Study Guide Block 4: Matrix Algebra

Unit 3: Inverse Matrices

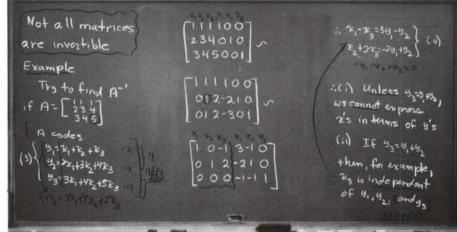
Lecture 4.030

1.

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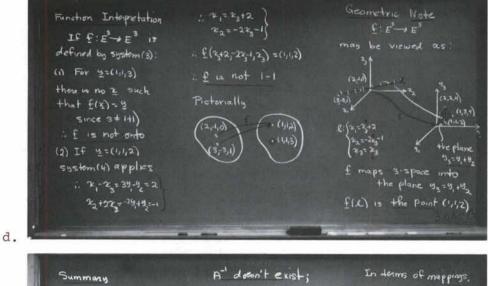
1111007 234010 346001 Inverting a 1002-21 Matrix 01003-2 Find A' IF [111100] 012-210 013-301 = [03-2] - 234 A -A-1 since A' codes 10-1 3-10 $x_1 = 2 \cdot y_1 - 2 \cdot y_2 + y_3 = x_2 = 3 \cdot y_2 - 2 \cdot y_3 = (2)$ 42=221+32+423 53= 32++422+62 23=-91-92+93 Solve for 25 001-1-11in terms of y's a. Function Interpretation : Existence of A" Matrix Algebra is equivalent to existence of f System(i) may be viewed as f:E3 = E³ Interpretation 4=2+22+23 where f(x, x, x, x)=(v,, v, v) e. 3 (-1,1,2) + (2,9,13) 5= 274,+32,+4x, (1) Lor £(x)= 1 3= 32,+42,+623 £(-1,1,2)=(2,9,13) is the single matrix equation f and A are "identifiable" A' need not 234 346 31 92 91 since y=f(z) Siven matrix, A and Y=AX contry Equivalent information - A'Y- A'(AX)=X; X - AY b.

(This lecture will also apply to Unit 4.)



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Summany Part I: Given the n by r matrix A, form the n by 2n matrix [A:In] i.e. [2,...2, 1...] Row - reduce this matrix. Then cither (i) the lefthalf conteins at least one row O's, whence

A⁻¹ doen't exist; or(ii) the left half reduces to In, where the right-half is A⁻¹. Part II

IF A' exists, then in terms of Matrix algebre, we can solve initial the equation Y=AX to Conclude X=A'Y let $\underline{\mathbf{f}}$ be defined by $\underline{\mathbf{f}}(\mathbf{x}_1, \dots, \mathbf{x}_n) = (\theta_{1,1}, \dots, \theta_n)$ where $\{\boldsymbol{y}_1 = \boldsymbol{a}_1, \boldsymbol{x}_1 + \boldsymbol{a}_1, \boldsymbol{x}_n \}$ $\{\boldsymbol{y}_n = \boldsymbol{a}_n, \boldsymbol{x}_1 + \boldsymbol{a}_n, \boldsymbol{x}_n \}$

Then É'exists ↔ A'exists, where A=[a,j] If A' doesn't exist then £ 12 neither 1-1 nor onto

e.

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- 2. Read Supplementary Notes, Chapter 6, Section E.
- 3. (Optional) Read Thomas, Section 13.3.
- 4. Exercises:

4.3.1

Let

$$A = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}, B = \begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix}, C = \begin{pmatrix} 5 & 4 \\ 6 & 5 \end{pmatrix}, 0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Solve each of the following matrix equations.

a. $\frac{1}{3}X - AB = C$.

b. AX - BC = 0.

Let

4.3.2

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \text{ and let } X = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}.$$

Assuming that ad - bc \neq 0, solve AX = I, where I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

4.3.3

Compute A^{-1} if $A = \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}$, imposing any necessary conditions on a or d to insure the existence of A^{-1} . Generalize this result to find A^{-1} if

 $A = \begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ \vdots & & & \vdots \\ \vdots & & & \vdots \\ 0 & 0 & \cdots & a_{nn} \end{bmatrix}$

(i.e., $A = [a_{ij}]$ where $a_{ij} = 0$ when $i \neq j$).

4.3.4(L)

a. Let A and B denote any <u>non-singular</u> 2 by 2 matrices (i.e., A^{-1} and B^{-1} exist). Show that AB is also non-singular and in particular $(AB)^{-1} = B^{-1}A^{-1}$. (Note the order of the factors.)

b. Check (a) when $A = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$.

4.3.5

Let A denote the matrix

 $\begin{pmatrix} 1 & 3 & 5 \\ 2 & 7 & 9 \\ 3 & 9 & 7 \end{pmatrix}$

a. Use the augmented-matrix technique described in the notes to find A^{-1} .

b. If

 $y_1 = x_1 + 3x_2 + 5x_3$ $y_2 = 2x_1 + 7x_2 + 9x_3$ $y_3 = 3x_1 + 9x_2 + 7x_3$

express x_1 , x_2 , x_3 as linear combinations of y_1 , y_2 , and y_3 . c. Solve the system of equations

 $x_1 + 3x_2 + 5x_3 = 8$ $2x_1 + 7x_2 + 9x_3 = 16$ $3x_1 + 9x_2 + 7x_3 = 32$

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4.3.6Let $A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 7 & 9 \\ 3 & 9 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 4 & 8 \\ 8 & 2 & 16 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 8 \\ 4 & 2 \\ 8 & 16 \end{bmatrix}$ a. Determine X if AX = C. b. Determine Y if YA = B. c. Repeat parts (a) and (b) with $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 3 & 7 & 8 \end{bmatrix}$. d. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$, $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, and $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Show that there exist infinitely many matrices X such that $AX = I_2$. e. With A, I_2 , and I_3 as in part (d), show that there is no matrix X such that $XA = I_3$.

4.3.7

Find the flaw in the following argument. "Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Replace the first row of A by the sum of the first and the second rows; and replace the second row of A by the sum of the second row and the first row. Call the resulting matrix B. Therefore, $B = \begin{pmatrix} 4 & 6 \\ 7 & 10 \end{pmatrix}$. Since B was obtained from A by the 'permitted' operations described in the notes, it follows that A ~ B."

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Resource: Calculus Revisited: Multivariable Calculus Prof. Herbert Gross

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