | 0.37 | 0.39 | 0.41 | 0.43 | 0.45 | 0.47 | 0.49 | 0.51 |
| 1 Male | 70\% | 0.05 | 0.08 | 0.12 | 0.15 | 0.17 | 0.20 | 0.22 | 0.25 | 0.27 | 0.29 | 0.31 | 0.33 | 0.35 | 0.37 | 0.39 | 0.41 | 0.43 | 0.45 | 0.46 | 0.48 |
|  | 75\% | 0.05 | 0.08 | 0.11 | 0.14 | 0.17 | 0.20 | 0.22 | 0.24 | 0.26 | 0.28 | 0.30 | 0.32 | 0.34 | 0.36 | 0.38 | 0.39 | 0.41 | 0.42 | 0.44 | 0.45 |
|  | 80\% | 0.05 | 0.08 | 0.11 | 0.14 | 0.17 | 0.19 | 0.21 | 0.24 | 0.26 | 0.27 | 0.29 | 0.31 | 0.33 | 0.34 | 0.36 | 0.37 | 0.39 | 0.40 | 0.41 | 0.43 |
|  | 85\% | 0.05 | 0.08 | 0.11 | 0.14 | 0.17 | 0.19 | 0.21 | 0.23 | 0.25 | 0.27 | 0.28 | 0.30 | 0.31 | 0.33 | 0.34 | 0.36 | 0.37 | 0.38 | 0.39 | 0.40 |
|  | 90\% | 0.05 | 0.08 | 0.11 | 0.14 | 0.16 | 0.18 | 0.21 | 0.22 | 0.24 | 0.26 | 0.27 | 0.29 | 0.30 | 0.31 | 0.33 | 0.34 | 0.35 | 0.36 | 0.37 | 0.38 |
|  | 95\% | 0.05 | 0.08 | 0.11 | 0.14 | 0.16 | 0.18 | 0.20 | 0.22 | 0.23 | 0.25 | 0.26 | 0.28 | 0.29 | 0.30 | 0.31 | 0.32 | 0.33 | 0.34 | 0.35 | 0.36 |
|  | 100\% | 0.05 | 0.08 | 0.11 | 0.14 | 0.16 | 0.18 | 0.20 | 0.21 | 0.23 | 0.24 | 0.26 | 0.27 | 0.28 | 0.29 | 0.30 | 0.31 | 0.31 | 0.32 | 0.33 | 0.33 |

## EV when M = 2

## Villain's Push Range

|  |  | 5\% | 10\% | 15\% | 20\% | 25\% | 30\% | 35\% | 40\% | 45\% | 50\% | 55\% | 60\% | 65\% | 70\% | 75\% | 80\% | 85\% | 90\% | 95\% | 100\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 5\% | 0.05 | 0.10 | 0.14 | 0.19 | 0.23 | 0.28 | 0.33 | 0.37 | 0.42 | 0.46 | 0.51 | 0.55 | 0.60 | 0.64 | 0.69 | 0.73 | 0.78 | 0.82 | 0.86 | 0.91 |
|  | 10\% | 0.05 | 0.09 | 0.14 | 0.18 | 0.22 | 0.27 | 0.31 | 0.35 | 0.39 | 0.44 | 0.48 | 0.52 | 0.56 | 0.60 | 0.64 | 0.68 | 0.72 | 0.76 | 0.80 | 0.85 |
|  | 15\% | 0.05 | 0.09 | 0.14 | 0.18 | 0.22 | 0.26 | 0.30 | 0.34 | 0.38 | 0.42 | 0.45 | 0.49 | 0.53 | 0.57 | 0.61 | 0.64 | 0.68 | 0.72 | 0.75 | 0.79 |
|  | 20\% | 0.05 | 0.09 | 0.13 | 0.17 | 0.21 | 0.25 | 0.29 | 0.33 | 0.36 | 0.40 | 0.43 | 0.47 | 0.50 | 0.54 | 0.57 | 0.61 | 0.64 | 0.68 | 0.71 | 0.74 |
|  | 25\% | 0.05 | 0.09 | 0.13 | 0.17 | 0.21 | 0.24 | 0.28 | 0.32 | 0.35 | 0.38 | 0.42 | 0.45 | 0.48 | 0.51 | 0.55 | 0.58 | 0.61 | 0.64 | 0.67 | 0.70 |
| TT ${ }^{9}$ | 30\% | 0.05 | 0.09 | 0.13 | 0.17 | 0.21 | 0.24 | 0.27 | 0.31 | 0.34 | 0.37 | 0.40 | 0.43 | 0.46 | 0.49 | 0.52 | 0.55 | 0.58 | 0.61 | 0.63 | 0.66 |
| -1er0 | 35\% | 0.05 | 0.09 | 0.13 | 0.17 | 0.20 | 0.24 | 0.27 | 0.30 | 0.33 | 0.36 | 0.39 | 0.42 | 0.44 | 0.47 | 0.50 | 0.52 | 0.55 | 0.58 | 0.60 | 0.63 |
|  | 40\% | 0.05 | 0.09 | 0.13 | 0.17 | 0.20 | 0.23 | 0.26 | 0.29 | 0.32 | 0.35 | 0.38 | 0.40 | 0.43 | 0.45 | 0.48 | 0.50 | 0.53 | 0.55 | 0.57 | 0.59 |
|  | 45\% | 0.05 | 0.10 | 0.13 | 0.17 | 0.20 | 0.23 | 0.26 | 0.29 | 0.32 | 0.34 | 0.37 | 0.39 | 0.41 | 0.44 | 0.46 | 0.48 | 0.50 | 0.52 | 0.54 | 0.56 |
| - | 50\% | 0.06 | 0.10 | 0.13 | 0.17 | 0.20 | 0.23 | 0.26 | 0.28 | 0.31 | 0.33 | 0.36 | 0.38 | 0.40 | 0.42 | 0.44 | 0.46 | 0.48 | 0.50 | 0.52 | 0.53 |
|  | 55\% | 0.06 | 0.10 | 0.14 | 0.17 | 0.20 | 0.23 | 0.26 | 0.28 | 0.30 | 0.33 | 0.35 | 0.37 | 0.39 | 0.41 | 0.43 | 0.44 | 0.46 | 0.48 | 0.49 | 0.51 |
|  | 60\% | 0.06 | 0.10 | 0.14 | 0.17 | 0.20 | 0.23 | 0.25 | 0.28 | 0.30 | 0.32 | 0.34 | 0.36 | 0.38 | 0.40 | 0.41 | 0.43 | 0.44 | 0.46 | 0.47 | 0.48 |
| $\square$ | 65\% | 0.06 | 0.10 | 0.14 | 0.17 | 0.20 | 0.23 | 0.25 | 0.28 | 0.30 | 0.32 | 0.33 | 0.35 | 0.37 | 0.38 | 0.40 | 0.41 | 0.42 | 0.44 | 0.45 | 0.46 |
| - | 70\% | 0.06 | 0.11 | 0.14 | 0.17 | 0.20 | 0.23 | 0.25 | 0.27 | 0.29 | 0.31 | 0.33 | 0.34 | 0.36 | 0.37 | 0.39 | 0.40 | 0.41 | 0.42 | 0.43 | 0.44 |
|  | 75\% | 0.06 | 0.11 | 0.14 | 0.18 | 0.20 | 0.23 | 0.25 | 0.27 | 0.29 | 0.31 | 0.32 | 0.34 | 0.35 | 0.36 | 0.38 | 0.39 | 0.39 | 0.40 | 0.41 | 0.42 |
|  | 80\% | 0.07 | 0.11 | 0.15 | 0.18 | 0.21 | 0.23 | 0.25 | 0.27 | 0.29 | 0.31 | 0.32 | 0.33 | 0.34 | 0.36 | 0.36 | 0.37 | 0.38 | 0.39 | 0.39 | 0.40 |
|  | 85\% | 0.07 | 0.11 | 0.15 | 0.18 | 0.21 | 0.23 | 0.25 | 0.27 | 0.29 | 0.30 | 0.32 | 0.33 | 0.34 | 0.35 | 0.36 | 0.36 | 0.37 | 0.37 | 0.38 | 0.38 |
|  | 90\% | 0.07 | 0.12 | 0.15 | 0.18 | 0.21 | 0.23 | 0.25 | 0.27 | 0.29 | 0.30 | 0.31 | 0.32 | 0.33 | 0.34 | 0.35 | 0.35 | 0.36 | 0.36 | 0.36 | 0.36 |
|  | 95\% | 0.07 | 0.12 | 0.16 | 0.19 | 0.21 | 0.24 | 0.26 | 0.27 | 0.29 | 0.30 | 0.31 | 0.32 | 0.33 | 0.33 | 0.34 | 0.34 | 0.35 | 0.35 | 0.35 | 0.35 |
|  | 100\% | 0.07 | 0.12 | 0.16 | 0.19 | 0.22 | 0.24 | 0.26 | 0.27 | 0.29 | 0.30 | 0.31 | 0.32 | 0.32 | 0.33 | 0.33 | 0.34 | 0.34 | 0.34 | 0.34 | 0.33 |

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## EV when M = 3

## Villain's Push Range

|  |  | 5\% | 10\% | 15\% | 20\% | 25\% | 30\% | 35\% | 40\% | 45\% | 50\% | 55\% | 60\% | 65\% | 70\% | 75\% | 80\% | 85\% | 90\% | 95\% | 100\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 5\% | 0.05 | 0.09 | 0.14 | 0.19 | 0.23 | 0.28 | 0.32 | 0.36 | 0.41 | 0.45 | 0.50 | 0.54 | 0.58 | 0.63 | 0.67 | 0.71 | 0.76 | 0.80 | 0.84 | 0.88 |
|  | 10\% | 0.05 | 0.09 | 0.14 | 0.18 | 0.22 | 0.26 | 0.30 | 0.34 | 0.38 | 0.42 | 0.46 | 0.50 | 0.54 | 0.58 | 0.62 | 0.66 | 0.69 | 0.73 | 0.77 | 0.81 |
|  | 15\% | 0.05 | 0.09 | 0.14 | 0.18 | 0.21 | 0.25 | 0.29 | 0.33 | 0.36 | 0.40 | 0.44 | 0.47 | 0.51 | 0.54 | 0.58 | 0.61 | 0.64 | 0.68 | 0.71 | 0.74 |
|  | 20\% | 0.05 | 0.09 | 0.13 | 0.17 | 0.21 | 0.25 | 0.28 | 0.32 | 0.35 | 0.38 | 0.42 | 0.45 | 0.48 | 0.51 | 0.54 | 0.57 | 0.60 | 0.63 | 0.66 | 0.69 |
|  | 25\% | 0.05 | 0.10 | 0.14 | 0.17 | 0.21 | 0.24 | 0.28 | 0.31 | 0.34 | 0.37 | 0.40 | 0.43 | 0.46 | 0.48 | 0.51 | 0.54 | 0.57 | 0.59 | 0.62 | 0.64 |
| TT ${ }^{9}$ | 30\% | 0.05 | 0.10 | 0.14 | 0.17 | 0.21 | 0.24 | 0.27 | 0.30 | 0.33 | 0.36 | 0.39 | 0.41 | 0.44 | 0.46 | 0.49 | 0.51 | 0.53 | 0.56 | 0.58 | 0.60 |
| -1er0 | 35\% | 0.06 | 0.10 | 0.14 | 0.17 | 0.21 | 0.24 | 0.27 | 0.30 | 0.32 | 0.35 | 0.37 | 0.40 | 0.42 | 0.44 | 0.47 | 0.49 | 0.51 | 0.53 | 0.55 | 0.57 |
|  | 40\% | 0.06 | 0.10 | 0.14 | 0.18 | 0.21 | 0.24 | 0.27 | 0.29 | 0.32 | 0.34 | 0.36 | 0.39 | 0.41 | 0.43 | 0.45 | 0.47 | 0.48 | 0.50 | 0.52 | 0.53 |
|  | 45\% | 0.06 | 0.11 | 0.15 | 0.18 | 0.21 | 0.24 | 0.27 | 0.29 | 0.32 | 0.34 | 0.36 | 0.38 | 0.40 | 0.41 | 0.43 | 0.45 | 0.46 | 0.48 | 0.49 | 0.50 |
| - | 50\% | 0.06 | 0.11 | 0.15 | 0.18 | 0.21 | 0.24 | 0.27 | 0.29 | 0.31 | 0.33 | 0.35 | 0.37 | 0.39 | 0.40 | 0.42 | 0.43 | 0.44 | 0.46 | 0.47 | 0.48 |
|  | 55\% | 0.07 | 0.11 | 0.15 | 0.19 | 0.22 | 0.24 | 0.27 | 0.29 | 0.31 | 0.33 | 0.35 | 0.36 | 0.38 | 0.39 | 0.40 | 0.42 | 0.43 | 0.44 | 0.45 | 0.45 |
|  | 60\% | 0.07 | 0.12 | 0.16 | 0.19 | 0.22 | 0.25 | 0.27 | 0.29 | 0.31 | 0.33 | 0.35 | 0.36 | 0.37 | 0.38 | 0.40 | 0.40 | 0.41 | 0.42 | 0.43 | 0.43 |
| $\square$ | 65\% | 0.07 | 0.12 | 0.16 | 0.20 | 0.23 | 0.25 | 0.28 | 0.30 | 0.31 | 0.33 | 0.34 | 0.36 | 0.37 | 0.38 | 0.39 | 0.39 | 0.40 | 0.41 | 0.41 | 0.41 |
| - | 70\% | 0.08 | 0.13 | 0.17 | 0.20 | 0.23 | 0.26 | 0.28 | 0.30 | 0.32 | 0.33 | 0.34 | 0.36 | 0.37 | 0.37 | 0.38 | 0.39 | 0.39 | 0.39 | 0.40 | 0.40 |
|  | 75\% | 0.08 | 0.13 | 0.17 | 0.21 | 0.24 | 0.26 | 0.28 | 0.30 | 0.32 | 0.33 | 0.35 | 0.35 | 0.36 | 0.37 | 0.38 | 0.38 | 0.38 | 0.38 | 0.38 | 0.38 |
|  | 80\% | 0.08 | 0.14 | 0.18 | 0.21 | 0.24 | 0.27 | 0.29 | 0.31 | 0.32 | 0.34 | 0.35 | 0.36 | 0.36 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 |
|  | 85\% | 0.09 | 0.14 | 0.19 | 0.22 | 0.25 | 0.28 | 0.30 | 0.31 | 0.33 | 0.34 | 0.35 | 0.36 | 0.36 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 0.36 | 0.36 |
|  | 90\% | 0.09 | 0.15 | 0.19 | 0.23 | 0.26 | 0.28 | 0.30 | 0.32 | 0.33 | 0.34 | 0.35 | 0.36 | 0.36 | 0.37 | 0.37 | 0.37 | 0.36 | 0.36 | 0.35 | 0.35 |
|  | 95\% | 0.09 | 0.15 | 0.20 | 0.24 | 0.27 | 0.29 | 0.31 | 0.33 | 0.34 | 0.35 | 0.36 | 0.36 | 0.37 | 0.37 | 0.37 | 0.36 | 0.36 | 0.36 | 0.35 | 0.34 |
|  | 100\% | 0.10 | 0.16 | 0.21 | 0.24 | 0.27 | 0.30 | 0.32 | 0.33 | 0.35 | 0.36 | 0.36 | 0.37 | 0.37 | 0.37 | 0.37 | 0.36 | 0.36 | 0.35 | 0.34 | 0.33 |

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## EV when M = 4

## Villain's Push Range

|  |  | 5\% | 10\% | 15\% | 20\% | 25\% | 30\% | 35\% | 40\% | 45\% | 50\% | 55\% | 60\% | 65\% | 70\% | 75\% | 80\% | 85\% | 90\% | 95\% | 100\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 5\% | 0.05 | 0.09 | 0.14 | 0.18 | 0.23 | 0.27 | 0.31 | 0.36 | 0.40 | 0.44 | 0.48 | 0.53 | 0.57 | 0.61 | 0.65 | 0.69 | 0.74 | 0.78 | 0.82 | 0.86 |
|  | 10\% | 0.05 | 0.09 | 0.14 | 0.18 | 0.22 | 0.26 | 0.30 | 0.33 | 0.37 | 0.41 | 0.45 | 0.48 | 0.52 | 0.56 | 0.59 | 0.63 | 0.66 | 0.70 | 0.73 | 0.77 |
|  | 15\% | 0.05 | 0.09 | 0.14 | 0.17 | 0.21 | 0.25 | 0.28 | 0.32 | 0.35 | 0.39 | 0.42 | 0.45 | 0.48 | 0.51 | 0.55 | 0.58 | 0.61 | 0.64 | 0.67 | 0.70 |
|  | 20\% | 0.05 | 0.10 | 0.14 | 0.17 | 0.21 | 0.24 | 0.28 | 0.31 | 0.34 | 0.37 | 0.40 | 0.43 | 0.45 | 0.48 | 0.51 | 0.54 | 0.56 | 0.59 | 0.61 | 0.64 |
|  | 25\% | 0.06 | 0.10 | 0.14 | 0.17 | 0.21 | 0.24 | 0.27 | 0.30 | 0.33 | 0.36 | 0.38 | 0.41 | 0.43 | 0.46 | 0.48 | 0.50 | 0.52 | 0.54 | 0.57 | 0.59 |
| TT ${ }^{\text {9 }}$ | 30\% | 0.06 | 0.10 | 0.14 | 0.18 | 0.21 | 0.24 | 0.27 | 0.30 | 0.32 | 0.35 | 0.37 | 0.39 | 0.41 | 0.43 | 0.45 | 0.47 | 0.49 | 0.51 | 0.53 | 0.54 |
| -1er0 | 35\% | 0.06 | 0.11 | 0.15 | 0.18 | 0.21 | 0.24 | 0.27 | 0.29 | 0.32 | 0.34 | 0.36 | 0.38 | 0.40 | 0.42 | 0.43 | 0.45 | 0.46 | 0.48 | 0.49 | 0.51 |
|  | 40\% | 0.07 | 0.11 | 0.15 | 0.19 | 0.22 | 0.24 | 0.27 | 0.29 | 0.31 | 0.33 | 0.35 | 0.37 | 0.39 | 0.40 | 0.42 | 0.43 | 0.44 | 0.45 | 0.46 | 0.47 |
|  | 45\% | 0.07 | 0.12 | 0.16 | 0.19 | 0.22 | 0.25 | 0.27 | 0.30 | 0.32 | 0.33 | 0.35 | 0.36 | 0.38 | 0.39 | 0.40 | 0.41 | 0.42 | 0.43 | 0.44 | 0.44 |
| - | 50\% | 0.07 | 0.12 | 0.16 | 0.20 | 0.23 | 0.25 | 0.28 | 0.30 | 0.32 | 0.33 | 0.35 | 0.36 | 0.37 | 0.38 | 0.39 | 0.40 | 0.41 | 0.41 | 0.42 | 0.42 |
|  | 55\% | 0.08 | 0.13 | 0.17 | 0.21 | 0.24 | 0.26 | 0.28 | 0.30 | 0.32 | 0.34 | 0.35 | 0.36 | 0.37 | 0.38 | 0.38 | 0.39 | 0.39 | 0.40 | 0.40 | 0.40 |
|  | 60\% | 0.08 | 0.14 | 0.18 | 0.21 | 0.24 | 0.27 | 0.29 | 0.31 | 0.33 | 0.34 | 0.35 | 0.36 | 0.37 | 0.37 | 0.38 | 0.38 | 0.38 | 0.38 | 0.38 | 0.38 |
| $\square$ | 65\% | 0.09 | 0.14 | 0.19 | 0.22 | 0.25 | 0.28 | 0.30 | 0.32 | 0.33 | 0.34 | 0.35 | 0.36 | 0.37 | 0.37 | 0.38 | 0.38 | 0.38 | 0.37 | 0.37 | 0.37 |
| - | 70\% | 0.09 | 0.15 | 0.20 | 0.23 | 0.26 | 0.29 | 0.31 | 0.32 | 0.34 | 0.35 | 0.36 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 0.36 | 0.36 |
|  | 75\% | 0.10 | 0.16 | 0.20 | 0.24 | 0.27 | 0.30 | 0.32 | 0.33 | 0.35 | 0.36 | 0.37 | 0.37 | 0.37 | 0.38 | 0.38 | 0.37 | 0.37 | 0.36 | 0.36 | 0.35 |
|  | 80\% | 0.10 | 0.17 | 0.21 | 0.25 | 0.28 | 0.31 | 0.33 | 0.34 | 0.36 | 0.37 | 0.37 | 0.38 | 0.38 | 0.38 | 0.38 | 0.37 | 0.37 | 0.36 | 0.35 | 0.34 |
|  | 85\% | 0.11 | 0.17 | 0.22 | 0.26 | 0.29 | 0.32 | 0.34 | 0.36 | 0.37 | 0.38 | 0.38 | 0.39 | 0.39 | 0.39 | 0.38 | 0.38 | 0.37 | 0.36 | 0.35 | 0.34 |
|  | 90\% | 0.11 | 0.18 | 0.23 | 0.27 | 0.31 | 0.33 | 0.35 | 0.37 | 0.38 | 0.39 | 0.39 | 0.40 | 0.40 | 0.39 | 0.39 | 0.38 | 0.37 | 0.36 | 0.35 | 0.33 |
|  | 95\% | 0.12 | 0.19 | 0.24 | 0.28 | 0.32 | 0.34 | 0.36 | 0.38 | 0.39 | 0.40 | 0.40 | 0.41 | 0.41 | 0.40 | 0.39 | 0.39 | 0.38 | 0.36 | 0.35 | 0.33 |
|  | 100\% | 0.12 | 0.20 | 0.25 | 0.30 | 0.33 | 0.36 | 0.38 | 0.39 | 0.41 | 0.41 | 0.42 | 0.42 | 0.42 | 0.41 | 0.40 | 0.39 | 0.38 | 0.37 | 0.35 | 0.33 |

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## EV when M = 5

## Villain's Push Range

|  |  | 5\% | 10\% | 15\% | 20\% | 25\% | 30\% | 35\% | 40\% | 45\% | 50\% | 55\% | 60\% | 65\% | 70\% | 75\% | 80\% | 85\% | 90\% | 95\% | 100\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 5\% | 0.05 | 0.09 | 0.14 | 0.18 | 0.22 | 0.27 | 0.31 | 0.35 | 0.39 | 0.43 | 0.47 | 0.51 | 0.56 | 0.60 | 0.64 | 0.68 | 0.72 | 0.76 | 0.80 | 0.84 |
|  | 10\% | 0.05 | 0.09 | 0.13 | 0.17 | 0.21 | 0.25 | 0.29 | 0.32 | 0.36 | 0.40 | 0.43 | 0.47 | 0.50 | 0.53 | 0.57 | 0.60 | 0.63 | 0.67 | 0.70 | 0.73 |
|  | 15\% | 0.05 | 0.10 | 0.14 | 0.17 | 0.21 | 0.24 | 0.28 | 0.31 | 0.34 | 0.37 | 0.40 | 0.43 | 0.46 | 0.49 | 0.52 | 0.54 | 0.57 | 0.60 | 0.62 | 0.65 |
|  | 20\% | 0.06 | 0.10 | 0.14 | 0.17 | 0.21 | 0.24 | 0.27 | 0.30 | 0.33 | 0.35 | 0.38 | 0.40 | 0.43 | 0.45 | 0.48 | 0.50 | 0.52 | 0.54 | 0.56 | 0.58 |
|  | 25\% | 0.06 | 0.10 | 0.14 | 0.18 | 0.21 | 0.24 | 0.27 | 0.29 | 0.32 | 0.34 | 0.36 | 0.38 | 0.41 | 0.43 | 0.44 | 0.46 | 0.48 | 0.50 | 0.51 | 0.53 |
|  | 30\% | 0.06 | 0.11 | 0.15 | 0.18 | 0.21 | 0.24 | 0.27 | 0.29 | 0.31 | 0.33 | 0.35 | 0.37 | 0.39 | 0.40 | 0.42 | 0.43 | 0.45 | 0.46 | 0.47 | 0.48 |
| 11 | 35\% | 0.07 | 0.12 | 0.15 | 0.19 | 0.22 | 0.24 | 0.27 | 0.29 | 0.31 | 0.33 | 0.35 | 0.36 | 0.38 | 0.39 | 0.40 | 0.41 | 0.42 | 0.43 | 0.44 | 0.44 |
|  | 40\% | 0.07 | 0.12 | 0.16 | 0.20 | 0.22 | 0.25 | 0.27 | 0.29 | 0.31 | 0.33 | 0.34 | 0.35 | 0.37 | 0.38 | 0.38 | 0.39 | 0.40 | 0.40 | 0.41 | 0.41 |
|  | 45\% | 0.08 | 0.13 | 0.17 | 0.20 | 0.23 | 0.26 | 0.28 | 0.30 | 0.32 | 0.33 | 0.34 | 0.35 | 0.36 | 0.37 | 0.37 | 0.38 | 0.38 | 0.38 | 0.39 | 0.39 |
| 入 | 50\% | 0.08 | 0.14 | 0.18 | 0.21 | 0.24 | 0.27 | 0.29 | 0.31 | 0.32 | 0.33 | 0.34 | 0.35 | 0.36 | 0.36 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 0.36 |
|  | 55\% | 0.09 | 0.15 | 0.19 | 0.22 | 0.25 | 0.28 | 0.30 | 0.31 | 0.33 | 0.34 | 0.35 | 0.35 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.35 | 0.35 |
|  | 60\% | 0.10 | 0.15 | 0.20 | 0.24 | 0.27 | 0.29 | 0.31 | 0.33 | 0.34 | 0.35 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.35 | 0.35 | 0.34 | 0.33 |
| anor | 65\% | 0.10 | 0.16 | 0.21 | 0.25 | 0.28 | 0.30 | 0.32 | 0.34 | 0.35 | 0.36 | 0.36 | 0.37 | 0.37 | 0.37 | 0.36 | 0.36 | 0.35 | 0.34 | 0.33 | 0.32 |
| - | 70\% | 0.11 | 0.17 | 0.22 | 0.26 | 0.29 | 0.32 | 0.34 | 0.35 | 0.36 | 0.37 | 0.37 | 0.38 | 0.38 | 0.37 | 0.37 | 0.36 | 0.35 | 0.34 | 0.33 | 0.31 |
|  | 75\% | 0.11 | 0.18 | 0.23 | 0.27 | 0.31 | 0.33 | 0.35 | 0.37 | 0.38 | 0.38 | 0.39 | 0.39 | 0.39 | 0.38 | 0.38 | 0.37 | 0.36 | 0.34 | 0.33 | 0.31 |
|  | 80\% | 0.12 | 0.19 | 0.25 | 0.29 | 0.32 | 0.35 | 0.37 | 0.38 | 0.39 | 0.40 | 0.40 | 0.40 | 0.40 | 0.39 | 0.38 | 0.37 | 0.36 | 0.35 | 0.33 | 0.31 |
|  | 85\% | 0.13 | 0.20 | 0.26 | 0.30 | 0.34 | 0.36 | 0.38 | 0.40 | 0.41 | 0.41 | 0.42 | 0.42 | 0.41 | 0.40 | 0.39 | 0.38 | 0.37 | 0.35 | 0.33 | 0.31 |
|  | 90\% | 0.13 | 0.21 | 0.27 | 0.32 | 0.35 | 0.38 | 0.40 | 0.42 | 0.43 | 0.43 | 0.43 | 0.43 | 0.43 | 0.42 | 0.41 | 0.39 | 0.38 | 0.36 | 0.34 | 0.32 |
|  | 95\% | 0.14 | 0.22 | 0.29 | 0.33 | 0.37 | 0.40 | 0.42 | 0.43 | 0.44 | 0.45 | 0.45 | 0.45 | 0.44 | 0.43 | 0.42 | 0.41 | 0.39 | 0.37 | 0.35 | 0.32 |
|  | 100\% | 0.15 | 0.23 | 0.30 | 0.35 | 0.39 | 0.42 | 0.44 | 0.45 | 0.46 | 0.47 | 0.47 | 0.47 | 0.46 | 0.45 | 0.44 | 0.42 | 0.40 | 0.38 | 0.36 | 0.33 |

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## EV when M = 6

## Villain's Push Range

|  |  | 5\% | 10\% | 15\% | 20\% | 25\% | 30\% | 35\% | 40\% | 45\% | 50\% | 55\% | 60\% | 65\% | 70\% | 75\% | 80\% | 85\% | 90\% | 95\% | 100\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 5\% | 0.05 | 0.09 | 0.14 | 0.18 | 0.22 | 0.26 | 0.30 | 0.34 | 0.38 | 0.42 | 0.46 | 0.50 | 0.54 | 0.58 | 0.62 | 0.66 | 0.70 | 0.73 | 0.77 | 0.81 |
|  | 10\% | 0.05 | 0.09 | 0.13 | 0.17 | 0.21 | 0.25 | 0.28 | 0.32 | 0.35 | 0.38 | 0.42 | 0.45 | 0.48 | 0.51 | 0.54 | 0.57 | 0.60 | 0.63 | 0.66 | 0.69 |
|  | 15\% | 0.05 | 0.10 | 0.14 | 0.17 | 0.21 | 0.24 | 0.27 | 0.30 | 0.33 | 0.36 | 0.38 | 0.41 | 0.44 | 0.46 | 0.49 | 0.51 | 0.54 | 0.56 | 0.58 | 0.60 |
|  | 20\% | 0.06 | 0.10 | 0.14 | 0.17 | 0.21 | 0.23 | 0.26 | 0.29 | 0.31 | 0.34 | 0.36 | 0.38 | 0.40 | 0.42 | 0.44 | 0.46 | 0.48 | 0.50 | 0.52 | 0.53 |
|  | 25\% | 0.06 | 0.11 | 0.14 | 0.18 | 0.21 | 0.24 | 0.26 | 0.28 | 0.31 | 0.33 | 0.35 | 0.36 | 0.38 | 0.40 | 0.41 | 0.42 | 0.44 | 0.45 | 0.46 | 0.47 |
| TT | 30\% | 0.07 | 0.11 | 0.15 | 0.19 | 0.21 | 0.24 | 0.26 | 0.28 | 0.30 | 0.32 | 0.34 | 0.35 | 0.36 | 0.37 | 0.39 | 0.40 | 0.40 | 0.41 | 0.42 | 0.42 |
| Heros | 35\% | 0.07 | 0.12 | 0.16 | 0.19 | 0.22 | 0.25 | 0.27 | 0.29 | 0.30 | 0.32 | 0.33 | 0.34 | 0.35 | 0.36 | 0.37 | 0.37 | 0.38 | 0.38 | 0.38 | 0.38 |
|  | 40\% | 0.08 | 0.13 | 0.17 | 0.20 | 0.23 | 0.26 | 0.28 | 0.29 | 0.31 | 0.32 | 0.33 | 0.34 | 0.35 | 0.35 | 0.35 | 0.36 | 0.36 | 0.36 | 0.35 | 0.35 |
|  | 45\% | 0.09 | 0.14 | 0.18 | 0.22 | 0.24 | 0.27 | 0.29 | 0.30 | 0.32 | 0.33 | 0.33 | 0.34 | 0.34 | 0.35 | 0.35 | 0.34 | 0.34 | 0.34 | 0.33 | 0.33 |
| - | 50\% | 0.09 | 0.15 | 0.19 | 0.23 | 0.26 | 0.28 | 0.30 | 0.31 | 0.32 | 0.33 | 0.34 | 0.34 | 0.34 | 0.34 | 0.34 | 0.34 | 0.33 | 0.32 | 0.32 | 0.31 |
|  | 55\% | 0.10 | 0.16 | 0.21 | 0.24 | 0.27 | 0.29 | 0.31 | 0.33 | 0.34 | 0.34 | 0.35 | 0.35 | 0.35 | 0.35 | 0.34 | 0.34 | 0.33 | 0.32 | 0.30 | 0.29 |
|  | 60\% | 0.11 | 0.17 | 0.22 | 0.26 | 0.29 | 0.31 | 0.33 | 0.34 | 0.35 | 0.36 | 0.36 | 0.36 | 0.36 | 0.35 | 0.35 | 0.34 | 0.33 | 0.31 | 0.30 | 0.28 |
| 入 | 65\% | 0.12 | 0.18 | 0.23 | 0.27 | 0.30 | 0.33 | 0.34 | 0.36 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 0.36 | 0.35 | 0.34 | 0.33 | 0.31 | 0.29 | 0.28 |
| - | 70\% | 0.12 | 0.19 | 0.25 | 0.29 | 0.32 | 0.35 | 0.36 | 0.38 | 0.38 | 0.39 | 0.39 | 0.39 | 0.38 | 0.37 | 0.36 | 0.35 | 0.33 | 0.32 | 0.30 | 0.27 |
|  | 75\% | 0.13 | 0.21 | 0.26 | 0.31 | 0.34 | 0.36 | 0.38 | 0.40 | 0.40 | 0.41 | 0.41 | 0.40 | 0.40 | 0.39 | 0.38 | 0.36 | 0.34 | 0.32 | 0.30 | 0.28 |
|  | 80\% | 0.14 | 0.22 | 0.28 | 0.32 | 0.36 | 0.38 | 0.40 | 0.42 | 0.43 | 0.43 | 0.43 | 0.42 | 0.42 | 0.40 | 0.39 | 0.37 | 0.35 | 0.33 | 0.31 | 0.28 |
|  | 85\% | 0.15 | 0.23 | 0.29 | 0.34 | 0.38 | 0.41 | 0.43 | 0.44 | 0.45 | 0.45 | 0.45 | 0.44 | 0.44 | 0.42 | 0.41 | 0.39 | 0.37 | 0.34 | 0.32 | 0.29 |
|  | 90\% | 0.16 | 0.25 | 0.31 | 0.36 | 0.40 | 0.43 | 0.45 | 0.46 | 0.47 | 0.48 | 0.47 | 0.47 | 0.46 | 0.44 | 0.43 | 0.41 | 0.39 | 0.36 | 0.33 | 0.30 |
|  | 95\% | 0.16 | 0.26 | 0.33 | 0.38 | 0.42 | 0.45 | 0.47 | 0.49 | 0.50 | 0.50 | 0.50 | 0.49 | 0.48 | 0.47 | 0.45 | 0.43 | 0.41 | 0.38 | 0.35 | 0.32 |
|  | 100\% | 0.17 | 0.27 | 0.35 | 0.40 | 0.44 | 0.48 | 0.50 | 0.51 | 0.52 | 0.53 | 0.53 | 0.52 | 0.51 | 0.49 | 0.47 | 0.45 | 0.43 | 0.40 | 0.37 | 0.33 |

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## EV when $\mathrm{M}=7$

## Villain's Push Range

|  |  | 5\% | 10\% | 15\% | 20\% | 25\% | 30\% | 35\% | 40\% | 45\% | 50\% | 55\% | 60\% | 65\% | 70\% | 75\% | 80\% | 85\% | 90\% | 95\% | 100\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 5\% | 0.05 | 0.09 | 0.14 | 0.18 | 0.22 | 0.26 | 0.30 | 0.34 | 0.38 | 0.41 | 0.45 | 0.49 | 0.53 | 0.57 | 0.60 | 0.64 | 0.68 | 0.71 | 0.75 | 0.79 |
|  | 10\% | 0.05 | 0.09 | 0.13 | 0.17 | 0.21 | 0.24 | 0.27 | 0.31 | 0.34 | 0.37 | 0.40 | 0.43 | 0.46 | 0.49 | 0.52 | 0.55 | 0.57 | 0.60 | 0.63 | 0.66 |
|  | 15\% | 0.05 | 0.10 | 0.14 | 0.17 | 0.20 | 0.23 | 0.26 | 0.29 | 0.32 | 0.34 | 0.37 | 0.39 | 0.41 | 0.44 | 0.46 | 0.48 | 0.50 | 0.52 | 0.54 | 0.56 |
|  | 20\% | 0.06 | 0.10 | 0.14 | 0.17 | 0.20 | 0.23 | 0.26 | 0.28 | 0.30 | 0.32 | 0.34 | 0.36 | 0.38 | 0.40 | 0.41 | 0.43 | 0.44 | 0.45 | 0.47 | 0.48 |
|  | 25\% | 0.07 | 0.11 | 0.15 | 0.18 | 0.21 | 0.23 | 0.26 | 0.28 | 0.30 | 0.31 | 0.33 | 0.34 | 0.35 | 0.37 | 0.38 | 0.39 | 0.40 | 0.40 | 0.41 | 0.42 |
|  | 30\% | 0.07 | 0.12 | 0.16 | 0.19 | 0.22 | 0.24 | 0.26 | 0.28 | 0.29 | 0.31 | 0.32 | 0.33 | 0.34 | 0.35 | 0.35 | 0.36 | 0.36 | 0.36 | 0.36 | 0.37 |
| 11 | 35\% | 0.08 | 0.13 | 0.17 | 0.20 | 0.23 | 0.25 | 0.27 | 0.28 | 0.30 | 0.31 | 0.32 | 0.32 | 0.33 | 0.33 | 0.33 | 0.33 | 0.33 | 0.33 | 0.33 | 0.32 |
|  | 40\% | 0.09 | 0.14 | 0.18 | 0.21 | 0.24 | 0.26 | 0.28 | 0.29 | 0.30 | 0.31 | 0.32 | 0.32 | 0.32 | 0.32 | 0.32 | 0.32 | 0.31 | 0.31 | 0.30 | 0.29 |
|  | 45\% | 0.09 | 0.15 | 0.19 | 0.23 | 0.25 | 0.28 | 0.29 | 0.31 | 0.32 | 0.32 | 0.33 | 0.33 | 0.33 | 0.32 | 0.32 | 0.31 | 0.30 | 0.29 | 0.28 | 0.27 |
| 入 | 50\% | 0.10 | 0.16 | 0.21 | 0.24 | 0.27 | 0.29 | 0.31 | 0.32 | 0.33 | 0.33 | 0.34 | 0.33 | 0.33 | 0.32 | 0.32 | 0.31 | 0.30 | 0.28 | 0.27 | 0.25 |
|  | 55\% | 0.11 | 0.18 | 0.22 | 0.26 | 0.29 | 0.31 | 0.33 | 0.34 | 0.34 | 0.35 | 0.35 | 0.35 | 0.34 | 0.33 | 0.32 | 0.31 | 0.29 | 0.28 | 0.26 | 0.24 |
|  | 60\% | 0.12 | 0.19 | 0.24 | 0.28 | 0.31 | 0.33 | 0.35 | 0.36 | 0.36 | 0.37 | 0.36 | 0.36 | 0.35 | 0.34 | 0.33 | 0.31 | 0.30 | 0.28 | 0.25 | 0.23 |
| anor | 65\% | 0.13 | 0.20 | 0.26 | 0.30 | 0.33 | 0.35 | 0.37 | 0.38 | 0.38 | 0.39 | 0.38 | 0.38 | 0.37 | 0.36 | 0.34 | 0.32 | 0.30 | 0.28 | 0.26 | 0.23 |
| - | 70\% | 0.14 | 0.22 | 0.27 | 0.32 | 0.35 | 0.37 | 0.39 | 0.40 | 0.41 | 0.41 | 0.41 | 0.40 | 0.39 | 0.37 | 0.36 | 0.34 | 0.31 | 0.29 | 0.26 | 0.23 |
|  | 75\% | 0.15 | 0.23 | 0.29 | 0.34 | 0.37 | 0.40 | 0.42 | 0.43 | 0.43 | 0.43 | 0.43 | 0.42 | 0.41 | 0.39 | 0.38 | 0.35 | 0.33 | 0.30 | 0.27 | 0.24 |
|  | 80\% | 0.16 | 0.25 | 0.31 | 0.36 | 0.40 | 0.42 | 0.44 | 0.45 | 0.46 | 0.46 | 0.46 | 0.45 | 0.43 | 0.42 | 0.40 | 0.37 | 0.35 | 0.32 | 0.29 | 0.25 |
|  | 85\% | 0.17 | 0.26 | 0.33 | 0.38 | 0.42 | 0.45 | 0.47 | 0.48 | 0.49 | 0.49 | 0.48 | 0.47 | 0.46 | 0.44 | 0.42 | 0.40 | 0.37 | 0.34 | 0.30 | 0.27 |
|  | 90\% | 0.18 | 0.28 | 0.35 | 0.41 | 0.45 | 0.48 | 0.50 | 0.51 | 0.52 | 0.52 | 0.51 | 0.50 | 0.49 | 0.47 | 0.45 | 0.42 | 0.39 | 0.36 | 0.32 | 0.29 |
|  | 95\% | 0.19 | 0.29 | 0.37 | 0.43 | 0.47 | 0.51 | 0.53 | 0.54 | 0.55 | 0.55 | 0.55 | 0.54 | 0.52 | 0.50 | 0.48 | 0.45 | 0.42 | 0.39 | 0.35 | 0.31 |
|  | 100\% | 0.20 | 0.31 | 0.39 | 0.45 | 0.50 | 0.53 | 0.56 | 0.57 | 0.58 | 0.58 | 0.58 | 0.57 | 0.55 | 0.53 | 0.51 | 0.48 | 0.45 | 0.41 | 0.38 | 0.33 |

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## EV when M = 8

## Villain's Push Range

|  |  | 5\% | 10\% | 15\% | 20\% | 25\% | 30\% | 35\% | 40\% | 45\% | 50\% | 55\% | 60\% | 65\% | 70\% | 75\% | 80\% | 85\% | 90\% | 95\% | 100\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 5\% | 0.05 | 0.09 | 0.13 | 0.17 | 0.21 | 0.25 | 0.29 | 0.33 | 0.37 | 0.40 | 0.44 | 0.48 | 0.51 | 0.55 | 0.59 | 0.62 | 0.66 | 0.69 | 0.73 | 0.76 |
|  | 10\% | 0.05 | 0.09 | 0.13 | 0.17 | 0.20 | 0.23 | 0.27 | 0.30 | 0.33 | 0.36 | 0.39 | 0.41 | 0.44 | 0.47 | 0.49 | 0.52 | 0.54 | 0.57 | 0.59 | 0.62 |
|  | 15\% | 0.06 | 0.10 | 0.14 | 0.17 | 0.20 | 0.23 | 0.25 | 0.28 | 0.30 | 0.33 | 0.35 | 0.37 | 0.39 | 0.41 | 0.43 | 0.45 | 0.46 | 0.48 | 0.50 | 0.51 |
|  | 20\% | 0.06 | 0.11 | 0.14 | 0.17 | 0.20 | 0.23 | 0.25 | 0.27 | 0.29 | 0.31 | 0.32 | 0.34 | 0.35 | 0.37 | 0.38 | 0.39 | 0.40 | 0.41 | 0.42 | 0.43 |
|  | 25\% | 0.07 | 0.11 | 0.15 | 0.18 | 0.21 | 0.23 | 0.25 | 0.27 | 0.28 | 0.30 | 0.31 | 0.32 | 0.33 | 0.34 | 0.34 | 0.35 | 0.35 | 0.36 | 0.36 | 0.36 |
|  | 30\% | 0.08 | 0.13 | 0.16 | 0.19 | 0.22 | 0.24 | 0.26 | 0.27 | 0.29 | 0.30 | 0.30 | 0.31 | 0.31 | 0.32 | 0.32 | 0.32 | 0.32 | 0.31 | 0.31 | 0.31 |
| 11 | 35\% | 0.08 | 0.14 | 0.18 | 0.21 | 0.23 | 0.25 | 0.27 | 0.28 | 0.29 | 0.30 | 0.30 | 0.31 | 0.31 | 0.30 | 0.30 | 0.30 | 0.29 | 0.28 | 0.27 | 0.26 |
|  | 40\% | 0.09 | 0.15 | 0.19 | 0.22 | 0.25 | 0.27 | 0.28 | 0.29 | 0.30 | 0.31 | 0.31 | 0.31 | 0.30 | 0.30 | 0.29 | 0.28 | 0.27 | 0.26 | 0.25 | 0.23 |
| $\Omega$ | 45\% | 0.10 | 0.16 | 0.21 | 0.24 | 0.27 | 0.28 | 0.30 | 0.31 | 0.32 | 0.32 | 0.32 | 0.31 | 0.31 | 0.30 | 0.29 | 0.28 | 0.26 | 0.25 | 0.23 | 0.21 |
| - | 50\% | 0.11 | 0.18 | 0.22 | 0.26 | 0.29 | 0.30 | 0.32 | 0.33 | 0.33 | 0.33 | 0.33 | 0.33 | 0.32 | 0.31 | 0.29 | 0.28 | 0.26 | 0.24 | 0.22 | 0.19 |
|  | 55\% | 0.12 | 0.19 | 0.24 | 0.28 | 0.31 | 0.33 | 0.34 | 0.35 | 0.35 | 0.35 | 0.35 | 0.34 | 0.33 | 0.32 | 0.30 | 0.28 | 0.26 | 0.24 | 0.21 | 0.18 |
|  | 60\% | 0.13 | 0.21 | 0.26 | 0.30 | 0.33 | 0.35 | 0.36 | 0.37 | 0.38 | 0.37 | 0.37 | 0.36 | 0.35 | 0.33 | 0.31 | 0.29 | 0.27 | 0.24 | 0.21 | 0.18 |
| anor | 65\% | 0.14 | 0.22 | 0.28 | 0.32 | 0.35 | 0.38 | 0.39 | 0.40 | 0.40 | 0.40 | 0.39 | 0.38 | 0.37 | 0.35 | 0.33 | 0.31 | 0.28 | 0.25 | 0.22 | 0.18 |
| - | 70\% | 0.15 | 0.24 | 0.30 | 0.35 | 0.38 | 0.40 | 0.42 | 0.43 | 0.43 | 0.43 | 0.42 | 0.41 | 0.39 | 0.37 | 0.35 | 0.32 | 0.30 | 0.26 | 0.23 | 0.19 |
|  | 75\% | 0.16 | 0.26 | 0.32 | 0.37 | 0.41 | 0.43 | 0.45 | 0.46 | 0.46 | 0.46 | 0.45 | 0.44 | 0.42 | 0.40 | 0.38 | 0.35 | 0.32 | 0.28 | 0.24 | 0.20 |
|  | 80\% | 0.18 | 0.27 | 0.34 | 0.40 | 0.43 | 0.46 | 0.48 | 0.49 | 0.49 | 0.49 | 0.48 | 0.47 | 0.45 | 0.43 | 0.40 | 0.37 | 0.34 | 0.30 | 0.26 | 0.22 |
|  | 85\% | 0.19 | 0.29 | 0.37 | 0.42 | 0.46 | 0.49 | 0.51 | 0.52 | 0.53 | 0.53 | 0.52 | 0.50 | 0.48 | 0.46 | 0.43 | 0.40 | 0.37 | 0.33 | 0.29 | 0.24 |
|  | 90\% | 0.20 | 0.31 | 0.39 | 0.45 | 0.49 | 0.53 | 0.55 | 0.56 | 0.56 | 0.56 | 0.55 | 0.54 | 0.52 | 0.50 | 0.47 | 0.44 | 0.40 | 0.36 | 0.32 | 0.27 |
|  | 95\% | 0.21 | 0.33 | 0.41 | 0.48 | 0.53 | 0.56 | 0.58 | 0.60 | 0.60 | 0.60 | 0.59 | 0.58 | 0.56 | 0.53 | 0.51 | 0.47 | 0.43 | 0.39 | 0.35 | 0.30 |
|  | 100\% | 0.22 | 0.35 | 0.44 | 0.51 | 0.56 | 0.59 | 0.62 | 0.63 | 0.64 | 0.64 | 0.63 | 0.62 | 0.60 | 0.57 | 0.55 | 0.51 | 0.47 | 0.43 | 0.38 | 0.33 |

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## EV when M = 9

## Villain's Push Range

|  |  | 5\% | 10\% | 15\% | 20\% | 25\% | 30\% | 35\% | 40\% | 45\% | 50\% | 55\% | 60\% | 65\% | 70\% | 75\% | 80\% | 85\% | 90\% | 95\% | 100\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 5\% | 0.05 | 0.09 | 0.13 | 0.17 | 0.21 | 0.25 | 0.29 | 0.32 | 0.36 | 0.40 | 0.43 | 0.47 | 0.50 | 0.54 | 0.57 | 0.60 | 0.64 | 0.67 | 0.70 | 0.74 |
|  | 10\% | 0.05 | 0.09 | 0.13 | 0.17 | 0.20 | 0.23 | 0.26 | 0.29 | 0.32 | 0.34 | 0.37 | 0.40 | 0.42 | 0.44 | 0.47 | 0.49 | 0.51 | 0.54 | 0.56 | 0.58 |
|  | 15\% | 0.06 | 0.10 | 0.14 | 0.17 | 0.20 | 0.22 | 0.25 | 0.27 | 0.29 | 0.31 | 0.33 | 0.35 | 0.37 | 0.38 | 0.40 | 0.41 | 0.43 | 0.44 | 0.45 | 0.46 |
|  | 20\% | 0.06 | 0.11 | 0.14 | 0.17 | 0.20 | 0.22 | 0.24 | 0.26 | 0.28 | 0.29 | 0.31 | 0.32 | 0.33 | 0.34 | 0.35 | 0.35 | 0.36 | 0.37 | 0.37 | 0.37 |
|  | 25\% | 0.07 | 0.12 | 0.15 | 0.18 | 0.21 | 0.23 | 0.25 | 0.26 | 0.27 | 0.28 | 0.29 | 0.30 | 0.30 | 0.31 | 0.31 | 0.31 | 0.31 | 0.31 | 0.31 | 0.30 |
|  | 30\% | 0.08 | 0.13 | 0.17 | 0.20 | 0.22 | 0.24 | 0.26 | 0.27 | 0.28 | 0.28 | 0.29 | 0.29 | 0.29 | 0.29 | 0.28 | 0.28 | 0.27 | 0.27 | 0.26 | 0.25 |
| 11 | 35\% | 0.09 | 0.14 | 0.18 | 0.21 | 0.24 | 0.25 | 0.27 | 0.28 | 0.28 | 0.29 | 0.29 | 0.29 | 0.28 | 0.28 | 0.27 | 0.26 | 0.25 | 0.23 | 0.22 | 0.20 |
|  | 40\% | 0.10 | 0.16 | 0.20 | 0.23 | 0.26 | 0.27 | 0.29 | 0.29 | 0.30 | 0.30 | 0.30 | 0.29 | 0.28 | 0.27 | 0.26 | 0.25 | 0.23 | 0.21 | 0.19 | 0.17 |
| $\bigcirc$ | 45\% | 0.11 | 0.17 | 0.22 | 0.25 | 0.28 | 0.29 | 0.31 | 0.31 | 0.32 | 0.31 | 0.31 | 0.30 | 0.29 | 0.28 | 0.26 | 0.24 | 0.22 | 0.20 | 0.18 | 0.15 |
| a | 50\% | 0.12 | 0.19 | 0.24 | 0.27 | 0.30 | 0.32 | 0.33 | 0.34 | 0.34 | 0.33 | 0.33 | 0.32 | 0.30 | 0.29 | 0.27 | 0.25 | 0.22 | 0.19 | 0.17 | 0.14 |
|  | 55\% | 0.13 | 0.21 | 0.26 | 0.30 | 0.32 | 0.34 | 0.36 | 0.36 | 0.36 | 0.36 | 0.35 | 0.34 | 0.32 | 0.30 | 0.28 | 0.25 | 0.23 | 0.20 | 0.16 | 0.13 |
|  | 60\% | 0.14 | 0.22 | 0.28 | 0.32 | 0.35 | 0.37 | 0.38 | 0.39 | 0.39 | 0.38 | 0.37 | 0.36 | 0.34 | 0.32 | 0.30 | 0.27 | 0.24 | 0.20 | 0.17 | 0.13 |
| 910 | 65\% | 0.16 | 0.24 | 0.30 | 0.35 | 0.38 | 0.40 | 0.41 | 0.42 | 0.42 | 0.41 | 0.40 | 0.39 | 0.37 | 0.34 | 0.32 | 0.29 | 0.25 | 0.22 | 0.18 | 0.14 |
| - | 70\% | 0.17 | 0.26 | 0.33 | 0.38 | 0.41 | 0.43 | 0.45 | 0.45 | 0.45 | 0.45 | 0.44 | 0.42 | 0.40 | 0.37 | 0.34 | 0.31 | 0.28 | 0.24 | 0.20 | 0.15 |
|  | 75\% | 0.18 | 0.28 | 0.35 | 0.40 | 0.44 | 0.47 | 0.48 | 0.49 | 0.49 | 0.48 | 0.47 | 0.45 | 0.43 | 0.41 | 0.38 | 0.34 | 0.30 | 0.26 | 0.22 | 0.17 |
|  | 80\% | 0.19 | 0.30 | 0.38 | 0.43 | 0.47 | 0.50 | 0.52 | 0.53 | 0.53 | 0.52 | 0.51 | 0.49 | 0.47 | 0.44 | 0.41 | 0.37 | 0.33 | 0.29 | 0.24 | 0.19 |
|  | 85\% | 0.21 | 0.32 | 0.40 | 0.46 | 0.51 | 0.54 | 0.56 | 0.57 | 0.57 | 0.56 | 0.55 | 0.53 | 0.51 | 0.48 | 0.45 | 0.41 | 0.37 | 0.32 | 0.27 | 0.22 |
|  | 90\% | 0.22 | 0.34 | 0.43 | 0.49 | 0.54 | 0.57 | 0.60 | 0.61 | 0.61 | 0.61 | 0.59 | 0.58 | 0.55 | 0.52 | 0.49 | 0.45 | 0.41 | 0.36 | 0.31 | 0.25 |
|  | 95\% | 0.23 | 0.36 | 0.46 | 0.53 | 0.58 | 0.61 | 0.64 | 0.65 | 0.65 | 0.65 | 0.64 | 0.62 | 0.60 | 0.57 | 0.53 | 0.49 | 0.45 | 0.40 | 0.35 | 0.29 |
|  | 100\% | 0.25 | 0.39 | 0.49 | 0.56 | 0.61 | 0.65 | 0.68 | 0.70 | 0.70 | 0.70 | 0.69 | 0.67 | 0.65 | 0.62 | 0.58 | 0.54 | 0.50 | 0.45 | 0.39 | 0.33 |

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## EV when M = 10

Good calling range

## Villain's Push Range

Hero's
Call
Range

|  | 5\% | 10\% | 15\% | 20\% | 25\% | 30\% | 35\% | 40\% | 45\% | 50\% | 55\% | 60\% | 65\% | 70\% | 75\% | 80\% | 85\% | 90\% | 95\% | 100\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5\% | 5ex | 0.09 | 0.13 | 0.17 | 0.21 | 0.24 | 0.28 | 0.32 | 0.35 | 0.39 | 0.42 | 0.45 | 0.49 | 0.52 | 0.55 | 0.59 | 0.62 | 0.65 | 0.68 | 0.71 |
| 10\% | 0.05 | 0.09 | 4 | 0.16 | 0.19 | 0.22 | 0.25 | 0.28 | 0.31 | 0.33 | 0.35 | 0.38 | 0.40 | 0.42 | 0.44 | 0.46 | 0.48 | 0.50 | 0.52 | 0.54 |
| 15\% | 0.06 | 0.10 | 0.14 | 0.17 | 0.19 | 0.22 | 0.24 | 0.26 | 0.28 | 0.30 | 0.31 | 0.33 | 0.34 | 0.36 | 0.37 | 0.38 | 0.39 | 0.40 | 0.41 | 0.42 |
| 20\% | 0.07 | 0.11 | 0.14 | 0.17 | 0.20 | 0.22 | 0.24 | 0.25 | 0.27 | 0.28 | 0.29 | 0.30 | 0.30 | 0.31 | 0.31 | 0.32 | 0.32 | 0.32 | 0.32 | 0.32 |
| 25\% | 0.08 | 0.12 | 0.16 | 0.19 | 0.21 | 0.23 | 0.24 | 0.2 | 0.26 | 0.27 | 0.27 | 0.28 | 0.28 | 0.28 | 0.28 | 0.27 | 0.27 | 0.26 | 0.25 | 0.25 |
| 30\% | 0.09 | 0.14 | 0.17 | 0.20 | 0.22 | 0.24 | 0.25 | 0.26 | 0.27 | -27 | 0.27 | 0.27 | 0.26 | 0.26 | 0.25 | 0.24 | 0.23 | 0.22 | 0.20 | 0.19 |
| 35\% | 0.10 | 0.15 | 0.19 | 0.22 | 0.24 | 0.26 | 0.27 | 0.27 | 0.28 | 0.28 | 0.27 | - | 0.26 | 0.25 | 0.24 | 0.22 | 0.20 | 0.19 | 0.16 | 0.14 |
| 40\% | 0.11 | 0.17 | 0.21 | 0.24 | 0.26 | 0.28 | 0.29 | 0.29 | 0.29 | 0.29 | 0.28 | 0.27 | 0.20 | - 2.25 | 0.23 | 0.21 | 0.19 | 0.16 | 0.14 | 0.11 |
| 45\% | 0.12 | 0.18 | 0.23 | 0.26 | 0.29 | 0.30 | 0.31 | 0.32 | 0.32 | 0.31 | 0.30 | 0.29 | 0.27 | 0.25 | 0.25 | 0.21 | 0.18 | 0.15 | 0.12 | 0.09 |
| 50\% | 0.13 | 0.20 | 0.25 | 0.29 | 0.31 | 0.33 | 0.34 | 0.34 | 0.34 | 0.33 | 0.32 | 0.31 | 0.29 | 0.27 | 0.24 | 0.21 | 0 | 0.15 | 0.12 | 0.08 |
| 55\% | 0.14 | 0.22 | 0.28 | 0.32 | 0.34 | 0.36 | 0.37 | 0.37 | 0.37 | 0.36 | 0.35 | 0.33 | 0.31 | 0.29 | 0.26 | 0.23 | 0.19 | 0.16 |  | 0.08 |
| 60\% | 0.16 | 0.24 | 0.30 | 0.34 | 0.37 | 0.39 | 0.40 | 0.41 | 0.40 | 0.39 | 0.38 | 0.36 | 0.34 | 0.31 | 0.28 | 0.25 | 0.21 | 0.17 | 0.13 | 0.08 |
| 65\% | 0.17 | 0.26 | 0.33 | 0.37 | 0.41 | 0.43 | 0.44 | 0.44 | 0.44 | 0.43 | 0.41 | 0.39 | 0.37 | 0.34 | 0.31 | 0.27 | 0.23 | 0.19 | 0.14 | 0.09 |
| 70\% | 0.18 | 0.28 | 0.35 | 0.40 | 0.44 | 0.46 | 0.47 | 0.48 | 0.48 | 0.47 | 0.45 | 0.43 | 0.40 | 0.37 | 0.34 | 0.30 | 0.26 | 0.21 | 0.16 | 0.11 |
| 75\% | 0.20 | 0.31 | 0.38 | 0.44 | 0.47 | 0.50 | 0.51 | 0.52 | 0.52 | 0.51 | 0.49 | 0.47 | 0.44 | 0.41 | 0.38 | 0.33 | 0.29 | 0.24 | 0.19 | 0.13 |
| 80\% | 0.21 | 0.33 | 0.41 | 0.47 | 0.51 | 0.54 | 0.56 | 0.56 | 0.56 | 0.55 | 0.54 | 0.51 | 0.49 | 0.45 | 0.42 | 0.37 | 0.33 | 0.28 | 0.22 | 0.16 |
| 85\% | 0.23 | 0.35 | 0.44 | 0.50 | 0.55 | 0.58 | 0.60 | 0.61 | 0.61 | 0.60 | 0.58 | 0.56 | 0.53 | 0.50 | 0.46 | 0.42 | 0.37 | 0.32 | 0.26 | 0.20 |
| 90\% | 0.24 | 0.38 | 0.47 | 0.54 | 0.59 | 0.62 | 0.64 | 0.66 | 0.66 | 0.65 | 0.63 | 0.61 | 0.58 | 0.55 | 0.51 | 0.46 | 0.41 | 0.36 | 0.30 | 0.24 |
| 95\% | 0.26 | 0.40 | 0.50 | 0.58 | 0.63 | 0.67 | 0.69 | 0.70 | 0.71 | 0.70 | 0.69 | 0.66 | 0.64 | 0.60 | 0.56 | 0.51 | 0.46 | 0.41 | 0.35 | 0.28 |
| 100\% | 0.27 | 0.42 | 0.53 | 0.61 | 0.67 | 0.71 | 0.74 | 0.76 | 0.76 | 0.75 | 0.74 | 0.72 | 0.69 | 0.66 | 0.62 | 0.57 | 0.52 | 0.46 | 0.40 | 0.33 |

## EV when M = 10

Good calling range

## Villain's Push Range

If SB Pushes Top 90\%
 <br> Good <br> Bad <br> Optimal (Most in Favor of BB)}

Massachusetts Institute of Technology

## EV when $\mathrm{M}=9$

Good calling range

## Villain's Push Range

|  |  | 5\% | 10\% | 15\% | 20\% | 25\% | 30\% | 35\% | 40\% | 45\% | 50\% | 55\% | 60\% | 65\% | 70\% | 75\% | 80\% | 85\% | 90\% | 95\% | 100\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 5\% | 0.05 | 0.09 | 0.13 | 0.17 | 0.21 | 0.25 | 0.29 | 0.32 | 0.36 | 0.40 | 0.43 | 0.47 | 0.50 | 0.54 | 0.57 | 0.60 | 0.64 | 0.67 | 0.70 | 0.74 |
|  | 10\% | 0.05 | 0.09 | 0.13 | 0.17 | 0.20 | 0.23 | 0.26 | 0.29 | 0.32 | 0.34 | 0.37 | 0.40 | 0.42 | 0.44 | 0.47 | 0.49 | 0.51 | 0.54 | 0.56 | 0.58 |
|  | 15\% | 0.06 | 0.10 | 0.14 | 0.17 | 0.20 | 0.22 | 0.25 | 0.27 | 0.29 | 0.31 | 0.33 | 0.35 | 0.37 | 0.38 | 0.40 | 0.41 | 0.43 | 0.44 | 0.45 | 0.46 |
|  | 20\% | 0.06 | 0.11 | 0.14 | 0.17 | 0.20 | 0.22 | 0.24 | 0.26 | 0.28 | 0.29 | 0.31 | 0.32 | 0.33 | 0.34 | 0.35 | 0.35 | 0.36 | 0.37 | 0.37 | 0.37 |
|  | 25\% | 0.07 | 0.12 | 0.15 | 0.18 | 0.21 | 0.23 | 0.25 | 0.26 | 0.27 | 0.28 | 0.29 | 0.30 | 0.30 | 0.31 | 0.31 | 0.31 | 0.31 | 0.31 | 0.31 | 0.30 |
|  | 30\% | 0.08 | 0.13 | 0.17 | 0.20 | 0.22 | 0.24 | 0.26 | 0.27 | 0.28 | 0.28 | 0.29 | 0.29 | 0.29 | 0.29 | 0.28 | 0.28 | 0.27 | 0.27 | 0.26 | 0.25 |
| 110 | 35\% | 0.09 | 0.14 | 0.18 | $\text { Call } \%=\operatorname{Push} \% * 1 / 2$ |  |  |  |  | 0.28 | 0.29 | 0.29 | 0.29 | 0.28 | 0.28 | 0.27 | 0.26 | 0.25 | 0.23 | 0.22 | 0.20 |
|  | 40\% | 0.10 | 0.16 | 0.20 |  |  |  |  |  | 0.30 | 0.30 | 0.30 | 0.29 | 0.28 | 0.27 | 0.26 | 0.25 | 0.23 | 0.21 | 0.19 | 0.17 |
| $\bigcirc 1$ | 45\% | 0.11 | 0.17 | 0.22 |  |  |  |  |  | 0.32 | 0.31 | 0.31 | 0.30 | 0.29 | 0.28 | 0.26 | 0.24 | 0.22 | 0.20 | 0.18 | 0.15 |
| $\boldsymbol{\square}$ | 50\% | 0.12 | 0.19 | 0.24 | - | \%.00 | -roz | -r.s) | \%.07 | 0.34 | 0.33 | 0.33 | 0.32 | 0.30 | 0.29 | 0.27 | 0.25 | 0.22 | 0.19 | 0.17 | 0.14 |
|  | 55\% | 0.13 | 0.21 | 0.26 | 0.30 | 0.32 | 0.34 | 0.36 | 0.36 | 0.36 | 0.36 | 0.35 | 0.34 | 0.32 | 0.30 | 0.28 | 0.25 | 0.23 | 0.20 | 0.16 | 0.13 |
|  | 60\% | 0.14 | 0.22 | 0.28 | 0.32 | 0.35 | 0.37 | 0.38 | 0.39 | 0.39 | 0.38 | 0.37 | 0.36 | 0.34 | 0.32 | 0.30 | 0.27 | 0.24 | 0.20 | 0.17 | 0.13 |
| 2anor | 65\% | 0.16 | 0.24 | 0.30 | 0.35 | 0.38 | 0.40 | 0.41 | 0.42 | 0.42 | 0.41 | 0.40 | 0.39 | 0.37 | 0.34 | 0.32 | 0.29 | 0.25 | 0.22 | 0.18 | 0.14 |
| Laile | 70\% | 0.17 | 0.26 | 0.33 | 0.38 | 0.41 | 0.43 | 0.45 | 0.45 | 0.45 | 0.45 | 0.44 | 0.42 | 0.40 | 0.37 | 0.34 | 0.31 | 0.28 | 0.24 | 0.20 | 0.15 |
|  | 75\% | 0.18 | 0.28 | 0.35 | 0.40 | 0.44 | 0.47 | 0.48 | 0.49 | 0.49 | 0.48 | 0.47 | 0.45 | 0.43 | 0.41 | 0.38 | 0.34 | 0.30 | 0.26 | 0.22 | 0.17 |
|  | 80\% | 0.19 | 0.30 | 0.38 | 0.43 | 0.47 | 0.50 | 0.52 | 0.53 | 0.53 | 0.52 | 0.51 | 0.49 | 0.47 | 0.44 | 0.41 | 0.37 | 0.33 | 0.29 | 0.24 | 0.19 |
|  | 85\% | 0.21 | 0.32 | 0.40 | 0.46 | 0.51 | 0.54 | 0.56 | 0.57 | 0.57 | 0.56 | 0.55 | 0.53 | 0.51 | 0.48 | 0.45 | 0.41 | 0.37 | 0.32 | 0.27 | 0.22 |
|  | 90\% | 0.22 | 0.34 | 0.43 | 0.49 | 0.54 | 0.57 | 0.60 | 0.61 | 0.61 | 0.61 | 0.59 | 0.58 | 0.55 | 0.52 | 0.49 | 0.45 | 0.41 | 0.36 | 0.31 | 0.25 |
|  | 95\% | 0.23 | 0.36 | 0.46 | 0.53 | 0.58 | 0.61 | 0.64 | 0.65 | 0.65 | 0.65 | 0.64 | 0.62 | 0.60 | 0.57 | 0.53 | 0.49 | 0.45 | 0.40 | 0.35 | 0.29 |
|  | 100\% | 0.25 | 0.39 | 0.49 | 0.56 | 0.61 | 0.65 | 0.68 | 0.70 | 0.70 | 0.70 | 0.69 | 0.67 | 0.65 | 0.62 | 0.58 | 0.54 | 0.50 | 0.45 | 0.39 | 0.33 |

Good Bad
Optimal (Most in Favor of BB)

Massachusetts Institute of Technology

## EV when M = 6

Good calling range

## Villain's Push Range


Good Bad
Optimal (Most in Favor of BB)

Massachusetts Institute of Technology

## EV when M = 4

Good calling range

## Villain's Push Range

|  |  | 5\% | 10\% | 15\% | 20\% | 25\% | 30\% | 35\% | 40\% | 45\% | 50\% | 55\% | 60\% | 65\% | 70\% | 75\% | 80\% | 85\% | 90\% | 95\% | 100\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 5\% | 0.05 | 0.09 | 0.14 | 0.18 | 0.23 | 0.27 | 0.31 | 0.36 | 0.40 | 0.44 | 0.48 | 0.53 | 0.57 | 0.61 | 0.65 | 0.69 | 0.74 | 0.78 | 0.82 | 0.86 |
|  | 10\% | 0.05 | 0.09 | 0.14 | 0.18 | 0.22 | 0.26 | 0.30 | 0.33 | 0.37 | 0.41 | 0.45 | 0.48 | 0.52 | 0.56 | 0.59 | 0.63 | 0.66 | 0.70 | 0.73 | 0.77 |
|  | 15\% | 0.05 | 0.09 | 0.14 | 0.17 | 0.21 | 0.25 | 0.28 | 0.32 | 0.35 | 0.39 | 0.42 | 0.45 | 0.48 | 0.51 | 0.55 | 0.58 | 0.61 | 0.64 | 0.67 | 0.70 |
|  | 20\% | 0.05 | 0.10 | 0.14 | 0.17 | 0.21 | 0.24 | 0.28 | 0.31 | 0.34 | 0.37 | 0.40 | 0.43 | 0.45 | 0.48 | 0.51 | 0.54 | 0.56 | 0.59 | 0.61 | 0.64 |
|  | 25\% | 0.06 | 0.10 | 0.14 | 0.17 | 0.21 | 0.24 | 0.27 | 0.30 | 0.33 | $\text { Callo } \% \text { Push } \%$ |  |  |  | - 46 | 0.48 | 0.50 | 0.52 | 0.54 | 0.57 | 0.59 |
| TT ${ }^{9}$ | 30\% | 0.06 | 0.10 | 0.14 | 0.18 | 0.21 | 0.24 | 0.27 | 0.30 | 0.32 |  |  |  |  | . 43 | 0.45 | 0.47 | 0.49 | 0.51 | 0.53 | 0.54 |
| 11 | 35\% | 0.06 | 0.11 | 0.15 | 0.18 | 0.21 | 0.24 | 0.27 | 0.29 | 0.32 |  |  |  |  | . 42 | 0.43 | 0.45 | 0.46 | 0.48 | 0.49 | 0.51 |
|  | 40\% | 0.07 | 0.11 | 0.15 | 0.19 | 0.22 | 0.24 | 0.27 | 0.29 | 0.31 | 0.33 | 0.35 | 0.37 | 0.39 | 0.40 | 0.42 | 0.43 | 0.44 | 0.45 | 0.46 | 0.47 |
| $\Omega$ | 45\% | 0.07 | 0.12 | 0.16 | 0.19 | 0.22 | 0.25 | 0.27 | 0.30 | 0.32 | 0.33 | 0.35 | 0.36 | 0.38 | 0.39 | 0.40 | 0.41 | 0.42 | 0.43 | 0.44 | 0.44 |
| d | 50\% | 0.07 | 0.12 | 0.16 | 0.20 | 0.23 | 0.25 | 0.28 | 0.30 | 0.32 | 0.33 | 0.35 | 0.36 | 0.37 | 0.38 | 0.39 | 0.40 | 0.41 | 0.41 | 0.42 | 0.42 |
|  | 55\% | 0.08 | 0.13 | 0.17 | 0.21 | 0.24 | 0.26 | 0.28 | 0.30 | 0.32 | 0.34 | 0.35 | 0.36 | 0.37 | 0.38 | 0.38 | 0.39 | 0.39 | 0.40 | 0.40 | 0.40 |
|  | 60\% | 0.08 | 0.14 | 0.18 | 0.21 | 0.24 | 0.27 | 0.29 | 0.31 | 0.33 | 0.34 | 0.35 | 0.36 | 0.37 | 0.37 | 0.38 | 0.38 | 0.38 | 0.38 | 0.38 | 0.38 |
| のno | 65\% | 0.09 | 0.14 | 0.19 | 0.22 | 0.25 | 0.28 | 0.30 | 0.32 | 0.33 | 0.34 | 0.35 | 0.36 | 0.37 | 0.37 | 0.38 | 0.38 | 0.38 | 0.37 | 0.37 | 0.37 |
| - | 70\% | 0.09 | 0.15 | 0.20 | 0.23 | 0.26 | 0.29 | 0.31 | 0.32 | 0.34 | 0.35 | 0.36 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 0.36 | 0.36 |
|  | 75\% | 0.10 | 0.16 | 0.20 | 0.24 | 0.27 | 0.30 | 0.32 | 0.33 | 0.35 | 0.36 | 0.37 | 0.37 | 0.37 | 0.38 | 0.38 | 0.37 | 0.37 | 0.36 | 0.36 | 0.35 |
|  | 80\% | 0.10 | 0.17 | 0.21 | 0.25 | 0.28 | 0.31 | 0.33 | 0.34 | 0.36 | 0.37 | 0.37 | 0.38 | 0.38 | 0.38 | 0.38 | 0.37 | 0.37 | 0.36 | 0.35 | 0.34 |
|  | 85\% | 0.11 | 0.17 | 0.22 | 0.26 | 0.29 | 0.32 | 0.34 | 0.36 | 0.37 | 0.38 | 0.38 | 0.39 | 0.39 | 0.39 | 0.38 | 0.38 | 0.37 | 0.36 | 0.35 | 0.34 |
|  | 90\% | 0.11 | 0.18 | 0.23 | 0.27 | 0.31 | 0.33 | 0.35 | 0.37 | 0.38 | 0.39 | 0.39 | 0.40 | 0.40 | 0.39 | 0.39 | 0.38 | 0.37 | 0.36 | 0.35 | 0.33 |
|  | 95\% | 0.12 | 0.19 | 0.24 | 0.28 | 0.32 | 0.34 | 0.36 | 0.38 | 0.39 | 0.40 | 0.40 | 0.41 | 0.41 | 0.40 | 0.39 | 0.39 | 0.38 | 0.36 | 0.35 | 0.33 |
|  | 100\% | 0.12 | 0.20 | 0.25 | 0.30 | 0.33 | 0.36 | 0.38 | 0.39 | 0.41 | 0.41 | 0.42 | 0.42 | 0.42 | 0.41 | 0.40 | 0.39 | 0.38 | 0.37 | 0.35 | 0.33 |

Good Bad
Optimal (Most in Favor of BB)

Massachusetts Institute of Technology

## EV when $\mathrm{M}=2$

Good calling range

## Villain's Push Range

|  |  | 5\% | 10\% | 15\% | 20\% | 25\% | 30\% | 35\% | 40\% | 45\% | 50\% | 55\% | 60\% | 65\% | 70\% | 75\% | 80\% | 85\% | 90\% | 95\% | 100\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 5\% | 0.05 | 0.10 | 0.14 | 0.19 | 0.23 | 0.28 | 0.33 | 0.37 | 0.42 | 0.46 | 0.51 | 0.55 | 0.60 | 0.64 | 0.69 | 0.73 | 0.78 | 0.82 | 0.86 | 0.91 |
|  | 10\% | 0.05 | 0.09 | 0.14 | 0.18 | 0.22 | 0.27 | 0.31 | 0.35 | 0.39 | 0.44 | 0.48 | 0.52 | 0.56 | 0.60 | 0.64 | 0.68 | 0.72 | 0.76 | 0.80 | 0.85 |
|  | 15\% | 0.05 | 0.09 | 0.14 | 0.18 | 0.22 | 0.26 | 0.30 | 0.34 | 0.38 | 0.42 | 0.45 | 0.49 | 0.53 | 0.57 | 0.61 | 0.64 | 0.68 | 0.72 | 0.75 | 0.79 |
|  | 20\% | 0.05 | 0.09 | 0.13 | 0.17 | 0.21 | 0.25 | 0.29 | 0.33 | 0.36 | 0.40 | 0.43 | 0.47 | 0.50 | 0.54 | 0.57 | 0.61 | 0.64 | 0.68 | 0.71 | 0.74 |
|  | 25\% | 0.05 | 0.09 | 0.13 | 0.17 | 0.21 | $\mathrm{Call} \%=\mathrm{Push} \% * 2$ |  |  |  |  | 0.42 | 0.45 | 0.48 | 0.51 | 0.55 | 0.58 | 0.61 | 0.64 | 0.67 | 0.70 |
| TTPen | 30\% | 0.05 | 0.09 | 0.13 | 0.17 | 0.21 |  |  |  |  |  | 0.40 | 0.43 | 0.46 | 0.49 | 0.52 | 0.55 | 0.58 | 0.61 | 0.63 | 0.66 |
| Hero | 35\% | 0.05 | 0.09 | 0.13 | 0.17 | 0.20 |  |  |  |  |  | 0.39 | 0.42 | 0.44 | 0.47 | 0.50 | 0.52 | 0.55 | 0.58 | 0.60 | 0.63 |
|  | 40\% | 0.05 | 0.09 | 0.13 | 0.17 | 0.20 | 0.23 | 0.26 | 0.29 | 0.32 | 0.35 | 0.38 | 0.40 | 0.43 | 0.45 | 0.48 | 0.50 | 0.53 | 0.55 | 0.57 | 0.59 |
|  | 45\% | 0.05 | 0.10 | 0.13 | 0.17 | 0.20 | 0.23 | 0.26 | 0.29 | 0.32 | 0.34 | 0.37 | 0.39 | 0.41 | 0.44 | 0.46 | 0.48 | 0.50 | 0.52 | 0.54 | 0.56 |
| - | 50\% | 0.06 | 0.10 | 0.13 | 0.17 | 0.20 | 0.23 | 0.26 | 0.28 | 0.31 | 0.33 | 0.36 | 0.38 | 0.40 | 0.42 | 0.44 | 0.46 | 0.48 | 0.50 | 0.52 | 0.53 |
|  | 55\% | 0.06 | 0.10 | 0.14 | 0.17 | 0.20 | 0.23 | 0.26 | 0.28 | 0.30 | 0.33 | 0.35 | 0.37 | 0.39 | 0.41 | 0.43 | 0.44 | 0.46 | 0.48 | 0.49 | 0.51 |
|  | 60\% | 0.06 | 0.10 | 0.14 | 0.17 | 0.20 | 0.23 | 0.25 | 0.28 | 0.30 | 0.32 | 0.34 | 0.36 | 0.38 | 0.40 | 0.41 | 0.43 | 0.44 | 0.46 | 0.47 | 0.48 |
| ح | 65\% | 0.06 | 0.10 | 0.14 | 0.17 | 0.20 | 0.23 | 0.25 | 0.28 | 0.30 | 0.32 | 0.33 | 0.35 | 0.37 | 0.38 | 0.40 | 0.41 | 0.42 | 0.44 | 0.45 | 0.46 |
| Malle | 70\% | 0.06 | 0.11 | 0.14 | 0.17 | 0.20 | 0.23 | 0.25 | 0.27 | 0.29 | 0.31 | 0.33 | 0.34 | 0.36 | 0.37 | 0.39 | 0.40 | 0.41 | 0.42 | 0.43 | 0.44 |
|  | 75\% | 0.06 | 0.11 | 0.14 | 0.18 | 0.20 | 0.23 | 0.25 | 0.27 | 0.29 | 0.31 | 0.32 | 0.34 | 0.35 | 0.36 | 0.38 | 0.39 | 0.39 | 0.40 | 0.41 | 0.42 |
|  | 80\% | 0.07 | 0.11 | 0.15 | 0.18 | 0.21 | 0.23 | 0.25 | 0.27 | 0.29 | 0.31 | 0.32 | 0.33 | 0.34 | 0.36 | 0.36 | 0.37 | 0.38 | 0.39 | 0.39 | 0.40 |
|  | 85\% | 0.07 | 0.11 | 0.15 | 0.18 | 0.21 | 0.23 | 0.25 | 0.27 | 0.29 | 0.30 | 0.32 | 0.33 | 0.34 | 0.35 | 0.36 | 0.36 | 0.37 | 0.37 | 0.38 | 0.38 |
|  | 90\% | 0.07 | 0.12 | 0.15 | 0.18 | 0.21 | 0.23 | 0.25 | 0.27 | 0.29 | 0.30 | 0.31 | 0.32 | 0.33 | 0.34 | 0.35 | 0.35 | 0.36 | 0.36 | 0.36 | 0.36 |
|  | 95\% | 0.07 | 0.12 | 0.16 | 0.19 | 0.21 | 0.24 | 0.26 | 0.27 | 0.29 | 0.30 | 0.31 | 0.32 | 0.33 | 0.33 | 0.34 | 0.34 | 0.35 | 0.35 | 0.35 | 0.35 |
|  | 100\% | 0.07 | 0.12 | 0.16 | 0.19 | 0.22 | 0.24 | 0.26 | 0.27 | 0.29 | 0.30 | 0.31 | 0.32 | 0.32 | 0.33 | 0.33 | 0.34 | 0.34 | 0.34 | 0.34 | 0.33 |

$\square$
Good
Bad

Optimal (Most in Favor of BB)

Massachusetts Institute of Technology

## EV when M = 1

Good calling range

## Villain's Push Range

|  |  | 5\% | 10\% | 15\% | 20\% | 25\% | 30\% | 35\% | 40\% | 45\% | 50\% | 55\% | 60\% | 65\% | 70\% | 75\% | 80\% | 85\% | 90\% | 95\% | 100\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 5\% | 0.05 | 0.10 | 0.14 | 0.19 | 0.24 | 0.28 | 0.33 | 0.38 | 0.42 | 0.47 | 0.52 | 0.56 | 0.61 | 0.66 | 0.70 | 0.75 | 0.80 | 0.84 | 0.89 | 0.93 |
|  | 10\% | 0.05 | 0.09 | 0.14 | 0.18 | 0.23 | 0.27 | 0.32 | 0.36 | 0.41 | 0.45 | 0.49 | 0.54 | 0.58 | 0.62 | 0.67 | 0.71 | 0.75 | 0.80 | 0.84 | 0.88 |
|  | 15\% | 0.05 | 0.09 | 0.14 | 0.18 | 0.22 | 0.26 | 0.31 | 0.35 | 0.39 | 0.43 | 0.47 | 0.51 | 0.55 | 0.59 | 0.64 | 0.68 | 0.72 | 0.76 | 0.80 | 0.84 |
|  | 20\% | 0.05 | 0.09 | 0.13 | 0.17 | 0.21 | 0.25 | 0.29 | 0.33 | 0.37 | 0.41 | 0.45 | 0.49 | 0.53 | 0.57 | 0.61 | 0.64 | 0.68 | 0.72 | 0.76 | 0.80 |
|  | 25\% | 0.05 | 0.09 | 0.13 | 0.17 | 0.21 | $\text { Call } \%=\text { Push } \% * 10$ |  |  |  |  | . 43 | 0.47 | 0.51 | 0.54 | 0.58 | 0.62 | 0.65 | 0.69 | 0.72 | 0.76 |
| T | 30\% | 0.05 | 0.09 | 0.13 | 0.17 | 0.20 |  |  |  |  |  | . 42 | 0.45 | 0.49 | 0.52 | 0.55 | 0.59 | 0.62 | 0.66 | 0.69 | 0.72 |
| Hero | 35\% | 0.05 | 0.09 | 0.12 | 0.16 | 0.20 |  |  |  |  |  | . 40 | 0.44 | 0.47 | 0.50 | 0.53 | 0.56 | 0.59 | 0.62 | 0.66 | 0.69 |
|  | 40\% | 0.05 | 0.09 | 0.12 | 0.16 | 0.19 | 0.23 | 0.26 | 0.29 | 0.33 | 0.36 | 0.39 | 0.42 | 0.45 | 0.48 | 0.51 | 0.54 | 0.57 | 0.60 | 0.62 | 0.65 |
|  | 45\% | 0.05 | 0.08 | 0.12 | 0.16 | 0.19 | 0.22 | 0.25 | 0.28 | 0.32 | 0.34 | 0.37 | 0.40 | 0.43 | 0.46 | 0.49 | 0.51 | 0.54 | 0.57 | 0.60 | 0.62 |
| - | 50\% | 0.05 | 0.08 | 0.12 | 0.15 | 0.19 | 0.22 | 0.25 | 0.28 | 0.31 | 0.33 | 0.36 | 0.39 | 0.41 | 0.44 | 0.47 | 0.49 | 0.52 | 0.54 | 0.57 | 0.59 |
|  | 55\% | 0.05 | 0.08 | 0.12 | 0.15 | 0.18 | 0.21 | 0.24 | 0.27 | 0.30 | 0.32 | 0.35 | 0.37 | 0.40 | 0.42 | 0.45 | 0.47 | 0.49 | 0.52 | 0.54 | 0.56 |
|  | 60\% | 0.05 | 0.08 | 0.12 | 0.15 | 0.18 | 0.21 | 0.23 | 0.26 | 0.29 | 0.31 | 0.34 | 0.36 | 0.38 | 0.41 | 0.43 | 0.45 | 0.47 | 0.49 | 0.51 | 0.53 |
|  | 65\% | 0.05 | 0.08 | 0.12 | 0.15 | 0.18 | 0.20 | 0.23 | 0.25 | 0.28 | 0.30 | 0.32 | 0.35 | 0.37 | 0.39 | 0.41 | 0.43 | 0.45 | 0.47 | 0.49 | 0.51 |
| Hante | 70\% | 0.05 | 0.08 | 0.12 | 0.15 | 0.17 | 0.20 | 0.22 | 0.25 | 0.27 | 0.29 | 0.31 | 0.33 | 0.35 | 0.37 | 0.39 | 0.41 | 0.43 | 0.45 | 0.46 | 0.48 |
|  | 75\% | 0.05 | 0.08 | 0.11 | 0.14 | 0.17 | 0.20 | 0.22 | 0.24 | 0.26 | 0.28 | 0.30 | 0.32 | 0.34 | 0.36 | 0.38 | 0.39 | 0.41 | 0.42 | 0.44 | 0.45 |
|  | 80\% | 0.05 | 0.08 | 0.11 | 0.14 | 0.17 | 0.19 | 0.21 | 0.24 | 0.26 | 0.27 | 0.29 | 0.31 | 0.33 | 0.34 | 0.36 | 0.37 | 0.39 | 0.40 | 0.41 | 0.43 |
|  | 85\% | 0.05 | 0.08 | 0.11 | 0.14 | 0.17 | 0.19 | 0.21 | 0.23 | 0.25 | 0.27 | 0.28 | 0.30 | 0.31 | 0.33 | 0.34 | 0.36 | 0.37 | 0.38 | 0.39 | 0.40 |
|  | 90\% | 0.05 | 0.08 | 0.11 | 0.14 | 0.16 | 0.18 | 0.21 | 0.22 | 0.24 | 0.26 | 0.27 | 0.29 | 0.30 | 0.31 | 0.33 | 0.34 | 0.35 | 0.36 | 0.37 | 0.38 |
|  | 95\% | 0.05 | 0.08 | 0.11 | 0.14 | 0.16 | 0.18 | 0.20 | 0.22 | 0.23 | 0.25 | 0.26 | 0.28 | 0.29 | 0.30 | 0.31 | 0.32 | 0.33 | 0.34 | 0.35 | 0.36 |
|  | 100\% | 0.05 | 0.08 | 0.11 | 0.14 | 0.16 | 0.18 | 0.20 | 0.21 | 0.23 | 0.24 | 0.26 | 0.27 | 0.28 | 0.29 | 0.30 | 0.31 | 0.31 | 0.32 | 0.33 | 0.33 |

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## When Hero is BB (second to act)

The dominating calling range is a proportion of the SB's pushing range

| Stack (M) | Call Multiple |
| :---: | :---: |
| 1 | 10 x |
| 2 | 2 x |
| 4 | 1 x |
| 6 | $2 / 3 \mathrm{x}$ |
| 9 | $1 / 2 \mathrm{x}$ |

Optimal BB Call $\%=$ SB Push $\% * \mathrm{CM}$

## Other Positions (Calling)



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## Other Positions (Calling)



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## Pre-flop Analysis

- Motivator
- Range Definition
- Basic Assumptions
- Heads Up
- Other Positions

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## Other Positions (Calling)

We need to make generous assumptions to help develop rules for other positions

The biggest unknown is the actions of players left to act
Assumptions are on the conservative side, so that the errors are folding small positive edges, rather than calling small negative edges

Conservative Assumptions
SDWinAmt = Stack (no discount for being SBRU\%\%)
Second caller range is TT + , $\mathrm{AQ}+$ or top $5 \%$
$\mathrm{Win} \%$ if called twice $=0 \%$
Minimum Win $\%=50 \%+5 \%$ for each remaining player
Since a second caller reduces our equity to $0 \%$

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## Other Positions (Calling)

Minimum Win $\%=50 \%+5 \%$ for each remaining player
HeroWin $\%=50 \%+.085 * \ln ($ VillainRange $)-.085 * \ln ($ HeroRange $)+\varepsilon$

| Players <br> Remaining | Minimum <br> Hero Win\% | Hero Range <br> Multiple | HeroRange = VillainRange * HRM |
| :---: | :---: | :---: | :--- |
| 5 | $75 \%$ | $5 \%$ | Example: Villain pushes top 50\% from |
| 4 | $70 \%$ | $10 \%$ | CO, Hero is BTN (2 remaining) |

## Other Positions (Calling) - Rules of Thumb

| Position | Hero Calling Range |
| :---: | :---: |
| MP | $5 \% * V$-Range |
| LP | $10 \% * V$-Range |
| CO | $30 \% * V$-Range |
| SB | $50 \% * V$-Range |

Thought process of the Cutoff facing a push:

Is his pushing range 3 x wider than my cards?
I have AQ , would he push A5?
Top 5\% vs Top 15\%
I have AT, would he push KJ? Top $10 \%$ vs Top $30 \%$

I have KQ , would he push 85 ?
Top $20 \%$ vs Top $60 \%$

Significant reliance on reading opponent push ranges
Reminder: These are conservative estimates, so a threshold call is likely to be very + EV

## Other Positions (Pushing)

Analytical solution is complex
Our goal is to develop a decision table based on conservative assumptions
Conservative Assumptions
SDWinAmt = Stack
First caller range is $55+$, AT+ or top $10 \%$
Second caller range is TT + , AQ+ or top $5 \%$
$\mathrm{Win} \%$ if called twice $=20 \%$
Third caller range is $0 \%$ [never called three times]
Questions:
When can we profitably push Broadway (Top 30\%)?
When can we profitably push any two cards (Top 100\%)?

## Pushing Top 30\% (Broadway)

Hero's EV (in terms
Remaining Players of M)

|  | $\mathbf{1}$ left | 2 left | 3 left | 4 left | 5 left | 6 left | 7 left | 8 left | 9 left |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $\mathbf{1 M}$ | 0.21 | 0.14 | 0.09 | 0.06 | 0.04 | 0.02 | 0.00 | -0.01 | -0.01 |
| $\mathbf{2 M}$ | 0.20 | 0.14 | 0.08 | 0.05 | 0.02 | 0.00 | -0.02 | -0.03 | -0.04 |
| $\mathbf{3 M}$ | 0.20 | 0.13 | 0.08 | 0.04 | 0.01 | -0.02 | -0.04 | -0.05 | -0.07 |
| $\mathbf{4 M}$ | 0.20 | 0.12 | 0.07 | 0.02 | -0.01 | -0.04 | -0.06 | -0.08 | -0.10 |
| $\mathbf{5 M}$ | 0.20 | 0.12 | 0.06 | 0.01 | -0.02 | -0.05 | -0.08 | -0.10 | -0.12 |
| $\mathbf{6 M}$ | 0.19 | 0.11 | 0.05 | 0.00 | -0.04 | -0.07 | -0.10 | -0.13 | -0.15 |
| $\mathbf{7 M}$ | 0.19 | 0.11 | 0.04 | -0.01 | -0.05 | -0.09 | -0.12 | -0.15 | -0.18 |
| $\mathbf{8 M}$ | 0.19 | 0.10 | 0.03 | -0.02 | -0.07 | -0.11 | -0.14 | -0.17 | -0.20 |
| $\mathbf{9 M}$ | 0.18 | 0.09 | 0.02 | -0.04 | -0.08 | -0.13 | -0.16 | -0.20 | -0.23 |
| $\mathbf{1 0 M}$ | 0.18 | 0.09 | 0.01 | -0.05 | -0.10 | -0.14 | -0.19 | -0.22 | -0.26 |

$\square$ Good $\square \mathrm{Bad}$

## Pushing Top 100\% (ATC)

Hero's EV (in terms
Remaining Players of M)

Stack
Size

|  | $\mathbf{1}$ left | 2 left | 3 left | 4 left | $\mathbf{5}$ left | $\mathbf{6}$ left | 7 left | 8 left | 9 left |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $\mathbf{1 M}$ | 0.63 | 0.37 | 0.18 | 0.05 | -0.05 | -0.12 | -0.17 | -0.21 | -0.24 |
| $\mathbf{2 M}$ | 0.56 | 0.25 | 0.02 | -0.14 | -0.26 | -0.35 | -0.42 | -0.47 | -0.52 |
| $\mathbf{3 M}$ | 0.49 | 0.13 | -0.14 | -0.33 | -0.48 | -0.59 | -0.67 | -0.74 | -0.80 |
| $\mathbf{4 M}$ | 0.42 | 0.01 | -0.30 | -0.52 | -0.69 | -0.82 | -0.92 | -1.01 | -1.08 |
| $\mathbf{5 M}$ | 0.35 | -0.11 | -0.46 | -0.71 | -0.90 | -1.05 | -1.17 | -1.27 | -1.36 |
| $\mathbf{6 M}$ | 0.28 | -0.23 | -0.62 | -0.90 | -1.12 | -1.29 | -1.43 | -1.54 | -1.64 |
| $\mathbf{7 M}$ | 0.21 | -0.36 | -0.78 | -1.09 | -1.33 | -1.52 | -1.68 | -1.81 | -1.92 |
| $\mathbf{8 M}$ | 0.15 | -0.48 | -0.94 | -1.28 | -1.55 | -1.76 | -1.93 | -2.07 | -2.20 |
| $\mathbf{9 M}$ | 0.08 | -0.60 | -1.10 | -1.47 | -1.76 | -1.99 | -2.18 | -2.34 | -2.48 |
| $\mathbf{1 0 M}$ | 0.01 | -0.72 | -1.26 | -1.66 | -1.98 | -2.23 | -2.43 | -2.61 | -2.76 |

$\square$ Good $\square \mathrm{Bad}$

## Pushing Top 10\% (AT+, 55+)

Hero's EV (in terms Remaining Players

$\square$ Good $\square \mathrm{Bad}$

## Pushing Rules of Thumb

AT+ 55+

|  | $\mathbf{1}$ left | $\mathbf{2}$ leff | $\mathbf{3}$ leff | $\mathbf{4}$ left | $\mathbf{5}$ left | $\mathbf{6}$ left | $\mathbf{7}$ left | $\mathbf{8}$ left | $\mathbf{9}$ left |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1 M}$ | 0.07 | 0.06 | 0.04 | 0.03 | 0.03 | 0.02 | 0.02 | 0.01 | 0.01 |
| $\mathbf{2 M}$ | 0.08 | 0.06 | 0.05 | 0.04 | 0.04 | 0.03 | 0.03 | 0.02 | 0.02 |
| $\mathbf{3 M}$ | 0.08 | 0.07 | 0.06 | 0.05 | 0.05 | 0.04 | 0.04 | 0.03 | 0.03 |
| $\mathbf{4 M}$ | 0.09 | 0.08 | 0.07 | 0.06 | 0.06 | 0.05 | 0.05 | 0.04 | 0.04 |
| $\mathbf{5 M}$ | 0.09 | 0.08 | 0.08 | 0.07 | 0.07 | 0.06 | 0.06 | 0.05 | 0.05 |
| $\mathbf{6 M}$ | 0.10 | 0.09 | 0.09 | 0.08 | 0.08 | 0.07 | 0.07 | 0.06 | 0.05 |
| $\mathbf{7 M}$ | 0.10 | 0.10 | 0.10 | 0.09 | 0.09 | 0.08 | 0.08 | 0.07 | 0.06 |
| $\mathbf{8 M}$ | 0.11 | 0.11 | 0.11 | 0.10 | 0.10 | 0.09 | 0.08 | 0.08 | 0.07 |
| $\mathbf{9 M}$ | 0.11 | 0.11 | 0.11 | 0.11 | 0.11 | 0.10 | 0.09 | 0.09 | 0.08 |
| $\mathbf{1 0 M}$ | 0.11 | 0.12 | 0.12 | 0.12 | 0.12 | 0.11 | 0.10 | 0.10 | 0.09 |

## Broadway

|  | 1 left | 2 left | 3 left | 4 left | 5 left | 6 left | 7 left | 8 left | 9 left |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1M | 0.21 | 0.14 | 0.09 | 0.06 | 0.04 | 0.02 | 0.00 | -0.01 | -0.01 |
| 2M | 0.20 | 0.14 | 0.08 | 0.05 | 0.02 | 0.00 | -0.02 | -0.03 | -0.04 |
| 3M | 0.20 | 0.13 | 0.08 | 0.04 | 0.01 | -0.02 | -0.04 | -0.05 | -0.07 |
| $\mathbf{4 M}$ | 0.20 | 0.12 | 0.07 | 0.02 | -0.01 | -0.04 | -0.06 | -0.08 | -0.10 |
| $\mathbf{5 M}$ | 0.20 | 0.12 | 0.06 | 0.01 | -0.02 | 0.05 | 0.08 | -0.10 | -0.12 |
| $\mathbf{6 M}$ | 0.19 | 0.11 | 0.05 | 0.00 | -0.04 | -0.07 | 0.10 | -0.13 | -0.15 |
| $\mathbf{7 M}$ | 0.19 | 0.11 | 0.04 | -0.01 | -0.05 | -0.09 | 0.12 | -0.15 | -0.18 |
| $\mathbf{8 M}$ | 0.19 | 0.10 | 0.03 | -0.02 | -0.07 | -0.11 | -0.14 | -0.17 | -0.20 |
| $\mathbf{9 M}$ | 0.18 | 0.09 | 0.02 | -0.04 | -0.08 | -0.13 | -0.16 | -0.20 | -0.23 |
| $\mathbf{1 0 M}$ | 0.18 | 0.09 | 0.01 | -0.05 | -0.10 | -0.14 | -0.19 | -0.22 | -0.26 |

## Any Two Cards

|  | 1 left | 2 left | 3 left | 4 left | 5 left | 6 left | 7 left | 8 left | 9 left |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1M | 0.63 | 0.37 | 0.18 | 0.05 | -0.05 | -0.12 | -0.17 | -0.21 | -0.24 |
| 2M | 0.56 | 0.25 | 0.02 | 0.14 | -0.26 | -0.35 | -0.42 | -0.47 | -0.52 |
| 3M | 0.49 | 0.13 | -0.14 | -0.33 | -0.48 | -0.59 | -0.67 | -0.74 | -0.80 |
| 4M | 0.42 | 0.01 | -0.30 | -0.52 | -0.69 | -0.82 | -0.92 | -1.01 | -1.08 |
| SM | 0.35 | -0.11 | -0.46 | -0.71 | -0.90 | -1.05 | -1.17 | -1.27 | -1.36 |
| $\mathbf{6 M}$ | 0.28 | -0.23 | -0.62 | -0.90 | -1.12 | -1.29 | -1.43 | -1.54 | -1.64 |
| 7M | 0.21 | -0.36 | -0.78 | -1.09 | -1.33 | -1.52 | -1.68 | -1.81 | -1.92 |
| 8M | 0.15 | -0.48 | -0.94 | -1.28 | -1.55 | -1.76 | -1.93 | -2.07 | -2.20 |
| 9M | 0.08 | -0.60 | -1.10 | -1.47 | -1.76 | -1.99 | -2.18 | -2.34 | -2.48 |
| $\mathbf{1 0 M}$ | 0.01 | -0.72 | -1.26 | -1.66 | -1.98 | -2.23 | -2.43 | -2.61 | -2.76 |

Pushing top $10 \%$ is profitable from any position with stack size up to 10 M

Pushing top $30 \%$ is profitable
From any position with 1M
From late position with 6M
From cutoff with 10M

Pushing top $100 \%$ is profitable
From late position with 1M
From button with 4M
From SB with 10M

## Other Positions (Pushing) - Rules of Thumb

Conservative + EV Pushing Ranges
Simplified (Not Optimal)

| Stack Size | Push ATC From | Push Broadway <br> From | Push AT+, 55+ <br> From |
| :---: | :---: | :---: | :---: |
| 1 M | Late | Always | Always |
| 5 M | Button | Late | Always |
| 10 M | Small Blind | Cutoff | Always |

Always push top $10 \%$ for $\mathrm{M}<10$
Push very wide from any position for $\mathrm{M}<1$
Push in late position with Broadway for $\mathrm{M}<5$

## Other Positions (Calling) - Rules of Thumb

| Position | Hero Calling Range |
| :---: | :---: |
| MP | $5 \% * V$-Range |
| LP | $10 \% * V$-Range |
| CO | $30 \% * V$-Range |
| SB | $50 \% * V$-Range |

Thought process of the Cutoff facing a push:

Is his pushing range 3 x wider than my cards?
I have AQ , would he push A5?
Top 5\% vs Top 15\%
I have AT, would he push KJ? Top $10 \%$ vs Top $30 \%$

I have KQ , would he push 85 ?
Top $20 \%$ vs Top $60 \%$

Significant reliance on reading opponent push ranges
Reminder: These are conservative estimates, so a threshold call is likely to be very + EV

## Heads Up (Pushing) - Rules of Thumb

If you think Villain is tight, top $100 \%$ is always optimal
Otherwise...

| Stack Size | Push |
| :---: | :---: |
| 1 M | Top $100 \%$ |
| 5 M | Top $75 \%$ |
| 10 M | Top $50 \%$ |

## Heads Up (Calling) - Rules of Thumb

The dominating calling range is a proportion of the SB's pushing range

| Stack (M) | Call Multiple |
| :---: | :---: |
| 1 | 10 x |
| 2 | 2 x |
| 4 | 1 x |
| 6 | $2 / 3 \mathrm{x}$ |
| 9 | $1 / 2 \mathrm{x}$ |

Optimal BB Call $\%=$ SB Push $\% * \mathrm{CM}$

## Range of Hands

- Simplified ranges you can memorize
- TT,$+ \mathrm{AQ}+=5 \%$
- $55+$, AT $+=10 \%$
$-22+, \mathbf{A 2 +}, \mathrm{KQ}=20 \%$
$-22+$, A2+, Broadway = 30\%
- Pairs and cards adding to $16=50 \%$
- Any two cards = 100\%

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## 15.S50 Poker Theory and Analytics

January IAP 2015

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