MIT OpenCourseWare
http://ocw.mit.edu

## Continuum Electromechanics

For any use or distribution of this textbook, please cite as follows:
Melcher, James R. Continuum Electromechanics. Cambridge, MA: MIT Press, 1981. Copyright Massachusetts Institute of Technology. ISBN: 9780262131650. Also available online from MIT OpenCourseWare at http://ocw.mit.edu (accessed MM DD, YYYY) under Creative Commons license Attribution-NonCommercial-Share Alike.

For more information about citing these materials or our Terms of Use, visit: http://ocw.mit.edu/terms.

## Appendix A

Differential Operators in
Cartesian, Cylindrical and
Spherical Coordinates

| APPENIX A. Differential Operators in Cartesian, Cylindrical and Spherical Coordinates |  |  |  |
| :---: | :---: | :---: | :---: |
| Operator |  | Cylindrical <br> coordinates | Spherical coordinates |
| ( $\mathrm{\nabla} \cdot \mathrm{~A}$ ) | $\frac{\partial A_{x}}{\partial x}+\frac{\partial A_{y}}{\partial y}+\frac{\partial A_{z}}{\partial z}$ | $\frac{1}{r} \frac{\partial}{\partial r}\left(\mathrm{rA}_{\mathrm{r}}\right)+\frac{1}{\mathrm{r}} \frac{\partial \mathrm{A}_{\theta}}{\partial \theta}+\frac{\partial \mathrm{A}_{z}}{\partial z}$ | $\frac{1}{r^{2}} \frac{\partial}{\partial r}\left(r^{2} A_{r}\right)+\frac{1}{r} \sin \theta \frac{\partial}{\partial \theta}\left(A_{\theta} \sin \theta\right)+\frac{1}{r \sin \theta} \frac{\partial A_{\phi}}{\partial \phi}$ |
| ${ }^{8}$ | $\frac{\partial \phi}{\partial \mathrm{x}} \vec{i}_{\mathrm{x}}+\frac{\partial \phi}{\partial \mathrm{y}} \overrightarrow{\mathrm{i}}_{\mathrm{y}}+\frac{\partial \phi}{\partial z} \vec{i}_{z}$ | $\frac{\partial \Phi}{\partial r} \vec{i}_{r}+\frac{1}{r} \frac{\partial \Phi}{\partial \theta} \vec{i}_{\theta}+\frac{\partial \phi}{\partial z} \vec{i}_{z}$ | $\frac{\partial \phi}{\partial r} \vec{i}_{r}+\frac{1}{r} \frac{\partial \phi}{\partial \theta} \vec{i}_{\theta}+\frac{1}{r \sin \theta} \frac{\partial \phi}{\partial \phi} \vec{i}_{\phi}$ |
| $\left(\nabla^{2}\right.$ ¢ $)$ | $\frac{\partial^{2} \phi}{\partial x^{2}}+\frac{\partial^{2} \phi}{\partial y^{2}}+\frac{\partial^{2} \phi}{\partial z^{2}}$ | $\frac{1}{r} \frac{\partial}{\partial r}\left(\mathrm{r} \frac{\partial \Phi}{\partial r}\right)+\frac{1}{r^{2}} \frac{\partial^{2} \Phi}{\partial \theta^{2}}+\frac{\partial^{2} \Phi}{\partial z^{2}}$ | $\frac{1}{r^{2}} \frac{\partial}{\partial r}\left(r^{2} \frac{\partial \Phi}{\partial r}\right)+\frac{1}{r^{2} \sin \theta} \frac{\partial}{\partial \theta}\left(\sin \frac{\partial \phi}{\partial \theta}\right)+\frac{1}{r^{2} \sin ^{2} \theta} \frac{\partial^{2} \phi}{\partial \phi^{2}}$ |
| (8× ${ }_{\text {® }}$ ) | $\begin{aligned} & \left(\frac{\partial A_{z}}{\partial y}-\frac{\partial A_{y}}{\partial z}\right) \vec{i}_{x} \\ & +\left(\frac{\partial A_{x}}{\partial z}-\frac{\partial A_{z}}{\partial x}\right) I_{y} \\ & +\left(\frac{\partial A_{y}}{\partial X}-\frac{\partial A_{x}}{\partial y}\right) \vec{i}_{z} \end{aligned}$ | $\begin{aligned} & \left(\frac{1}{r} \frac{\partial A_{z}}{\partial \theta}-\frac{\partial A_{\theta}}{\partial z}\right){\stackrel{\rightharpoonup}{I_{r}}} \\ & +\left(\frac{\partial A_{r}}{\partial z}-\frac{\partial A_{z}}{\partial r}\right) \dot{I}_{\theta} \\ & +\left(\frac{1}{r} \frac{\partial}{\partial r}\left(r A_{\theta}\right)-\frac{1}{r} \frac{\partial A_{r}}{\partial \theta}\right) \stackrel{I}{z}^{z} \end{aligned}$ | $\begin{aligned} & \left(\frac{1}{r \sin \theta} \frac{\partial}{\partial \theta}\left(A_{\phi} \sin \theta\right)-\frac{1}{r} \sin \theta \frac{\partial A_{\theta}}{\partial \phi}\right) \\ + & \left(\frac{1}{\mathbf{i}_{r}}\right. \\ + & \left.\left(\frac{1}{r \sin \theta} \frac{\partial A_{r}}{\partial \phi}-\frac{1}{r} \frac{\partial}{\partial r}\left(r A_{\theta}\right)-\frac{1}{\partial r} \frac{\partial A_{r}}{\partial \theta}\right) \vec{i}_{\phi}\right) \end{aligned}$ |

## Appendix B

## Vector and Operator Identities

$$
\begin{align*}
& \vec{A} \times \vec{B} \cdot \vec{C}=\vec{A} \cdot \vec{B} \times \vec{C}  \tag{1}\\
& \vec{A} \times(\vec{B} \times \vec{C})=\vec{B}(\vec{A} \cdot \vec{C})-\vec{C}(\vec{A} \cdot \vec{B})  \tag{2}\\
& \nabla(\phi+\psi)=\nabla \phi+\nabla \psi  \tag{3}\\
& \nabla \cdot(\vec{A}+\vec{B})=\nabla \cdot \vec{A}+\nabla \cdot \vec{B}  \tag{4}\\
& \nabla \times(\vec{A}+\vec{B})=\nabla \times \vec{A}+\nabla \times \vec{B}  \tag{5}\\
& \nabla(\phi \psi)=\phi \nabla \psi+\psi \nabla \phi  \tag{6}\\
& \nabla \cdot(\psi \vec{A})=\vec{A} \cdot \nabla \psi+\psi \nabla \cdot \vec{A}  \tag{7}\\
& \nabla \cdot(\vec{A} \times \vec{B})=\vec{B} \cdot \nabla \times \vec{A}-\vec{A} \cdot \nabla \times \vec{B}  \tag{8}\\
& \nabla \cdot \nabla \phi=\nabla^{2} \phi  \tag{9}\\
& \nabla \cdot \nabla \times \vec{A}=0  \tag{10}\\
& \nabla \times \nabla \phi=0  \tag{11}\\
& \nabla \times(\nabla \times \vec{A})=\nabla(\nabla \cdot \vec{A})-\nabla^{2} \vec{A}  \tag{12}\\
& (\nabla \times \vec{A}) \times \vec{A}=(\vec{A} \cdot \nabla) \vec{A}-1 / 2 \nabla(\vec{A} \cdot \vec{A})  \tag{13}\\
& \nabla(\vec{A} \cdot \vec{B})=(\vec{A} \cdot \nabla) \vec{B}+(\vec{B} \cdot \nabla) \vec{A}+\vec{A} \times(\nabla \times \vec{B})+\vec{B} \times(\nabla \times \vec{A})  \tag{14}\\
& \nabla \times(\vec{A})=\nabla \phi \times \vec{A}+\phi \nabla \times \vec{A}  \tag{15}\\
& \nabla \times(\vec{A} \times \vec{B})=\vec{A}(\nabla \cdot \vec{B})-\vec{B}(\nabla \cdot \vec{A})+(\vec{B} \cdot \nabla) \vec{A}-(\vec{A} \cdot \nabla) \vec{B} \tag{16}
\end{align*}
$$

## Appendix C

[^0]Developed for educational purposes with the support of the National Science Foundation at the Education Development Center, films cited fall in one of two series.

Produced by the National Committee for Fluid Mechanics Films and distributed by Encyclopedia Britannica Educational Corp., 425 N. Michigan Ave., Chicago, Illinois (60611) are:
(1) Channel Flow of a Compressible Fluid
(2) Current-induced Instability of a Mercury Jet
(3) Eulerian and Lagrangian Descriptions in Fluid Mechanics
(4) Flow Instabilities
(5) Fundamentals of Boundary Layers
(6) Low-Reynolds Number Flows
(7) Magnetohydrodynamics
(8) Pressure Fields and Fluid Acceleration
(9) Surface Tension and Fluid Mechanics
(10) Waves in Fluids

Produced by the National Committee for Electrical Engineering Films and distributed by Education Development Center, 39 Chapel Street, Newton, Mass. 02160 are:
(11) Complex Waves I and Complex Waves II
(12) Electric Fields and Moving Media

## Index

Absolute from convective instability, distinguishing,
Absolute instability, 11.32
Absolute instability, boundary conditions and, 11.41
Accelerator, linear particle, 4.18
Accelerator operation, $\mathrm{d}-\mathrm{c}$ machine, 4.39
Acoustic dynamics, nonlinear, 11.16
Acoustic guides, 7.15
Acoustic impedance, numerical values of, 7.14
Acoustic surface waves, 7.48
Acoustic transmission lines, 7.15, 7.17
Acoustic velocity, ideal gas, 7.40
Acoustic velocity, numerical values of, 7.14
Acoustic wave spatial modes, 7.17
Acoustic wave transit time, 1.5
Acoustic wave velocity, 7.13
Acoustic waves, 7.13, 7.15
Acoustic waves in solids, 7.48
Aerosol, dynamics of stratified charged, 8.46
Aerosol, impact charging of, 8.50
Aerosol, space-charge static equilibrium of, 8.8
Agglomeration, self-, 5.27
Alfvén waves, 8.16
Alfvén waves, demonstration of, 8.20
Alfvén waves, effect of finite conductivity and viscosity on, 8.16
Ampère's law, differential form of, 2.2
Amplifying from evanescent modes, distinguishing, 11.46
Amplifying wave, $11.32,11.41,11.44$
Amplitude parameter expansion (see also linearization), 1.4
Anemometer, ion drag, 5.7
Anisotropic conductor, charge relaxation in deformable, 8.20
Antiduals, 8.12
Applications, 1.2
Atomization, 8.37
Average of periodic functions, 2.31
Averaging theorem, complex amplitude, 2.31
Averaging theorem, Fourier series, 2.31
Axisymmetric spherical creep flow, 7.33
Bénard instability, hydromagnetic, 10.15
Bernoulli's equation, 7.9
Bernoulli's equation, streamline form of, 7.9
Bessel functions, 2.36
Bessel functions modified, 2.36
Biharmonic equation, 7,32
Biological locomotion, 10.32
Bipolar migration, 5.26
Bipolar migration, laws for, 5.27
Blasius boundary layer, 9.18
Boltzmann constant, 5.3
Boundary condition, thin conducting permeable sheet, 6.35

Boundary condition, thin conductor, 6.4
Boundary condition, thin permeable sheet, 6.35
Boundary conditions, causality and, $11.35,11.40,11.45$
Boundary conditions, electromagnetic, 2.14, 2.19
Boundary conditions, fluid mechanics, 7.8
Boundary conditions, magnetoquasistatic, 2.18, 2.19
Boundary layer, Blasius, 9.18
Boundary layer, irrotational force density and viscous, 9.18
Boundary layer, linear viscous, 9.17
Boundary layer, magnetic, 6.22
Boundary layer, stress-constrained, 9.20
Boundary layer, viscous diffusion, 9.16
Boundary layer equations, 9.16
Boundary layer equations, integration of, 9.18
Boundary layer equations, streamfunction form of, 9.17
Brake, d-c electric ohmic, 5.43
Brake, Hartmann magnetic, 9.28
Brake, unipolar electric, 5.22
Brake operation, d-c machine, 4.39
Breakdown strength of gases, 4.55
Bulk electromechanical interactions, homogeneous, 8.16
Buoyancy, 10.12
Busch's theorem for electron beam, 11.2

Capacitance, 2.1, 2.20
Capillary instability of cylindrical jet, 8.53, 8.73
Capillary ripples, 7.4
Capillary rise, demonstration of, 7.6
Capillary static equilibrium, 7.10
Cauchy integral theorem, 5.66
Causality, characteristics and boundary conditions, 11.27
Causality and boundary conditions, two-stream systems, 11.35
Cellular convection, magnetic fieldinduced liquid metal, 9.23
Channel flow, compressible fluid, 9.41
Channel flow of a compressible fluid, film Reference 1, Appendix C, 9.45
Channel flows, 9.35
Characteristic equations, 11.13, 11.16, $11.21,11.23$
Characteristic equations, charge relaxation in terms of, 5.39
Characteristic equations, determinantal conditions, 11.14
Characteristic equations, first and second, 11.14
Characteristic equations, first order systems, 5.5, 5.7, 5.17, 5.26, 5.33, 5.38

Characteristic equations, method of undetermined coefficients, 11.14

Characteristic time, capillary, 9.1
Characteristic time, charge relaxation, 2.3, 8.23
Characteristic time, electroinertial, 8.49, 9.32
Characteristic time, electroviscous, 8.23, 9.8
Characteristic time, gravity, 9.1
Characteristic time, Hartmann flow establishment, 9.31
Characteristic time, magnetic diffusion, 6.3, 8.11, 9.25
Characteristic time, magnetoinertial, 8.17, 9.25
Characteristic time, migration, 10.19
Characteristic time, molecular diffusion, 10.3, 10.19
Characteristic time, reciprocal cyclotron frequency, 11.4
Characteristic time, reciprocal plasma frequency, $11.10,11.23$
Characteristic time, thermal diffusion, $10.2,10.13$
Characteristic time, thermal relaxation, 10.13
Characteristic time, viscous diffusion, 7.27, 7.32, 7.42, 8.17, 8.23, 9.25

Characteristic time, viscous relaxation, 7.42
Characteristic times, 1.4, 1.5
Characteristic times, ambipolar diffusion and, 10.4
Characteristics, boundary condition and, 5.6
Charge, 2.1
Charge, conservation of, 2.2, 5.2
Charge, lumped parameter variable of, 2.20
Charge conservation boundary conditions, 5.44, 5.45, 5.50
Charge conserving continua, 8.8, 8.57
Charge conserving continua, transfer relations for, 8.46
Charge conserving interfacial dynamics, 8.54
Charge convection, 5.1
Charge convection, instability and, 8.49
Charge density, free, 2.1
Charge density, magnetic, 2.13
Charge density, polarization, 2.12
Charge diffusion, heterogeneous ohmic, 5.56
Charge generation, 5.2
Charge migration, 5.1
Charge monolayer driven convection, 9.7
Charge monolayer induced cellular convection, 9.24
Charge recombination, 5.2
Charge relaxation, 5.1
Charge relaxation, deforming ohmic conductor, 5.38
Charge relaxation, heterogeneous system of uniform conductors, 5.56
Charge relaxation temporal modes, 5.54
Charge relaxation time, 1.5, 2.3, 5.34
Charge relaxation time, imposed field flow approximation, 9.32

Charge relaxation time, particle, 5.76
Charge relaxation transmission line, 4.51
Charged aerosol equilibrium, stability
of, 8.9
Charged drop dynamics, 8.44
Charged particle beam, kinematics of, 4.17
Charged particle migration, 5.5
Charged particles in vacuum, 11.1
Charging diagram, negative ion impact, 5.13
Charging diagram, positive ion impact, 5.12
Charging of macroscopic particle, ion diffusion, 10.19
Choking, compressible channel flow, 9.44
Chu formulation, 2.1
Classification, energy converter, 4.3
Coefficient of heat transfer, surface, 10.11
Coenergy, electroquasistatic, 2.24
Coenergy, magnetoquasistatic, 2.28
Coenergy density, electroquasistatic, 2.24
Coenergy density, magnetoquasistatic, 2.28
Coenergy function, electrocapillary, 10.29
Collision frequency, 3.2
Combustor, energy conversion cycle, 9.53
Complex amplitudes, definition of, 2.29
Complex amplitudes, polyphase currents
represented by, 4.21
Complex waves, 11.37
Complex waves, second order, 11.37
Complex waves I, film Reference 11, Appendix C, 11.42
Complex waves II, film Reference 11, Appendix C, 8.32, 11.43, 11.44
Compressibility constitutive law, weak, 7.13
Compressible channel flow, conservative transition in, 9.44
Compressible channel flow, electrogasdynamic, 9.48, 9.62
Compressible flow, channel, 9.41
Compressible flow, free surface flow analogous to, 9.43, 9.60
Compressible fiow, velocity-area diagram, 9.44, 9.45
Compressible fluid, weakly, 7.13
Compressible quasistatic, 7.42
Compressible quasistatic limit (CQS), 7.16
Compressional waves, 8.25
Compressional waves in solids, 7.48
Compressional waves, magnetization, 8.27
Conduction, electrical, 3.2
Conduction machines, 4.33
Conductivity, evolution of bipolar, 5.33
Conductivity, numerical values of electrical, 6.3
Conductivity, surface electrical, 5.44, 5.45, 6.4

Conductivity, temperature dependent
electrical, $10.3,10.8$
Confinement, plasma, 8.40
Conservation of energy, internal and kinetic energy combined, 7.39
Conservation of energy, surface, 7.5
Conservation of free charge, boundary condition for, 2.17, 2.19
Conservation of internal energy, highly compressible fluid, 7.38

Conservation of internal energy, lumped parameter, 7.37
Conservation of kinetic energy, fluid, 7.25
Conservation of mass, fluid, 7.1
Conservation of momentum, fluid, 7.2
Conservative transition, compressible flow, 9.44
Conservative transitions, piecewise homogeneous flow, 9.37
Constitutive laws, polarization, 3.9
Continua, current conserving, 8.70, 8.71
Convection, natural, 10.10
Convection, rotor model for therma1, 10.10
Convective derivative, 2.6, 2.7
Convective derivative, fluid acceleration and, 7.2
Convective from absolute instabilities, distinguishing, 11.54
Convective instability, 11.32
Convective magnetic diffusion, 6.2
Convective magnetic diffusion, skineffect approximation to, 6.21
Coordinates, Eulerian, 2.6
Coordinates, Lagrangian, 2.6
Corona discharge, electrostatic precipitator, 5.9
Couette flow, generalized, 9.6
Couette mixer, electrohydrodynamic, 8.24
CQS (see compressible quasistatic)
Creep flow, 7.27
Creep flow, cellular electromechanical, 9.22
Critical angle, particle charging, 5.13
Critical charge (also saturation charge), 5.10
Critical depth of gravity flows, 9.40
Critical flow, free surface, 9.39
Critical lines (or points), charge trajectory, 5.8, 5.17
Current, 2.1
Current, electroquasistatic lumped parameter, 2.20
Current, Hall, 3.3
Current-charge relation, lumped parameter, 2.20
Current conserving continua, 8.70, 8.71
Current density, free, 2.1
Current-induced instability of a mercury jet, film Reference 2, Appendix C, 8.51
Current tube, 2.26
Curvature, radius of, 7.5
Cycilotron frequency, 11.4
Cylindrical shell, boundary condition for rotating conducting, 6.5
Cylindrical she11, boundary condition for translating conducting, 6.5

D-c interactions, conditions for, 4.5
D-c interactions, magnetohydrodynamic, 9.45
D-c interactions, ohmic electric, 5.42
D-c machine, energy conservation in, 4.39

D-c machine, magnetic, 4.33
D-c machine, model for, 4.33
D-c machine, unipolar electric, 5.22
D-c machine, winding for, 4.35
D-c machines, classification of, 4.2
D-c pump, unipolar electric, 5.22
Debye length, 10.22
Diffuser, compressible flow, 9.44
Diffusion, ambipolar, 10.3
Diffusion, charged particle, 5.2
Diffusion, magnetic traveling wave, 6.18
Diffusion, migration relative to, 5.3
Diffusion, transient magnetic, 6.25
Diffusion, unipolar ion, 10.19
Diffusion charging of macroscopic particle, 10.20
Diffusion coefficient, mobility related to, 5.3
Diffusion coefficient, numerical values of molecular, 10.4
Diffusion equation, magnetic, 6.2
Diffusion time, numerical values of magnetic, 6.4
Diffusion time, viscous, 7.27
Diffusion wave, phase of, 6.18
Dimensionless numbers, 1.5
Dipole, force on, 3.7
Direction cosine, 3.16
Dissipation, calculation of electrical, 6.20
Dissipation, relation of magnetic stress to, 6.22
Dissipation density, electrical, 9.39
Dissipation density, electrical to thermal, 10.5
Dissipation density, fluid, 7.25
Dissociation, formation of charge by, 5.27
Dorn effect, 10.27
Double layer, 2.15, 10.21
Double layer, boundary conditions for, 2.16, 2.19
Double layer, ideally polarized, 10.11, 10.32

Double layer charge conservation, ideally polarized, 10.33
Double layer incremental capacitance, 10.30
Double layer surface force density, 3.20 10.31

Drag, rigid sphere viscous, 7.36
Drag, Stokes's, 7.36
Drag-cup tachometer, 6.11
Drop charging, ion impact, 5.10
Drop formation, 8.53
Drop formation, electric field and, 8.73
Dynamical processes, characteristic times and, 1.4
Dynamics in space and time, 11.13
EGD (see electrogasdynamic)
EHD (see electrohydrodynamic)
Eigenfrequencies, temporal mode Alfvén wave, 8.20
Eigenfrequencies, integral condition on, 8.66
Einstein relation, 5.3
Elastic modulus, numerical values of, 7.46
Elastic properties, numerical values of, 7.46

Electric displacement, definition of, 2.1, 2.2
Electric field intensity, 2.1
Electric field intensity, breakdown, 4.55
Electric field intensity, limits on, 4.53
Electric fields and moving media, film Reference 12, Appendix C, 5.53, 8.5, 9.9

Electric Hartmann number, 1.5, 5.76, 9.9

Electric Prandtl number, 5.76
Electric Reynolds number, 1.5, 9.8
Electric Reynolds number, unipolar, 5.20, 5.23
Electrical energy subsystem, 8.60
Electrical dissipation, magnetoquasistatic ohmic, 6.19
Electrical dissipation density, 7.39
Electrification, thunderstorm, 5.10
Electro-acoustic compressional waves, 8.25
Electro-acoustic velocity, 8.26
Electrocapillarity, 10.27
Electrocapillary mobility, 10.40
Electrocapillary motions of drop, 10.32
Electrodynamic laws, differential, 2.1
Electrogasdynamic compressible energy converter, 9.48
Electrohydrodynamic imposed field approximation, 9.32
Electro-inertial time, 1.5, 8.23, 8.49

Electrokinetics, $10.23,10.25$
Electromagnetic flight, 6.23
Electromagnetic wave transit time, 1.5, 2.3
Electromagnetic waves, quasistatics and, 1.1
Electromagnetics, branches of, 1.1
Electromechanical coupling, thermal or molecular subsystem and, 10.1
Electromechanical energy conversion configurations, 4.3
Electromechanical kinematics, 1.1, 2.1, 4.1

Electron beam, equation of motion for, 11.1
Electron beam, magnetic confinement of, 4.17
Electron beam, resistive wall amplification on, 11.68
Electron beam, transfer relations for, 11.10
Electron beam, traveling-wave interaction with, 11.78
Electron beam devices, 4.17
Electron beam dynamics, non1inear, 11.23
Electron beam energy converters, 4.56
Electro-osmosis, 10.23, 10.24
Electrophoresis, 10.25

Electroquasistatic jump conditions, 2.14
Electroquasistatics, 2.3
Electrostatic precipitator, 5.9
Electroviscous time, 1.5, 5.76, 8.23, 9.8
Energy, conservation of, 2.22
Energy, electroquasistatic, 2.22
Energy, gas internal, 7.36
Energy, magnetoquasistatic, 2.28
Energy and coenergy, relation between, 2.24
Energy conservation, electrocapillary surface, 10.29
Energy conservation, electroquasistatic subsystem, 2.25
Energy conservation, lumped parameter electroquasistatic, 2.24
Energy conservation, magnetoquasistatic, 2.26
Energy conservation, magnetoquasistatic subsystem, 2.28
Energy conversion, d-c machine, 4.39
Energy conversion 1imitations, 4.53
Energy conversion limitations, electrogasdynamic, 9.53
Energy conversion limitations, magnetohydrodynamic, 9.48
Energy conversion processes, 1.3
Energy converters, classification of, 1.4
Energy density, electroquasistatic, 2.24
Energy density, magnetoquasistatic, 2.28
Energy equation, electron beam, 11.2
Energy function, permanent polarization, 4.12
Energy state equation, fluid, 7.37
Enthalpy, electrical power generated and change in, $9.48,9.55$
Enthalpy, specific, 7.38
Entropy, 7.37
Entropy, MHD generator increase in, 9.48
Entropy, specific, 7.38
Entropy flow, 7.40
EQS (see electroquasistatic)
Equation of state, mechanical fluid, 7.37
Equations of elasticity, 7.46, 7.47
Equations of motion for inviscid fluid, 7.2
Equivalent circuit, charge relaxation diffusion line, 5.56
Equivalent circuit, magnetic induction machine, 6.10
Error function, solution to diffusion equation, 6.24
Eulerian and Lagrangian coordinates, 2.48
Eulerian and Lagrangian descriptions in fluid mechanics, film Reference 3, Appendix C, 2.7
Eulerian coordinates, 2.6
Evanescent from amplifying modes, distinguishing, 11.46
Evanescent wave, 11.37, 11.41, 11.42
Exchange modes, MHD, 8.40
Exchange of stabilities, smoothly inhomogeneous temporal modes, 8.66
Expansion, space-rate, 4.41

Faraday's integral law, ohmic deformable media and, 6.2
Faraday's law, Chu formulation, 2.14
Faraday's law, differential form of, 2.1
Ferrofluid, 8.35
Ferrofluid, demonstration illustrating, 8.7
Ferrofluid interfacial instability, 8.37
Flow, low magnetic Reynolds number, $9.2,9.10,9.14,9.17,9.25$
Flow development, Hartmann magnetic, 9.28
Flow development, temporal, 9.13
F1ow instabilities, film Reference 4, Appendix C, 10.16
Flows, electromechanical, 9.1
Flows, fully developed, 9.5, 9.6, $9.7,9.11,9.26,9.28,9.33$
Flows, imposed surface and volume force density, 9.5
Fluid interface, Eulerian description of, 7.3
Fluid mechanics, laws of, 7.1
Fluid mechanics boundary and jump conditions, 7.8
Fluid power flow, 7.25
Flux, 1umped parameter variable of, 2.21
Flux conservation, surface of fixed identity and, 6.2
Flux conservation in conducting fluid, 8.16
F1ux conserving continua, compressible, 8.25
Flux conserving continua, homogeneous bulk, 8.16
Flux conserving continua, static equilibrium of, 8.11
Flux conserving continua, z-theta pinch, 8.40
Flux-potential transfer relations, 2.16
Force, 2.1
Force, Lorentz, 3.1
Force, time-average, 5.60
Force, time-average in Induction machine, $6.8,6.16$
Force coenergy relations, electroquasistatic, 3.5
Force coenergy relations, magnetoquasistatic, 3.6
Force densities, electromagnetic, 3.1
Force densities, tenuous dipole, 3.6
Force density, 2.1
Force density, electroquasistatic density dependent, 3.12
Force density, electroquasistatic free-charge, 3.4
Force density, flows with irrotational, 9.2
Force density, gradient of "pressure" as, 7.9
Force density, gravitational, 7.9

Force density, incompressible electrically linear electroquasistatic, 3.12, 3.18

Force density, incompressible electrically linear magnetoquasistatic, $3.15,3.18$
Force density, incompressible electroquasistatic, $3.11,3.18$
Force density, Kelvin magnetization, 3.8, 3.18
Force density, Kelvin polarization, 3.7, 3.18
Force density, Korteweg-Helmholtz electroquasistatic, 3.9, 3.11, 3.18
Force density, Korteweg-He1mholtz, magnetoquasistatic, 3.13, 3.18
Force density, Lorentz, 3.1
Force density, macroscopic vs. microscopic, 3.1
Force density, magnetic irrotational, 9.2
Force density, magnetoquasistatic density dependent, $3.15,3.18$
Force density, magnetoquasistatic free current, 3.4
Force density, quasistatic, 3.4
Force density, rotational, 9.3
Force density, viscous, 7.24
Force-energy relations, electroquasistatic, 3.5
Force-energy relations, magnetoquasistatic, 3.6
Force equation, fluid, 7.2
Force equation, isotropic perfectly elastic solid, 7.47
Force equation for viscous fluid, 7.24
Forces, electromagnetic, 3.1
Forces, 1umped parameter, 3.4
Fourier amplitudes, definition of, 2.30
Fourier amplitudes, synchronous machine application of, 4.9
Fourier averaging theorem, synchronous machine application of, 4.10
Fourier series, 2.30
Fourier series complex amplitudes, time average of, 5.60
Fourier transform complex amplitudes, time average of, 5.60
Fourier transforms, definition of, 2.30
Frozen charge model, 8.57
Fully developed flow, mass conservation in, 9.7
Fully developed flows, surface coupled, 9.7
Fundamentals of boundary layers, film Reference 5, Appendix C, 9.19
Fusion experiments, 8.40
Galilean transformation, 2.7
Gas constant, 7.37
Gas constant, specific heats and, 7.38
Gasdynamic energy converters, 9.41
Gasdynamic flows, 9.41
Gauss' integral theorem, 2.9
Gauss' law, differential form of, 2.2
Gauss' law, multiple charge species and, 5.3

Gauss' theorem, tensor form, 3.15
Generalized Leibnitz rule, 2.10
Generating systems, open cycle, 9.53
Generation, bipolar, 5.33
Generation, charge, 5.2
Generator, d-c electric ohmic, 5.43
Generator, Hartmann magnetic, 9.28
Generator, inviscid magnetohydrodynamic, 9.4
Generator, magnetohydrodynamic gas-dynamic, 9.46
Generator, unipolar electric, 5.22
Generator, variable capacitance, 4.45
Generator operation, d-c machine, 4.39
Glass thickness control, 8.2, 8.10
Gradient field stabilization, 8.38
Gradient integral theorem, 2.9
Gravitational force density, 7.9
Gravitational subsystem, 8.60
Gravity-capillary dynamics, 8.28
Gravity-capillary modes of charge monolayer, 8.56
Gravity-capillary spatial modes, 8.31
Gravity-capillary wave, phase velocity, 8.30
Gravity-capillary waves, dispersion equation for, 8.29
Gravity flow, 9.35
Green's function field representations, 4.40

Hall current, 3.3
Hankel functions, 2.36
Hartmann channel flow, 9.26
Hartmann flow, electrohydrodynamic, 9.33
Hartmann layer, 9.29, 9.59
Hartmann number, Alfvén waves and magnetic, 8.18
Hartmann number, electric, 1.5, 9.9, 9.34

Hartmann number, ideally polarized double layer, 10.35
Hartmann number, magnetic, 1.5, 9.27
Hartmann profile, magnetic, 9.27
Hartmann type flows, magnetic, 9.25
Hartmann velocity profile, electric, 9.34
Head diagram, free surface f1ow, 9.39
Heat conduction and convection, imposed dissipation, 10.5
Heat transfer, 10.1
Helmholtz equation, 4.16
Hyperbolic systems, streaming, 11.27
Hysteresis loop, 6.30
Hysteresis motor, 6.30
Ideal gas equations of state, 7.37
Impact charging of macroscopic particles, 5.9
Impedance, acoustic, 7.18
Imposed vs. self-fields, 8.38
Imposed gradient surface interactions, 8.38
Incompressibility, fluid, 7.1
Incompressible elastic solid, 7.48
Incompressible inertialess solid, 7.49

Incompressible quasistatic, 7.42
Independent variables, electroquasistatic, 3.5
Inductance, 2.1, 2.21
Inductance matrix, 2.21, 4.21, 4.32
Induction heating, thermal response to magnetic, 10.5
Induction interactions, magnetic, 6.1
Induction machine, balanced twophase, 6.8
Induction machine, end effect in magnetic Iinear, 6.36, 6.37
Induction machine, single-phase magnetic, 6.10
Induction motor, deep conductor, 6.15
Induction motor, electroquasistatic, 5.46
Induction motor, magnetic, 6.6
Induction pump, liquid metal magnetic, 9.11
Intertial quasistatic laws, 7.42
Inertial reference frame, 2.7
Inhomogeneity, mass density, 7.1
Initial value problem, method of characteristics and, 11.16
Initial value problem, single stream, 11.30
Ink jet printing, 8.44
Instability, absolute, 11.32, 11.37, 11.41
Instability, bulk electrohydrodynamic, 8.24
Instability, convective (also, amplifying wave), $11.32,11.37,11.41,11.46,11.54$
Instability, critical conditions for, 8.36
Instability, electrohydrodynamic equipotential surface, 8.37
Instability, heavy fluid on top of light, 8.30
Instability, incipience of, 8.36
Instability, internal, 8.62
Instability, Kelvin-Helmholtz type, 11.56
Instability, nonlinear stages of surface, $8.31,8.32,8.37$
Instability, Rayleigh-P1ateau, 8.53
Instability, self-field interfacial, 8.33
Instability, two-stream, 11.34
Instability, $z$-pinch at low magnetic Reynolds number, 8.54
Instability of glycerin interface stressed by electric field, 8.37
Integral law, mass conservation, 7.1
Integral law, momentum conservation, 7.2
Integral laws, electroquasistatic, 2.10
Integral laws, magnetoquasistatic, 2.10
Integral theorem, Gauss', 2.9
Integral theorem, generalized Leibnitz, 2.9
Integral theorem, gradient, 2.9
Integral theorem, Stokes's, 2.9, 3.26
Integral theorem, Stokes's type, 3.25
Interface, fluid, 7.3
Interface, fluid velocity at an, 7.4
Interfacial coupling, charge monolayer, 8.54
Internal energy, specific, 7.37
Internal energy differential law 7.39

Internal energy integral law, 7.39
Internal energy of gas, 7.36
Internal energy storage, weakly compressible fluid, 7.26
Internal instabilities, 8.62
Internal waves, 8.62
Inviscid fluid, equations of motion for, 7.2
Inviscid irrotational flow, 9.2
Ion diffusion time, 5.34
Ion drag anemometer, 5.7
Ion drag brake, 5.22
Ion drag generator, 5.22
Ion drag pump, 5.22, 9.33
Ion mobility in gases, 5.4
Ion mobility in liquids, 5.4
Ionization, liquid, 5.27
IQS (see inertial or incompressible quasistatic)
Irrotational flow, fluid, 7.9
Irrotational force densities, homogeneous flows and, 9.2
Isentropic flow through nozzles and diffusers, 9.42
Isotropy relations, stress-strainrate, 7.23

Jump conditions, electromagnetic, 2.14, 2.19
Jump conditions, fluid mechanic, 7.8
Jump conditions, magnetoquasistatic, $2.18,2.19$

Kelvin and Korteweg-Helmholtz force densities compared, 8.4
Kelvin force density, static equilibrium in terms of the, 8.4
Kelvin-Helmholtz type instability, 11.56
Kelvin magnetization force density interaction between dipoles, 3.15
Kelvin polarization force density, 3.7, 3.12, 3.18
Kelvin polarization force density, interaction between dipoles and, $3.12,3.18$
Kelvin theorem, Busch's theorem and, 11.2
Kinematics, electromechanical, 1.1, 2.1, 4.1

Kinematics, mechanical, 2.1
Kinetic energy storage, fluid, 7.25
Kinetic energy subsystem, 8.60
Klystron, 11.24
Korteweg-Helmholtz and Kelvin force densities compared, 8.4
Korteweg-Helmholtz electroquasistatic force density, 3.9, 3.11, 3.18

Korteweg-Helmholtz magnetoquasistatic force density, $3.13,3.18$
Kronecker delta, 3.17

Lagrangian and Eulerian coordinates, 2.48
Lagrangian coordinates, 2.6
Lagrangian coordinates, electron motion in, 11.1
Langevin recombination coefficient, 5.26
Laplace-Fourier transform representation, 11.47
Laplace's equation, irrotational flow and, 7.10
Laplace's equation, numerical solution of, 8.14
Laval nozzle, 9.44
Legendre functions, associated, 2.40
Legendre polynomials, $2.40,7.34$
Leibnitz rule, generalized line integral, 2.49
Leibnitz rule, generalized surface integral, 2.49
Lens, electric electron, 11.6, 11.8
Lens, magnetic electron, 11.6, 11.8
Levitation, magnetic, 6.24, 8.2
Levitation force, relation of dissipation to, 6.22
Linearized model, limitations of acoustic wave, 11.20
Linearized models, 4.42
Lippmann equation, electrocapillary, 10.30
Liquid metal, motions in uniform magnetic field, 8.16
Liquid metal, static equilibrium of of, 8.11
Liquid metal levitation, 8.2
Liquid metal $z$-pinch, 8.51
Longitudinal coordinate, 1.6, 4.53, 9.35

Long-wave free surface models, 9.35
Long-wave model, 4.42
Long-wave model, field coupled membrane, 11.37
Long-wave models, 1.4
Lorentz force, 3.1
Lorentz force density, 3.1
Low Reynolds number cellular convection, 9.22
Low Reynolds number flow, 7.27
Low Reynolds number flow, velocitystress functions in, 7.35
Low Reynolds number flow in spherical coordinates, 7.33
Low Reynolds number flows, film Reference 6, Appendix C, 7.32
Lumped parameter forces, 3.4
Lumped parameters, electroquasistatic, 2.19
Lumped parameters, magnetoquasistatic, 2.20

Mach number, 1.5, 9.1
Mach number, channel flow, 9.43
Mach number, streaming membrane, 11.29
Magneplane, 6.24
Magnetic diffusion, 6.1
Magnetic diffusion, boundary layer and, 6.22

Magnetic diffusion, conducting sheet and, 6.4, 6.6
Magnetic diffusion, deep conductor, 6.15
Magnetic diffusion, instantaneous, 8.50
Magnetic diffusion, laws of, 6.1
Magnetic diffusion, temporal modes of, 6.28
Magnetic diffusion, thick conductor translation and rotation, 6.12
Magnetic diffusion, vector potential and, 6.13
Magnetic diffusion equation, normalized, 6.3
Magnetic diffusion time, 1.5, 2.3, 6.3, 9.25

Magnetic diffusion time, Alfvén waves and the, 8.17
Magnetic diffusion transfer relations, 6.12
Magnetic drag, boundary layer and, 6.23
Magnetic field intensity, 2.1
Magnetic flight, 6.24
Magnetic flux, 2.1
Magnetic flux density, 2.1
Magnetic flux density, definition of, 2.1, 2.2
Magnetic flux density, limits on, 4.53
Magnetic Hartmann number, 1.5
Magnetic lift, boundary layer and, 6.23
Magnetic propulsion, rail, 6.24
Magnetic Reynolds number, 1.5, 6.3, 9.12

Magnetic Reynolds number, free surface flow with low, 9.38
Magnetic Reynolds number, MHD flow and, 9.48
Magnetic saturation, magnetization force density with, 8.6
Magnetic-viscous Prandt1 number, 1.5
Magnetization continua, compressible, 8.27
Magnetization continua, instability of bulk, 8.27
Magnetization density, 2.1, 2.13
Magnetization dilatational waves, 8.27
Magnetization force density, static equilibrium with, 8.6
Magnetization hysteresis motor, 6.30
Iagnetization of moving media, 2.13
Magnetization surface instability, 8.33
Magnetization surface interaction dispersion equation, 8.35
Magneto-acoustic velocity, 8.26
Magneto-acoustic waves, Iinear, 8.25
Magneto-acoustic waves, nonlinear, 11.21
Magnetohydrodynamic compressible flow, 9.45, 9.61

Magnetohydrodynamic energy conversion, 9.41, 9.45
Magnetohydrodynamic flow development, 9.28
Magnetohydrodynamic induction pump, 9.11
Magnetohydrodynamics, film Reference 7, Appendix C, 8.20, 9.3
Magneto-inertial time, Alfvén wave and the, 8.18
Magnetoquasistatic jump conditions, 2.18, 2.19

Magnetoquasistatics, 2.3
Magneto-viscous time, 1.5
Magnetron flow, 11.3
Mass conservation, fluid, 7.1
Mass conservation, free surface quasi-one-dimensional, 9.36
Mass conservation, incompressible, 7.2
Mass conservation jump condition, 7.8
Mass density, numerical values of, 7.14
Mass density, numerical values of fluid, 7.19
Mass density, solids, 7.46
Mass density, surface, 10.13
Maxwell's capacitor, bipolar model for, 5.35
Maxwell's equations, Chu formulation of, 2.1
Mechanical kinematics, 2.1
Mercury drop electrocapillary migration, 10.27
Mercury-electrolyte double layer, 10.28
Method of characteristics, 11.13
Method of characteristics, first order systems, 5.5, 5.7, 5.17, 5.26, 5.33, 5.38

Method of characteristics, higher order systems, 11.21
MHD (see magnetohydrodynamic)
Microwave generator, magnetron, 11.3, 11.5
Migration, bipolar, 5.26
Migration, diffusion relative to, 5.3
Migration, electrochemically induced, 10.32
Migration, imposed field and flow, 5.5, 5.7, 5.9

Migration time, 5.34
Migration time, imposed field flow approximation, 9.32
Migration time, particle, 1.5
Migration with convection, 5.6
Migration with convection, quasistationary, 5.7
Mixing, electrically induced, 8.24
Mobility, diffusion coefficient relative to, 5.3
Mobility, electrocapillary, 10.40
Mobility, macroscopic particle, 5.3, 5.4
Mobility in gases, ion, 5.4
Mobility in liquids, ion, 5.4
Mode1s, charge conserving, 8.1
Models, electromechanical, 8.1
Models, flux conserving, 8.1, 8.2
Models, instantaneous charge relaxation, 8.1, 8.2
Models, instantaneous magnetic diffusion, 8.1
Molecular diffusion, 10.1, 10.19

Molecular diffusion, neutral particle, 10.2
Molecular diffusion time, 1.5, 10.3, 10.19

Molecular Peclet number, 1.5
Molecular-viscous Prandtl number, 1.5
Momentum conservation, fluid, 7.2
Momentum conservation jump condition, 7.9
Motor operation, d-c machine, 4.39
Moving media, Ohm's law and, 6.1
MQS (see magnetoquasistatic)
Navier-Stokes equation, 7.24
Normal vector, surface deformation related to, 7.7
Normal vector, surface geometry and, 7.3
Normalization, convention for equations, 2.3
Normalization, convention for symbols, 2.2
Nozzle, compressible flow, 9.44
Numerical integration by method of characteristics, 5.30, 5.36, $11.30,11.32$
Numerical solution, superposition integral approach to, 8.14

Ohmic conduction transfer relations, 5.44
Ohmic conduction with convection, 5.42
Ohmiç conductor, constitutive law for moving, 5.38
Ohmic conductor, dynamics of, 5.38
Ohmic limit, bipolar, 5.33
Ohmic model, hierarchy of characteristic times for, 5.35
Ohm's law, moving conductor, 3.3
Ohm's law, moving media and, 6.1
Orthogonal modes, representation of source distributions, 4.16
Orthogonality, Helmholtz equation and, 4.16
Orthogonality, magnetic temporal mode, 6.29
Orthogenality, principal axes, 7.22
Overview, text, 2.1
Overview of energy conversion processes, 4.53

Paint spraying, electrostatic, 8.44
Paraxial ray equation for electron beam, 11.6
Particle charging, impact, 5.9
Particle charging, ion diffusion, 10.19
Particular solution, Poisson's equation, 4.13, 4.14
Particular solution, vector Poisson's equation, 4.26
Peclet number, molecular, 1.5, 10.3
Peclet number, thermal, 1.5, $10.2,10.9$
Periodic systems, stress, force and torque in, 4.1

Permanent polarization motor, 6.30
Permeability, free space, 2.1
Permittivity, free space, 2.1
Phase-plane, electron beam pictured in, 11.27
Phase velocity, 2.30
Piecewise homogeneous systems, 8.28
Pinch, instantaneous magnetic diffusion, 8.50
Pinch, low magnetic Reynolds number, 8.50
Pinch, MHD z-theta, 8.40
Pinch, sheet, 8.72
Planar sheet conductor, boundary condition for translating conducting, 6.5
P1asma, cold, 11.10
Plasma column, stability of, 840
Plasma frequency, 11.10, 11.23
Plasma stability, z-theta pinch, 8.40
Poiseuille flow, generalized, 9.6
Poisson's equation, Green's function for, 4.40
Poisson's equation, particular solutions of, 4.13, 4.14
Poisson's equation, scalar, 4.13
Poisson's equation, transfer relations for vector, 4.26
Poisson's equation, vector, 2.45
Poisson's ratio, numerical values of, 7.46
Polarization, moving media, 2.11
Polarization charge, conservation of, 2.12
Polarization charge density, 2.12
Polarization current density, 2.12, 2.13
Polarization density, 2.1, 2.12
Polarization force density, illustration of, 8.5
Polarization stabilization of RayleighTaylor instability, 8.31
Polarization surface instability, 8.33
Polarization surface interaction dispersion equation, 8.35
Pollution control, 8.44
Potential, velocity, 7.10
Potential conserving continua, charged drop, 8.44
Potential conserving continua, compressible, 8.25
Potential conserving continua, homogeneous anisotropic bulk, 8.20
Potential conserving continua, static equilibrium of, 8.11
Potential well, magnetron, 11.4
Power conversion, electromagnetic-tointernal, 7.39
Power dissipation, vector potential magnetic field intensity evaluation of, 6.20
Power flow, electroquasistatic, 2.24
Power flow, magnetoquasistatic, 2.28
Power flow density, magnetoquasistatic, 2.29
Power flux density, electroquasistatic, 2.25
Prandtl number, 10.13
Prandtl number, magnetic-viscous, 1.5
Prandtl number, molecular-viscous, 1.5
Prandtl number, numerical values of molecular, 10.4

Prandt1 number, thermal magnetic, 10.13
Prandtl number, thermal viscous, 1.5
Precipitator, electrostatic, 5.9
Precipitator, space-charge, 5.20
Pressure, force density and, 7.3
Pressure, irrotational flow and, 7.10
Pressure, stress tensor and, 7.3
Pressure fields and fluid acceleration, filn Reference 8, Appendix C, 7.10
Pressure in inviscid fluid, 7.3
Pressure-velocity transfer rela-
tions for inviscid fluid, 7.11
Principal axes of tensor, 7.22
Principal coordinate relations, stress-strain-rate, 7.23
Principal mode, acoustic waveguide, 7.18
Principle of virtual power, 3.21
Pump, d-c electric ohmic, 5.43
Pump, Hartmann electric, 9.33
Pump, Hartmann magnetic, 9.28
Pump, inviscid magnetohydrodynamic, 9.4
Pump, ion drag, 9.33
Pump, traveling-wave surface MHD and EHD, 9.10
Pumpimg, electroquasistatic backward, 5.51
Pumping, electroquasistatic thermally induced, 10.8

Quasi-one-dimensional model, boundary layer as a, 9.16
Quasi-one-dimensional model, compressible flow, 9.41
Quasi-one-dimensional model, electrogasdynamic generator, 9.48, 9.62

Quasi-one-dimensional model, electrokinetic, 10.23
Quasi-one-dimensional model, free surface, $9.35,9.37,9.60$
Quasi-one-dimensional models, 4.41
Quasi-one-dimensional models, streaming, 11.28 , 11.32
Quasistatic, compressible, 7.42
Quasistatic integral laws, 2.10
Quasistatic laws, electromagnetic, 2.2
Quasistatic laws, electromagnetic differential, 2.5
Quasistatic limit, fluid compressible (CQS), 7.16
Quasistatics, electromagnetic waves and, $2.3,2.47$
Quasistatics, fluid mechanics, 7.41
Quasistatics, inertial, 7.42
Quasistatics, instantaneous charge relaxation, 4.51
Quasistatics, time-rate expansion and, 2.2

Radii of curvature, double layer surface force density and, 3.20
Radii of curvature, interfacial, 7.5
Rayleigh number, $10.13,10.17$
Rayleigh number, magnetic, 10.13, 10.17
Rayleigh-Plateau instability, 8.53

Rayleigh-Taylor instability, 8.30
Raylaigh-Taylor instability, polarization stabilization of, 8.38
Rayleigh-Taylor instability in smooth1y inhomogeneous systems, 8.57
Rayleigh waves, 7.48
Rayleigh's limit of charge on a drop, 8.44
Reciprocity, rotating machine model and, 4.12
Reciprocity and energy conservation in smooothly inhomogeneous systems, 8.60
Reciprocity conditions, inductance matrix and the, 4.26
Reciprocity relations, lumped parameter electroquasistatic, 3.5
Reciprocity relations, lumped parameter magnetoquasistatic, 3.6
Recombination, bipolar, 5.26, 5.33
Recombination, charge, 5.2
Red Sea, Moses' parting of, 8.1
Reentrant flows, turn-on transient, 9.13
Reflection coefficient, acoustic, 7.18
Residue theorem, Cauchy, 5.66
Resistive wall electron beam amplification, 11.68
Reyno1ds number, 1.5, 7.27
Reynolds number, boundary layer and, 9.16
Rotating incompressible inviscid fluid, 7.45
Rotational flow, magnetohydrodynamic, 9.3
Rotor model, MHD thermal convection, $10.10,10.37$
Rotor model, natural convection, 10.10, 10.37

Rotor model, single-phase induction machine, 6.10, 6.37
Rotor model, two-phase induction machine, 6.8
Rotor model, Von Quincke's, 5.49, 5.75
Salient pole machines, 4.3, 4.5
Salient pole machines, force from stress tensor in, 4.6
Salt in solvent, 5.27
Saturation charge (see also critical charge), 5.10
Scrubbers, charged drop, 8.44
Seal, magnetic fluid, 8.2
Sedimentation potential, 10.25
Self-precipitation, 5.17
Shear modulus, numerical values of, 7.46
Shear stress, electric bulk instability and, 8.20
Shear stress, propagation of magnetic, 8.16
Shear waves in solids, 7.48
Sheets, boundary conditions for thin conducting, 6.4
Shell, heat balance in rotating, 10.11
Shells, boundary conditions for thin conducting, 6.4
Shock, compressible gas-dynamic, 9.45
Shock formation, 11.18
Similarity parameter, 6.24
Similarity solution, boundary layer, 9.18, 9.20
Similarity solution, linear diffusion, 6.24

Simple waves, method of characteristics and, 11.18
Skin depth, magnetic, 6.3
Skin depth, molecular, 10.3
Skin depth, moving frame of reference, 6.16
Skin depth, numerical values of magnetic, 6.4
Skin depth, therma1, 10.2, 10.3
Skin depth, viscous, 7.28, 9.16
Skin effect, 2.48
Skin effect, magnetic levitation and, 8.3
Skin effect, moving conductor, 6.16
Skin effect, transfer relations for magnetic, 6.21
Skin-effect induced cellular convection, 9.22
Skin-effect model, magnetic, 6.20
Skin-effect model, stress in magnetic, 6.20, 6.25
Smoothly inhomogeneous systems, 8.57
Solenoidal fields, representation of, 2.42
Space-average theorem, force and torque from, 4.2, 4.4, 4.5, 4.7, 4.10, 4.19, 4.24, 4.30, 4.31, 4.36, 4.47

Space-charge dynamics, particles in vacuum, 11.10
Space-charge dynamics, smoothly inhomogeneous fluid, 5.17, 8.59
Space-charge dynamics, unipolar migration, 5.17
Space-rate expansion, 1.4, 4.41
Space-rate expansion, boundary layer and, 9.16
Space-rate expansion, free surface flow, 9.60
Spatial modes, 5.61
Spatial modes, acoustic wave, 7.12
Spatial modes, electron beam and cold plasma, 11.11
Spatial modes, Fourier transform and, 5.66
Spatial modes, gravity-capillary, 8.31
Spatial modes, internal charge conserving, 8.66
Spatial modes, internal mass conserving, 8.66
Spatial modes, moving charged thin sheet, 5.62
Spatial modes, numerical solution for, 5.65
Spatial transients, sinusoidal steady state, 5.61
Specific entropy, ideal gas, 7.40
Specific heat, constant pressure, 7.38
Specific heat, constant volume, 7.37
Specific heat, numerical values, 10.2
Specific heats, ratio of, 7.40
Specific volume, 7.40
Specific volume of fluid, 7.37
Spherical shell, boundary condition for rotating conducting, 6.5
Stability, synchronous machine, 4.3
State equation, isentropic, 7.40

State space integration, energy
function, 3.5, 4.12
Static equilibria, capillary, 7.10
Static equilibria, charge conserving, 8.8
Static equilibria, conditions for, 8.1
Static equilibria, constant potential, 8.2
Static equilibria, examples of, 8.2, 8.4, $8.6,8.8,8.10,8.11$
Static equilibria, flux conserving, 8.2.
Static equilibria, force density conditions for, 8.2
Static equilibria, magnetization, 8.2, 8.4, 8.6

Static equilibria, numerical solution of, 8.14
Static equilibria, polarization, 8.2, 8.4, 8.31

Static equilibria, stability of, 8.9
Static equilibria, surface force density conditions for, 8.3
Static equilibria, uniform current, 8.8
Static equilibria, viscous fluid perturbations from, 7.27
Stokes's drag, 7.36
Stokes's integral theorem, 2.9
Strain-displacement relations, 7.47
Strain rate, normal, 7.20
Strain rate, shear, 7.20
Strain rate, viscous stress and 7.18
Strain-rate principal axes same as for stress, 7.22
Strain rate proved a tensor, 7.32
Strain-rate tensor, 7.20
Strain-stress relations, 7.47
Stratified media, smoothly, 8.57
Stream functions, 2.42
Stream functions, convective migration, 5.6
Stream functions, polar, 2.43, 5.6
Stream functions, spherical, $2.43,5.6$
Streaming interactions, 11.1
Streaming potential, 10.25
Streaming systems, single-stream prototype models, 11.28
Streaming systems, two-stream prototype models, 11.32
Streamlines unaltered by irrotational force density, 9.5
Stress-energy conversion relations, 4.53
Stress-strain-rate relation for isotropic fluid, 7.24
Stress-strain-rate relations, physical motivation for, 7.19
Stress-strain relations, 7.47
Stress-strain relations, general linear, 7.21, 7.41
Stress tensor, components defined, 3.16
Stress tensor, divergence of, 3.15
Stress tensor, force density related to, 3.15
Stress tensor, force found from, 4.1
Stress tensor, force in salient pole machine found from, 4.48
Stress tensor, physical significance of, 3.16
Stress tensor, torque found from, 4.1
Stress tensors, 3.15

Stress tensors, electromagnetic, 3.1
Stress tensors, electromechanical, 3.17
Stress tensors, summary of electromagnetic, 3.18
Subcritical free surface flow, 9.39
Subsystems, electrical, kinetic and gravitational, 8.60
Superconducting machine, power output of, 4.54
Supercritical free surface flow, 9.39
Supercritical waves, 11.40
Superposition integral field solution, 4.40
Surface acoustic waves, 7.48
Surface charge density, free, 2.1, 2.15
Surface charge density, polarization, 2.15
Surface coupling, shearstress, 8.54
Surface current density, 2.1
Surface dilatational modes of charge monolayer, 8.56
Surface double layer density, 2.16
Surface energy conservation, 7.5
Surface force density, 3.19
Surface force density, double layer, 10.28
Surface force density, interfacial curvature and, 7.5
Surface force density, surface tension, 7.4
Surface force density, Young and Laplace, 7.5
Surface heat transfer coefficient, 10.11
Surface shaping, magnetic, 8.11
Surface tension, 7.4
Surface tension, clean interface and, 7.4
Surface tension, energy constitutive law for, 7.4
Surface tension, nonlinear static equilibrium with, 8.13
Surface tension, numerical values of, 7.4
Surface tension in fluid mechanics, film Reference 9, Appendix C, 7.6
Surface tension surface force density, deformation related to, 7.6, 7.7
Surface tension, voltage dependence of $\mathrm{Hg}-\mathrm{KNO}_{3}$ interface, 10.30
Surface torque density, 3.17
Synchronous alternator, power output of 4.54
Synchronous interactions, conditions for, 4.4
Synchronous machine, classification of, 4.2
Synchronous machine, exposed winding, 4.28
Synchronous machine, hysteresis, 6.30
Synchronous machine, model for, 4.28
Synchronous machine, model for smooth air-gap, 4.21

Synchronous machine, permanent magnet, 4.3
Synchronous machine, permanent polarization, 4.8
Synchronous machine, salient pole, 4.3
Synchronous machine, superconducting, 4.28
Synchronous machine, variable capacitance, $4.42,4.44$

Tachometer, drag-cup, 6.11
Tachometer, electroquasistatic, 5.45
Tachometer, magnetic induction, 6.6, 6.36

Taylor pump, 9.9
Taylor wavelength, 8.30
Temporal flow development, imposed surface and volume forces and, 9.13
Temporal mode orthogonality, 6.29
Temporal modes, charge relaxation, 5.54
Temporal modes, conducting fluid in uniform field, 8.19
Temporal modes, constant potential continua, 8.23
Temporal modes, eigenvalues of, 6.28
Temporal modes, electron beam and cold plasma, 11.11
Temporal modes, field-gradient coupled interfacial, 8.39
Temporal modes, gravitycapillary, 8.30
Temporal modes, hydromagnetic Bénard, 10.18
Temporal modes, internal charge conserving, 8.66
Temporal modes, magnetic diffusion thick conductor, 6.27
Temporal modes, magnetic diffusion thin sheet, 6.26
Temporal transient, Hartmann flow established by, 9.30
Temporal transient, stress constrained flow, 9.14
Temporal transient, velocity constrained flow, 9.14
Tensor, strain-rate, 7.20
Tensor, transformation of, 3.17
Tensor integral theorem of Gauss, 3.15
Terminal relations, electric d-c machine, 4.52
Terminal relations, electrical rotating machine, $4.11,4.21,4.31$
Terminal relations, Van de Graaff machine, 4.52
Thermal conductivity, definition of, 10.1
Thermal conductivity, numerical values of, 10.2
Thermal convection in a magnetic field, 10.10, 10.15
Thermal diffusion, $10.1,10.5$
Thermal diffusion time, 1.5, 10.2
Thermal-electromechanical energy conversion, 9.53
Thermal energy conversion cycle, 9.53, 9.63

Thermal diffusivity, definition of, 10.1
Thermal energy conversion efficiency, 9.55
Thermal expansion, coefficient of, 10.15
Thermal expansion, numerical values of coefficient of, 10.16
Thermal generation time, 5.34
Thermal Peclet number, 1.5
Thermal-viscous Prandtl number, 1.5
Thermal voltage, 5.3, 10.22
Thermodynamics, electroquasistatic, 2.22
Thermodynamics, equilibrium, 7.38
Thermodynamics, lumped parameter, of highly compressible fluid, 7.36
Thermodynamics, magnetoquasistatic subsystem, 2.26
Thermonuclear experiments, 8.40
Theta pinch, 8.43
Three-phase machine, 4.21
Thin sheet, charge relaxation on, 5.45, 5.55
Thin-sheet limit, magnetic diffusion in the, 6.17
Thunderstorm electrification, 5.10
Time-average force, 5.60
Time-average force, spatial transient, 5.67
Time-average torque, 5.60
Time-rate expansion, 1.4, 2.2
Time-rate parameter, 2.4
Time-rate parameter, acoustic, 7.42
Torque, surface torque density and, 3.17
Torque, time-average, 5.60
Torque-speed characteristic, hysteresis motor, 6.33
Torque-speed characteristic, induction motor, $6.9,6.10,6.16$
Traction, 3.16
Transfer relations, 1.6, 2.46
Transfer relations, anisotropic ohmic conductor, 5.74
Transfer relations, cold plasma, 11.10
Transfer relations, conducting fluid in magnetic field, 8.18
Transfer relations, constrained charge, 4.13
Transfer relations, constrained current, 4.26
Transfer relations, electromagnetic planar, 2.52
Transfer relations, electron beam, 11.10
Transfer relations, electroquasistatic, 2.16
Transfer relations, electroquasistatic inhomogeneous dielectric planar, 2.53
Transfer relations, fluxpotential, 2.16
Transfer relations, fluxpotential cylindrical, 2.35

Transfer relations, flux-potential planar layer, 2.33
Transfer relations, flux-potential spherical, 2.38
Transfer relations, half-space of viscous fluid, 7.31
Transfer relations, implications of energy conservation for quasistatic, 2.40
Transfer relations, imposed force density fluid, 7.49
Transfer relations, incompressible elastic solid, 7.48
Transfer relations, incompressible inertialess solid, 7.49
Transfer relations, infinite halfspace elastic material, 7.48
Transfer relations, inviscid fluid pressure-velocity, 7.11, 7.12
Transfer relations, Laplacian fields, 2.32
Transfer relations, low magnetic Reynolds number, 8.52
Transfer relations, low Reynolds number flow, 7.32, 7.33, 7.36
Transfer relations, magnetic diffusion, 6.12
Transfer relations, magnetoquasistatic, 2.16
Transfer relations, method of denoting variables, 2.46
Transfer relations, ohmic conduction, 5.44
Transfer relations, rotating incompressible inviscid fluid, 7.44
Transfer relations, smoothly inhomogeneous system, 8.57
Transfer relations, thermal diffusion, 10.5
Transfer relations, thermal diffusion with source, 10.6
Transfer relations, uniformly charged fluid layer, 8.46
Transfer relations, vector potential Laplacian field, 2.42
Transfer relations, viscous diffusion, 7.28
Transfer relations, viscous layer of arbitrary thickness, 7.30
Transfer relations, volume source, 4.13
Transfer relations, weak compressibility, 7.13
Transformation, Galilean, 2.7
Transformations, electroquasistatic, 2.9
Transformations, magnetoquasistatic, 2.9
Transformations between frames of reference, 2.7
Transverse coordinate, $1.6,4.53,9.35$
Traveling space-charge wave, kinematics of, 4.17
Traveling-wave amplifier, 4.18
Traveling-wave induced convection, 5.51, 5.53
Traveling-wave-induced convection, surface, 9.10
Turn-on transient, electron beam in gap, 11.26

Turn-on transient, reentrant flow, 9.13
Two-phase surface currents, 6.6

Unipolar space-charge dynamics, 5.17
Units, electromagnetic, 2.1

Van de Graaff generator, 4.49
Van de Graaff machine, d-c machines and 4.53
Van de Graaff machine, energy conversion in, 4.53
Variable capacitance machine, 4.42, 4.44
Variable capacitance machine, output power of, 4.55
Vector, transformation of, 3.16
Vector potential, 2.42
Vector potential, magnetic diffusion and, 6.12, 6.13
Vector potential, velocity, 7.26
Velocity potential, 7.10
Virtual power, 3.21
Virtual work, 3.21
Viscometer, 7.18
Viscosity, absolute, 7.19
Viscosity, kinematic, 7.19
Viscosity, numerical values of, 7.19
Viscosity, unit conversion for, 7.19
Viscous diffusion, 7.26
Viscous diffusion, Alfvén waves and, 8.18
Viscous diffusion, boundary layer, 9.16
Viscous diffusion time, 1.5, $7.27,7.42,9.25,9.32$
Viscous diffusion time, Alfvén waves and, 8.17
Viscous diffusion transfer relations, 7.28
Viscous dissipation density, 7.25
Viscous force density, 7.24
Viscous relaxation time, 1.5, 7.42

Viscous skin depth, 7.28
Viscous skin depth, numerical values of, 7.29
Viscous stress, strain rate and, 7.18
Voltage, 2.1
Voltage, lumped parameter variable of, 2.20
Voltage-flux relation, lumped parameter, 2.22
Von Quincke's rotor, 5.49
Von Quincke's rotor, equations of motion for, 5.75
Vorticity, convective diffusion of, 7.26
Vorticity, fluid, 7.9
Vorticity, generation of, 7.26
Vorticity, surface of fixed identity conservation of, 7.10

Wall-less pipes, 9.35, 9.38
Wavelength, Taylor, 8.30
Waves, acoustic, 7.13
Waves, Alfvén, 9.8
Waves, amplifying, 11.31, 11.37, $11.41,11.42$
Waves, capillary-gravity surface, 8.28
Waves, charge conserving, 8.46
Waves, charge monolayer surface, 8.54, 8.75
Waves, complex, 11.37
Waves, current conserving, 8.71
Waves, elastic isotropic solid, 7.48
Waves, electro-acoustic, 8.25
Waves, evanescent, 11.31, 11.37, 11.41, 11.42

Waves, field-coupled surface, 8.33
Waves, imposed gradient polarization surface, 8.38
Waves, internal, 8.62
Waves, internal charge and mass conserving, 8.62
Waves, internal magnetization, 8.77
Waves, magnetic diffusion, 6.17
Waves, magneto-acoustic, 8.25, 11.21
Waves, magnetization dilatational, 8.27
Waves, prototype single-stream, 11.27
Waves, Rayleigh surface, 7.48
Waves, shock, 9.45, 11.19
Waves, space-charge gravity, 8.62
Waves, supercritical, 11.40
Waves, surface Alfvén, $8.40,8.72$
Waves, thermal diffusion, 10.5
Waves, viscous diffusion, 7.26, 8.16, 9.13

Waves in fluids, film Reference 10, Appendix C, 11.16
Weak-gradient imposed field model, 8.59, 8.64
Wetting, surface tension and liquid-solid, 7.6
Whipple and Chalmers model for particle charging, 5.9
Windings, two-phase, 6.6
Young and Laplace surface force density, 7.5

Zero-gravity liquid orientation, 8.2
Zeta potential, $10.22,10.24$
z pinch, MHD, 8.42
z-theta pinch, feedback stabilization of, 8.44


[^0]:    Films

