PROBLEMS

5.1. Two parallel, perfectly-conducting plates are constrained as shown in Figure 5P.1 in such a way that the bottom plate is fixed and the top one is free to move only in the x-direction. A field is applied between the plates by the voltage source $v(t)$. When $x = 0$, the spring is in its equilibrium position.

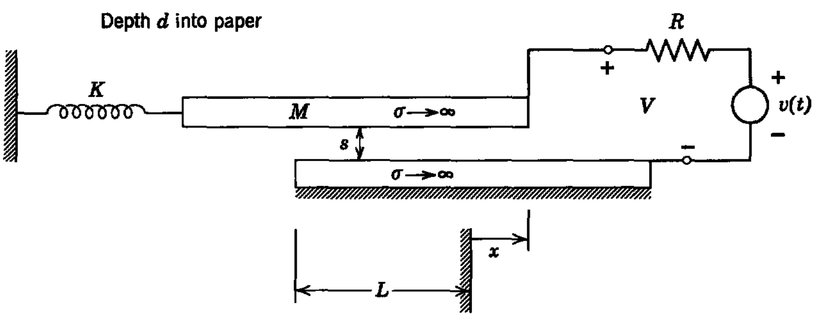


Fig. 5P.1

(a) What is the force of electric origin exerted on the upper plate?
(b) Write the complete equations of motion for this system.
(c) If $R = 0$ and $v(t) = (V_o \sin \omega t)u_{-1}(t)$, where $u_{-1}(t)$ is the unit step function, what is $x(t)$? Assume that the system is in static equilibrium when $t < 0$.

5.2. The system illustrated in Fig. 5P.2 is a schematic model of a differential transformer, which is a device for measuring small changes in mechanical position electrically. The movable core is constrained by bearings (not shown) to move in the x-direction. The two excitation windings, each having N_1 turns, are connected in series with relative polarity such that, when the movable core is centered as shown, there is no coupling between the excitation circuit and the signal winding. When the core moves from the center position in either direction, a voltage is induced in the signal winding. In the analysis neglect fringing fields and assume that the magnetic material is infinitely permeable.

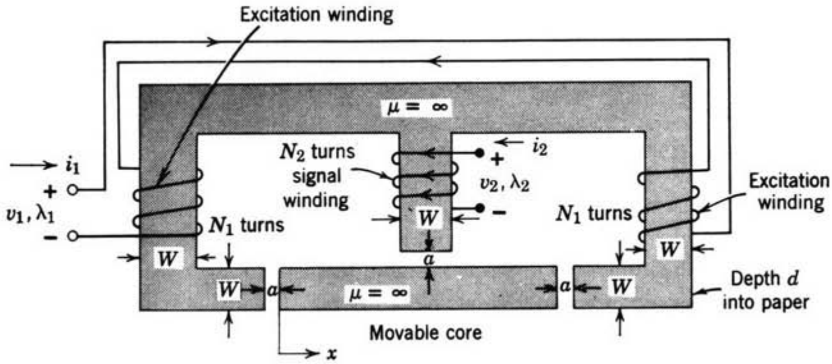


Fig. 5P.2

(a) Calculate the lumped-parameter equations of state, $\lambda_1(i_1, i_2, x)$, $\lambda_2(i_1, i_2, x)$, for this system.
(b) Terminal pair 1 is constrained by a current source $i_1 = I_o \cos \omega t$ and the system operates in the steady state. The open-circuit voltage v_2 is measured. Calculate the amplitude and phase angle of v_2 as a function of displacement x for the range $-a < x < a$.

5.3. A pair of highly conducting plates is mounted on insulating sheets, as shown in Fig. 5P.3*a*. The bottom sheet is immobile and hinged to the top sheet along the axis A. The top sheet is therefore free to rotate through an angle ψ. A torsion spring tends to make $\psi = \psi_o$ so that there is a spring torque in the $+\psi$-direction $T^s = K(\psi_o - \psi)$. A source of charge $Q(t)$ is shunted by a conductance G and connected by flexible leads between the conducting plates. We wish to describe mathematically the motion of the upper plate [e.g., find $\psi(t)$]. To do this complete the following steps:

(a) Find the static electric field $\mathbf{E}$ between the two flat, perfectly conducting plates shown in Fig. 5P.3*b*. Assume that each of the plates extends to infinity in the r- and z-directions.
(b) If the angle ψ is small [$\psi a \ll D$, $\psi a \ll (b - a)$] so that fringing fields are not important, the electric field of part (a) can be used to approximate $\mathbf{E}$ between the metal plates of Fig. 5P.3*a*. Under this assumption find the charge q on the upper metal plate. Your answer should be in the form of $q = q(V, \psi)$ and is the electrical terminal relation for the block diagram of Fig. 5P.3*c*.

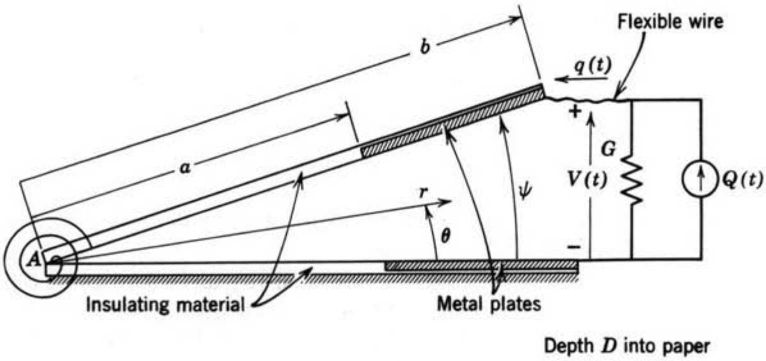


Fig. 5P.3a

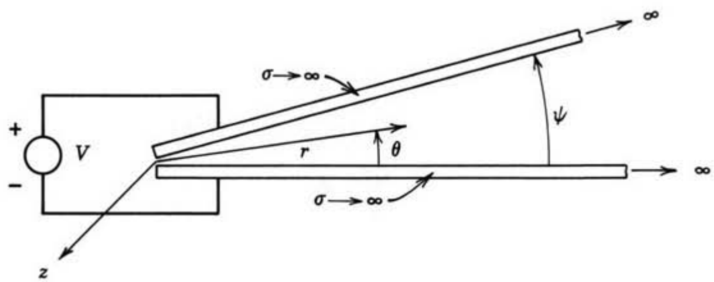


Fig. 5P.3b

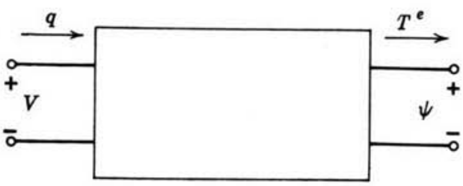


Fig. 5P.3c

(c) Use part (b) to find the electrical energy stored between the plates $W = W(q, \psi)$ and, using W, find the torque of electrical origin T^e exerted by the field on the movable plate.

(d) Write two differential equations that, with initial conditions, define the motion of the top plate. These equations should be written in terms of the two dependent variables $\psi(t)$ and $q(t)$. The driving function $Q(t)$ is known and the movable plate has a moment of inertia J.

(e) Use the equations of part (d) to find the sinusoidal steady-state deflection $\psi(t)$ if $G = 0$ and $Q(t) = Q_o \cos \omega t$. You may wish to define ψ as $\psi = \psi_1 + \psi'(t)$, where ψ_1 is a part of the deflection which is independent of time. Identify the steady-state frequency at which the plate vibrates and give a physical reason why this answer would be expected.

5.4. This is a continuation of Problem 3.4, in which the equations of motion for the system shown in Fig. 3P.4 were developed.

(a) The resistance R is made large enough to be ignorable and the current $I(t) = I_o$, where I_o is a constant. Write the equation of motion for $x(t)$.

(b) Use a force diagram (as in Example 5.1.1) to determine the position $x = x_o$, where the mass can be in static equilibrium, and show whether this equilibrium is stable.

(c) With the mass M initially in static equilibrium, $x(0) = x_o$, it is given an initial velocity v_o. Find $x(t)$ for $x \approx x_o$.

5.5. Two small spheres are attached to an insulating rod, and a third sphere is free to slide between them. Each of the outside spheres has a charge Q_1, whereas the inside sphere has a charge Q_o and a mass M. Hence the equation of motion for the inside sphere is

$$M\frac{d^2x}{dt^2} = \frac{Q_oQ_1}{4\pi\epsilon(d+x)^2} - \frac{Q_1Q_o}{4\pi\epsilon(d-x)^2}.$$

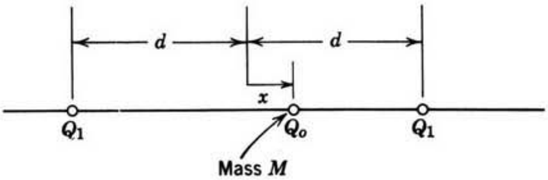


Fig. 5P.5

(a) For what values of Q_o and Q_1 will the movable sphere have a stable static equilibrium at $x = 0$? Show your reasoning.

(b) Under the conditions of (a), what will be the response of the sphere to an initial small static deflection $x = x_o$? $x_o \ll d$. (When $t = 0$, $x = x_o$, $dx/dt = 0$.)

5.6. Figure 5P.6 shows a sphere of magnetic material in the magnetic field of a coil. The coenergy of the coil is

$$W'(i, x) = \frac{L_o}{2}\left[1 - \left(\frac{x}{b}\right)^4\right]i^2,$$

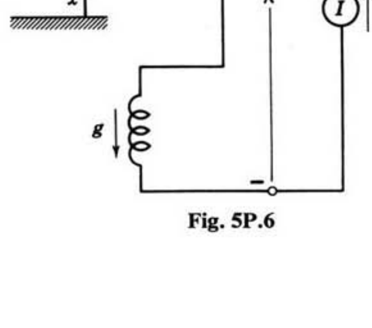


Fig. 5P.6

where L_o is positive. The sphere has a mass M and is subject to a gravitational force Mg (as shown).

(a) Write the differential equation that determines $x(t)$.
(b) Find the equilibrium position(s).
(c) Show whether this (these) equilibrium position(s) is (are) stable.

5.7. In Problem 3.15 we developed the equation of motion for a magnetic wedge, and it was shown that, with a constant current $i = I_o$ applied, the wedge could be in static equilibrium at $\theta = 0$. Under what conditions is this equilibrium stable?

5.8. The system shown in Fig. 5P.8 is one third of a system (governing vertical motion only) for suspending an airfoil or other test vehicle in a wind tunnel without mechanical support. The mass M_o, which represents the airfoil, contains magnetizable material and is constrained by means not shown to move in the vertical direction only. The system is designed so that the main supporting field is generated by current i_1 and the stabilizing field is generated by current i_2. Over the range of positions (x) of interest, the electrical terminal relations may be expressed as:

$$\lambda_1(i_1, i_2, x) = \frac{L_1 i_1}{(1 + x/a)^3} + \frac{M i_2}{(1 + x/a)^3},$$

$$\lambda_2(i_1, i_2, x) = \frac{M i_1}{(1 + x/a)^3} + \frac{L_2 i_2}{(1 + x/a)^3},$$

where a, L_1, L_2, and M are positive constants and $M^2 < L_1 L_2$.

(a) Find the force of electric origin $f^e(i_1, i_2, x)$ acting on mass M_o.
(b) Set $i_1 = I$, a constant current, and set $i_2 = 0$. Find the equilibrium position X_o where f^e is just sufficient to balance the gravitational force on the mass M_o.
(c) With the currents as specified in part (b), write the linear incremental differential equation that describes the motion of mass M_o for small excursions $x'(t)$ from the equilibrium X_o. If an external force $f(t) = I_o u_o(t)$ (an impulse) is applied to the mass in the positive x-direction with the mass initially at rest, find the response $x'(t)$.
(d) For stabilization of the equilibrium at X_o a feedback system, which uses a light source, photoelectric sensor, and amplifiers, supplies a current i_2 such that $i_2(t) = \alpha x'(t)$, where α is a real constant. Keeping $i_1 = I$, write the equation of motion for $x'(t)$. For what range of α is the impulse response $x'(t)$ bounded?

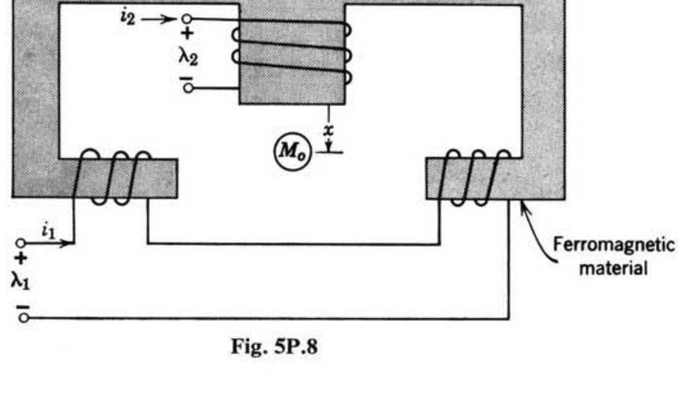


Fig. 5P.8

(e) To make the impulse response tend to zero as $t \to \infty$, the signal from the photoelectric sensors is operated on electronically to produce a current i_2 such that

$$i_2 = \alpha x'(t) + \beta \frac{dx'}{dt},$$

where α and β are real constants. Again, write the equation of motion for $x'(t)$. For what ranges of α and β does the impulse response $x'(t)$ tend to zero as $t \to \infty$?

5.9. A conservative magnetic field transducer for which variables are defined in Fig. 5P.9 has the electrical equation of state $\lambda = Ax^3 i$, for $x > 0$, where A is a positive constant. The system is loaded at its mechanical terminals by a spring, whose spring constant is K

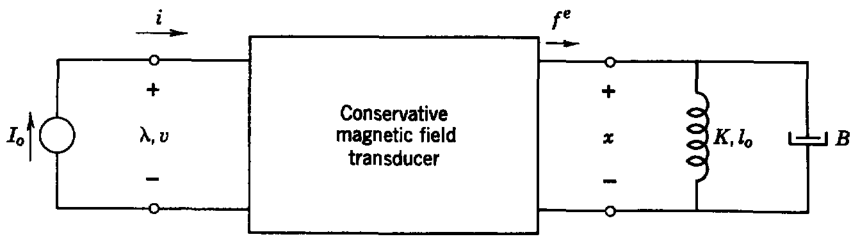


Fig. 5P.9

and whose force is zero when $x = l_0$, and a mechanical damper with the constant B. The electrical terminals are excited by a direct-current source I_0 with the value

$$I_0 = \left(\frac{K}{8Al_0}\right)^{1/2}.$$

(a) Write the mechanical equation in terms of f^e.
(b) Find f^e in terms of data given above.
(c) Find by *algebraic* techniques the possible equilibrium positions for the system and show whether each equilibrium point is stable.
(d) Check the results of part (c) by using *graphical* techniques to investigate the stability of the equilibrium points.

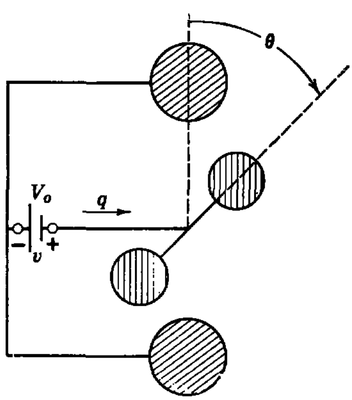


Fig. 5P.10

5.10. An electric field system has a single electrical terminal pair and one mechanical degree of freedom θ (Fig. 5P.10). The electrical terminal variables are related by $q = C_o(1 + \cos 2\theta)v$, where θ is the angular position of a shaft. The only torques acting on this shaft are of electrical origin. The voltage $v = -V_o$, where V_o is a constant.

(a) At what angles θ can the shaft be in static equilibrium?
(b) Which of these cases represents a stable equilibrium? Show your reasoning.

5.11. Figure 5P.11 shows a diagrammatic cross section of a two-phase, salient-pole synchronous machine. The windings in an actual machine are distributed in many slots

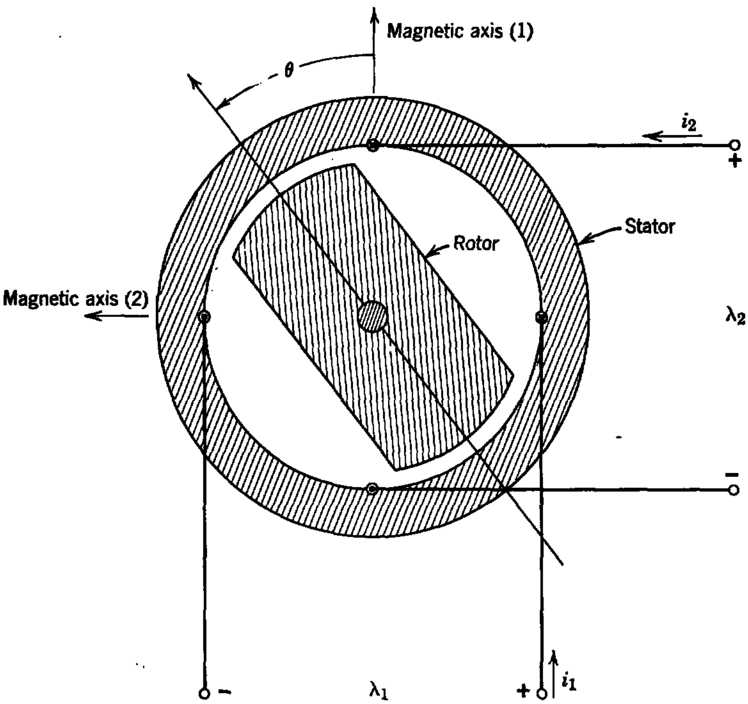


Fig. 5P.11

along the periphery of the stator, rather than as shown. The rotor is made of magnetically soft iron which has no residual permanent magnetism. The electrical terminal relations are given by

$$\lambda_1 = (L_o + M\cos 2\theta)i_1 + M\sin 2\theta\, i_2,$$

$$\lambda_2 = M\sin 2\theta\, i_1 + (L_o - M\cos 2\theta)i_2.$$

(a) Determine the torque of electrical origin $T^e(i_1, i_2, \theta)$.
(b) Assume that the machine is excited by sources such that $i_1 = I\cos\omega_s t$, $i_2 = I\sin\omega_s t$, and the rotor has the constant angular velocity ω_m such that $\theta = \omega_m t + \gamma$. Evaluate the instantaneous torque T^e. Under what conditions is it constant?

(c) The rotor is subjected to a mechanical torque (acting on it in the $+\theta$-direction): $T = T_o + T'(t)$, where T_o is a constant. The time-varying part of the torque perturbs the steady rotation of (b) so that $\theta = \omega_m t + \gamma_o + \gamma'(t)$. Assume that the rotor has a moment of inertia J but that there is no damping. Find the possible equilibrium angles γ_o between the rotor and the stator field. Then write a differential equation for $\gamma'(t)$, with $T'(t)$ as a driving function.

(d) Consider small perturbations of the rotation $\gamma'(t)$, so that the equation of motion found in (c) can be linearized. Find the response to an impulse of torque $T'(t) = I_o u_o(t)$, assuming that before the impulse in torque the rotation velocity is constant.

(e) Which of the equilibrium phase angles γ_o found in (c) is stable?

5.12. An electromechanical model for a magnetic transducer is shown in Fig. 5P.12. A force $f(t)$ is to be transduced into a signal $v_o(t)$ which appears across the resistance R. The

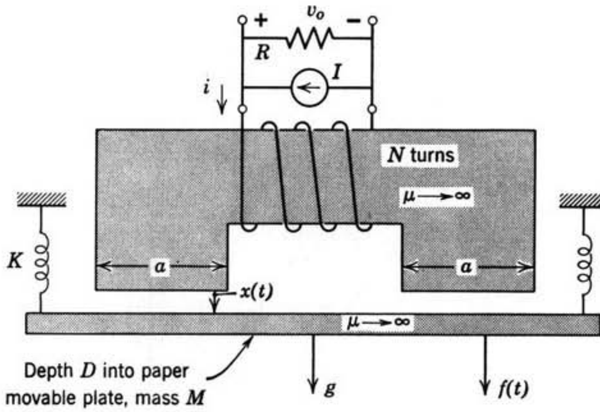


Fig. 5P.12

system is designed to provide linear operation about an equilibrium where the coil is excited by a constant current I. The plate is constrained at each end by springs that exert no force when $x = 0$.

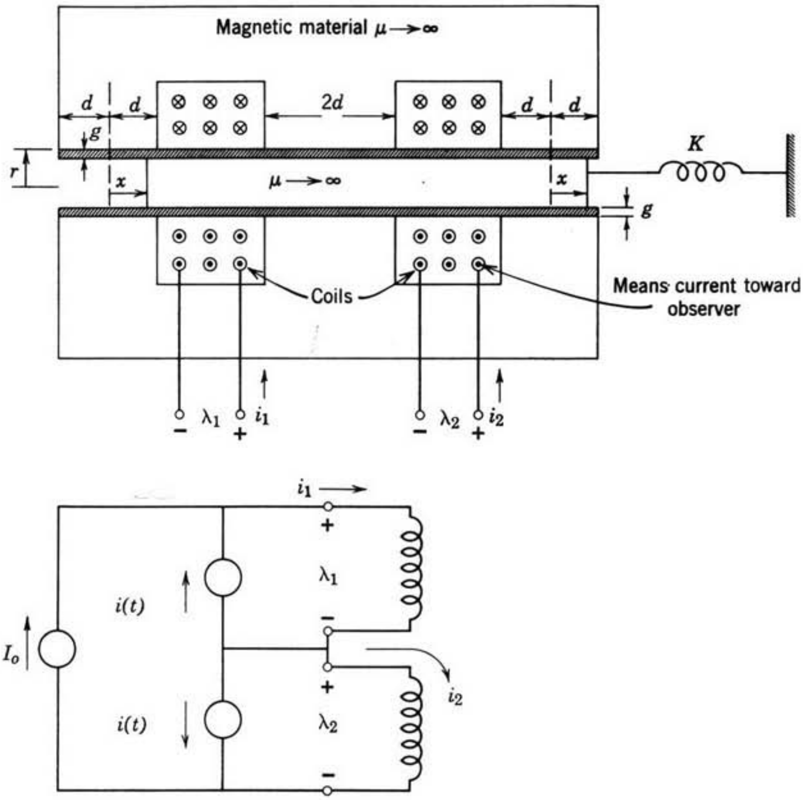
(a) Find the force of electrical origin $f^e(x, i)$ on the plate in the x-direction.

(b) Write the equations of motion for the system. These should be two equations in the dependent variables (i, x).

(c) The static equilibrium is established with $f(t) = 0$, $x = X$, and $i = I$. Write the equilibrium force equation that determines X. Use a graphical sketch to indicate the equilibrium position X at which the system is stable. Assume in the following that the system is perturbed from this stable static equilibrium.

(d) The resistance R is made large enough so that the voltage drop across the resistance is much larger than that across the self-inductance of the coil. Use this fact as the basis for an approximation in the electrical equation of motion. Assume also that perturbations from the equilibrium conditions of (c) are small enough to justify linearization of the equations. Given that $f(t) = \text{Re}\,[\hat{f}e^{j\omega t}]$, $v_o(t) = \text{Re}\,[\hat{v}_o e^{j\omega t}]$, solve for the frequency response $\hat{v}_o/\hat{f}$.

5.13. The cross-section of a cylindrical solenoid used to position the valve mechanism of a hydraulic control system is shown in Fig. 5P.13. When the currents i_1 and i_2 are equal, the plunger is centered horizontally ($x = 0$). When the coil currents are unbalanced, the plunger



Note. When $x = 0$, the spring force is zero!

Fig. 5P.13

moves a distance x. The nonmagnetic sleeves keep the plunger centered radially. The mass of the plunger is M, the spring constant K, and the viscous friction coefficient is B. The displacement x is limited to the range $-d < x < d$. You are given the terminal conditions

$$\begin{bmatrix} \lambda_1 = L_{11}i_1 + L_{12}i_2 \\ \lambda_2 = L_{12}i_1 + L_{22}i_2 \end{bmatrix}$$

where

$$L_{11} = L_o\left(3 - 2\frac{x}{d} - \frac{x^2}{d^2}\right),$$

$$L_{22} = L_o\left(3 + 2\frac{x}{d} - \frac{x^2}{d^2}\right),$$

$$L_{12} = L_o\left(1 - \frac{x^2}{d^2}\right).$$

(a) Write the mechanical equation of motion.
(b) Assume that the system is excited by the bias current I_0 and the two signal current sources $i(t)$ in the circuit of Fig. 5P.13 with the restriction that $|i(t)| \ll I_0$. Linearize the mechanical equation of motion obtained in part (a) for this excitation.
(c) Is the system stable for all values of I_0?
(d) The system is under damped. Find the response $x(t)$ to a step of signal current $i(t) = Iu_{-1}(t)$.
(e) Find the steady-state response $x(t)$ to a signal current $i(t) = I \sin \omega t$.

5.14. A plane rectangular coil of wire can be rotated about its axis as shown in Fig. 5P.14. This coil is excited electrically through sliding contacts and the switch S by the constant-current source I in parallel with the conductance G. A second coil, not shown, produces a

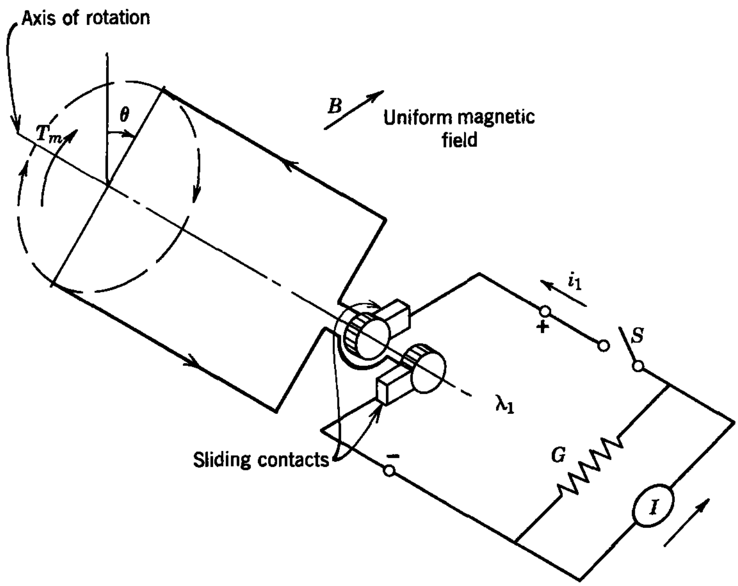


Fig. 5P.14

uniform magnetic field perpendicular to the plane of the rectangular coil when $\theta = 0$. Assume that the terminals of the second coil are described by the variables i_2, λ_2, so that we can write the electrical terminal relations as $\lambda_1 = L_1 i_1 + i_2 M \cos \theta$, $\lambda_2 = i_1 M \cos \theta + L_2 i_2$, where L_1, M, and L_2 (the self- and maximum mutual inductances of the coils) are constants. Concentrate attention on the electrical variables of the rotating coil by assuming that $i_2 = I_2 =$ constant. This is the excitation that provides the uniform constant magnetic field. In addition, assume that the mechanical position is constrained by the source $\theta = \Omega t$.

(a) Write the electrical equation for the coil. This equation, together with initial conditions, should determine $i_1(t)$.

(b) Assume that the switch S is closed at $t = 0$; that is, the initial conditions are, when $t = 0$, $\theta = 0$ and $i_1 = 0$. Find the current $i_1(t)$.
(c) Find the flux $\lambda_1(t)$ that links the rotating coil.
(d) Consider the limiting case of (b) and (c) in which the current i_1 can be considered as constrained by the current source.

$$\Omega GL_1 \ll 1 \quad \text{and} \quad \Omega GL_1 \ll \frac{L_1 I}{MI_2}.$$

Sketch $i_1(t)$ and $\lambda_1(t)$.
(e) Consider the limiting case in which the electrical terminals can be considered to be constrained to constant flux $\Omega GL_1 \gg 1$ and sketch $i_1(t)$ and $\lambda_1(t)$.
(f) Compute the instantaneous torque of electrical origin on the rotating coil.
(g) Find the average power required to rotate the coil. Sketch this power as a function of the normalized conductance ΩGL_1. For what value of ΩGL_1 does G absorb the maximum power?

5.15. In the system illustrated in Fig. 5P.15 the lower capacitor plate is fixed and the upper plate is constrained to move only in the x-direction. The spring force is zero when $x = l$, and the damping with coefficient B is so large that we can neglect the mass of the movable

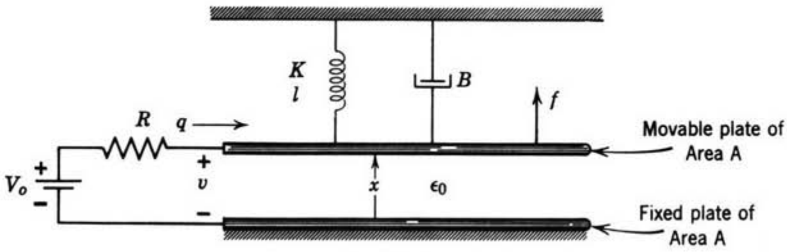


Fig. 5P.15

plate. The capacitor plates are excited by a constant voltage source in series with a resistance R. Neglect fringing fields. The voltage V_o is adjusted so that a stable, static equilibrium occurs at $X_o = 0.7l$. With the system at rest at this equilibrium position, a small step of mechanical force f is applied: $f = Fu_{-1}(t)$. In all of your analyses assume that the perturbations from equilibrium are small enough to allow use of linear incremental differential equations.

(a) Calculate the resulting transient in position x.
(b) Specify the condition that must be satisfied by the parameters in order that the mechanical transient may occur essentially at constant voltage. Sketch and label the transient under this condition.
(c) Specify the condition that must be satisfied by the parameters in order that the initial part of the mechanical transient may occur essentially at constant charge. Sketch and label the transient under this condition.

5.16. A mass M has the position $x(t)$. It is subjected to forces f_1 and f_2 which have the dependence on x shown in Fig. 5P.16. The mass is released at $x = 0$ with the velocity v_o. In terms of F_o and K, what is the largest value of v_o that will lead to bounded displacements of M?

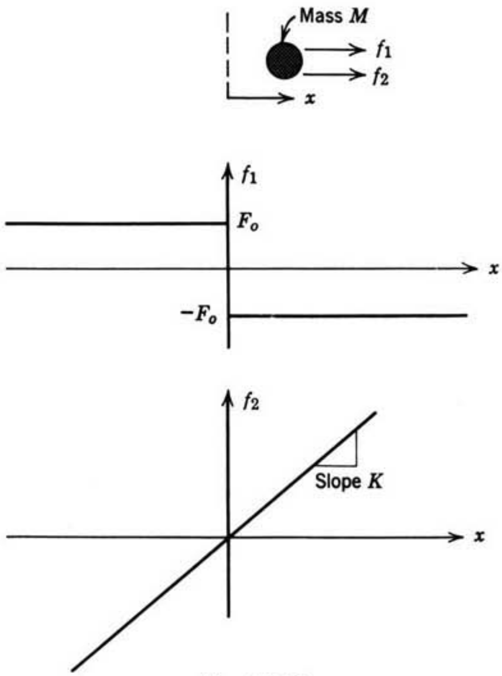


Fig. 5P.16

5.17. The electric field transducer shown in Fig. 5P.17 has two electrical terminal pairs and a single mechanical terminal pair. Both plates and movable elements can be regarded as perfectly conducting.

(a) Find the electrical terminal relations $q_1 = q_1(v_1, v_2, x)$, $q_2 = q_2(v_1, v_2, x)$.

(b) Now the terminals are constrained so that $v_2 = V_o =$ constant and $q_1 = 0$. Find the energy function $U(x)$ such that the force of electrical origin acting in the x-direction on the movable element is

$$f^e = -\frac{\partial U}{\partial x}.$$

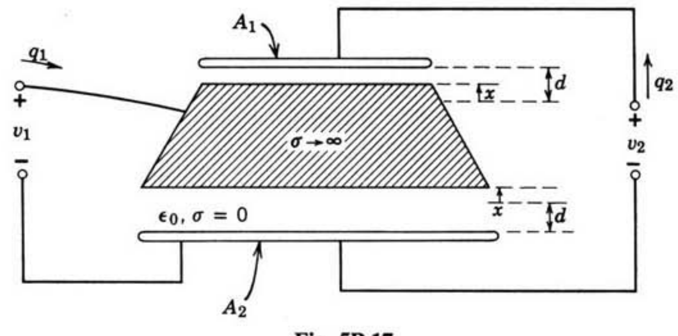


Fig. 5P.17

5.18. The central plate of a three-plate capacitor system (Fig. 5P.18) has one mechanical degree of freedom, x. The springs are relaxed in the equilibrium position $x = 0$ and fringing can be neglected. For a long time the system is maintained with the central plate fixed at $x = 0$ and the switch closed. At $t = 0$ the switch is opened and the center plate is released simultaneously.

(a) Find, in terms of given parameters, a hybrid energy function W'' such that $f^e = -\partial W''/\partial x$ for $t > 0$.

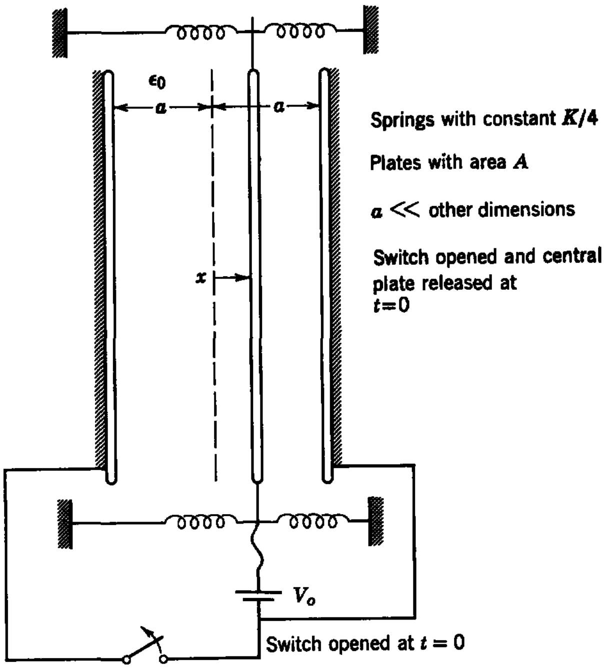


Fig. 5P.18

(b) Determine the criterion that the central plate be in stable equilibrium at $x = 0$.

(c) In the case in which the criterion of part (b) is satisfied, sketch a potential well diagram for $-a \leq x \leq a$, indicating all static equilibrium points, and whether they are stable or unstable.

5.19. An electromechanical system with one electrical and one mechanical terminal pair is shown in Fig. 5P.19. The electrical terminal relation is

$$\lambda = \frac{L_0 i}{(1 - x/a)^4},$$

where L_0 and a are given constants. The system is driven by a voltage $V_o + v(t)$, where V_o is constant. The mass of the plunger can be ignored. Gravity acts on M as shown.

(a) Write the complete equations of motion for the system. There should be two equations in the unknowns i and x.

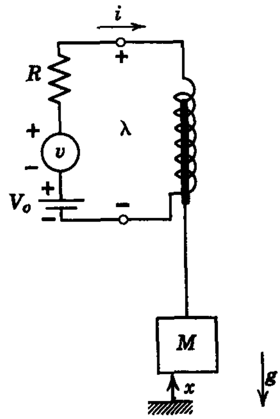


Fig. 5P.19

(b) With $v(t) = 0$, the current produced by V_0 holds the mass M in static equilibrium at $x = x_0$. Write the linearized equations of motion for the perturbations from this equilibrium that result because of $v(t)$.

5.20. The upper of the three plane-parallel electrodes shown in the Fig. 5P.20 is free to move in the x-direction. Ignore fringing fields, and find the following:

(a) The electrical terminal relations $q_1(v_1, v_2, x)$ and $q_2(v_1, v_2, x)$.

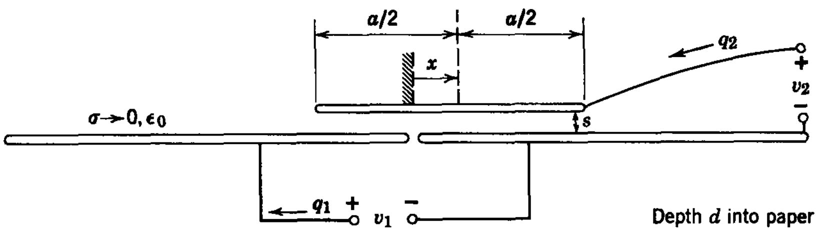


Fig. 5P.20

(b) Now the top plate is insulated from the lower plates after a charge $q_2 = Q$ has been established. Also, the potential difference between the lower plates is constrained by the voltage source $v_1 = V_o$. Find a hybrid energy function $W''(V_o, Q, x)$ such that

$$f^e = \frac{\partial W''(V_o, Q, x)}{\partial x}.$$

5.21. In Problem 3.8 the equation of motion was found for a superconducting coil rotating in the field of a fixed coil excited by a current source. This problem is a continuation of that development in which we consider the dynamics of the coil in a special case. The current I is constrained to be I_o = constant.

(a) Write the equation of motion in the form

$$J\frac{d^2\theta}{dt^2} + \frac{\partial V}{\partial \theta} = 0.$$

(see Section 5.2.1) and sketch the potential well.

(b) Indicate on the potential-well sketch the angular positions at which the rotor can be in stable static equilibrium and in unstable static equilibrium.

(c) With the rotor initially at rest at $\theta = 0$, how much kinetic energy must be imparted to the rotor to make it rotate continuously?

5.22. The system of Fig. 5P.22 contains a simple pendulum with mass M and length l. The pivot has viscous (linear) friction of coefficient B. The mass is made of ferromagnetic material. It causes a variation of coil inductance with angle θ that can be represented

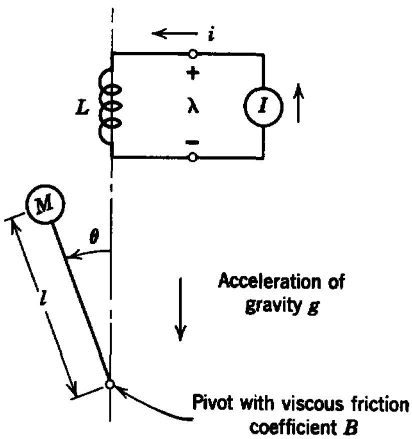


Fig. 5P.22

approximately by the expression $L = L_0(1 + 0.2\cos\theta + 0.05\cos 2\theta)$, where L_0 is a positive constant. The coil is excited by a constant-current source I at a value such that $I^2L_0 = 6Mgl$ with no externally applied forces other than gravity g.

(a) Write the mechanical equation of motion for the system.

(b) Find all of the possible static equilibria and show whether or not each one is stable.

5.23. The one-turn inductor shown in Fig. 5P.23 is made from plane parallel plates with a spacing w and depth (into the paper) D. The plates are short-circuited by a sliding plate in the position $x(t)$. This movable plate is constrained by a spring (constant K) and has a mass M.

(a) Find the equation of motion for the plate, assuming that the electrical terminals are constrained to constant flux $\lambda = \Lambda =$ constant.

(b) Find the position(s) $x = X_0$ at which the plate can be in static equilibrium. Determine if each point represents a stable equilibrium. Can you assign an equivalent spring constant to the magnetic field for small-signal (linear) motions?

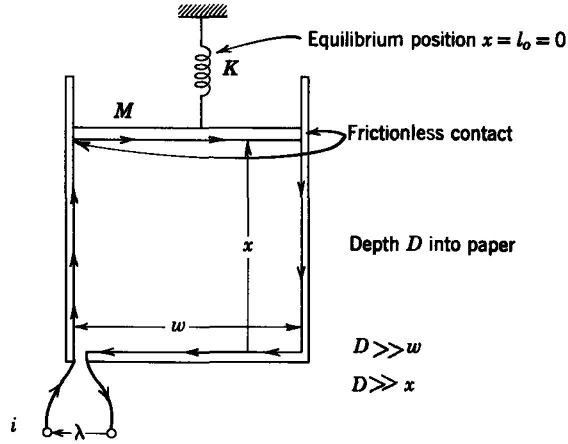


Fig. 5P.23

(c) Use a potential-well argument to describe the nonlinear motions of the plate. Include in your discussion how you would use the initial conditions to establish the constant of the motion E.

(d) Briefly describe the motions of the plate constrained so that $i = I =$ constant.

5.24. The terminals of the device shown in Problem 4.23 are now constrained to a constant value of λ: $\lambda = \Lambda_0 =$ constant. The rotor has a moment of inertia J and is free of damping.

(a) When $\theta = 0$, the angular velocity $d\theta/dt = \Omega$. Find an analytical expression for $d\theta/dt$ at each value of θ. (Given the angle θ, this expression should provide the angular velocity.)

(b) What is the minimum initial angular velocity required to make the rotor rotate continuously in one direction?

(c) For what values of θ can the rotor be in static equilibrium? Which of these equilibria is stable?

(d) Describe quantitatively the angular excursion of the rotor when it is given an initial angular velocity less than that found in (b).

5.25. A mass M_1 attached to a weightless string rotates in a circle of radius r on a fixed frictionless surface as illustrated in Fig. 5P.25. The other end of the string is passed through a frictionless hole in the surface and is attached to a movable capacitor plate of mass M_2. The other capacitor plate is fixed and the capacitor is excited by a voltage source $v(t)$. The necessary dimensions are defined in the figure. The length of the string is such that when $x = 0$, $r = 0$. You may assume that $a \gg x$ and ignore the effects of gravity and electrical resistance. With $v(t) = V_0 =$ constant and $r = l$, the mass M_1 is given an angular velocity ω_m necessary for equilibrium.

(a) Find the force of electromagnetic origin exerted on the capacitor plate.

(b) Determine the equilibrium value of ω_m.

(c) Show that the angular momentum $M_1 r^2\, d\theta/dt$ is constant, even if $r = r(t)$ and $\theta = \theta(t)$. [See Problem 2.8 for writing force equations in (r, θ) coordinates.]

(d) Use the result of (c) to write the equation of motion for $r(t)$. Write this equation in a form such that potential well arguments can be used to deduce the dyamics.

(e) Is the equilibrium found in (b) stable?

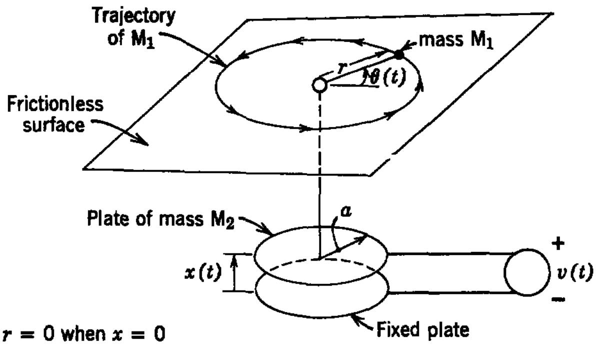


Fig. 5P.25

5.26. As an example of a lossy nonlinear system, consider the basic actuator for an electrically damped time-delay relay* illustrated in Fig. 5P.26. The transducer is designed to operate as follows. With switch S open, the spring holds the plunger against a mechanical stop at $x = x_o$. When switch S is closed, the magnetic field is excited, but the winding that is short-circuited through resistance R_2 limits the rate of buildup of flux to a low value. As the flux builds up slowly, the magnetic force increases. When the magnetic force equals the spring force, the plunger starts to move and close the air gap. The velocity of the plunger is so low that inertia and friction forces can be neglected; thus, when the plunger is moving, the spring force is at all times balanced by the magnetic force (see Section 5.2.2).

(a) Write the electrical circuit equations. The magnetic flux Φ is defined such that $\lambda_1 = N_1\Phi$ and $\lambda_2 = N_2\Phi$. Use these equations to find a single equation involving (Φ, x) with V as a driving function.

(b) Define two constants: the flux Φ_o linking the coils with the air gap closed ($x = 0$), $\Phi_o = 2\mu_0 wdN_1V/gR_1$, and the time constant τ_o for flux buildup when the air gap is closed,

$$\tau_o = \frac{2\mu_o wd}{g}\left(\frac{N_1^2}{R_1} + \frac{N_2^2}{R_2}\right).$$

Show that the result of (a) can be written in the form

$$\Phi_o = \left(1 + \frac{x}{g}\right)\Phi + \tau_o\frac{d\Phi}{dt}.$$

The transient behavior of this device can be divided into three intervals:

1. The switch S is closed with the plunger at $x = x_0$ and with zero initial flux Φ. The flux builds up to a value necessary to provide a magnetic force equal to the spring force that is holding the plunger against the stop at $x = x_o$.

2. The plunger moves from the stop at $x = x_o$ to the stop at $x = 0$. During this motion the spring force is the only appreciable mechanical force and is balanced by the magnetic force.

3. The plunger is held against the stop at $x = 0$ by the magnetic force, whereas the flux Φ continues to build up to Φ_o.

* *Standard Handbook for Electrical Engineers*, 9th ed., McGraw-Hill, New York, 1957, Sections 5-150 and 5-168.

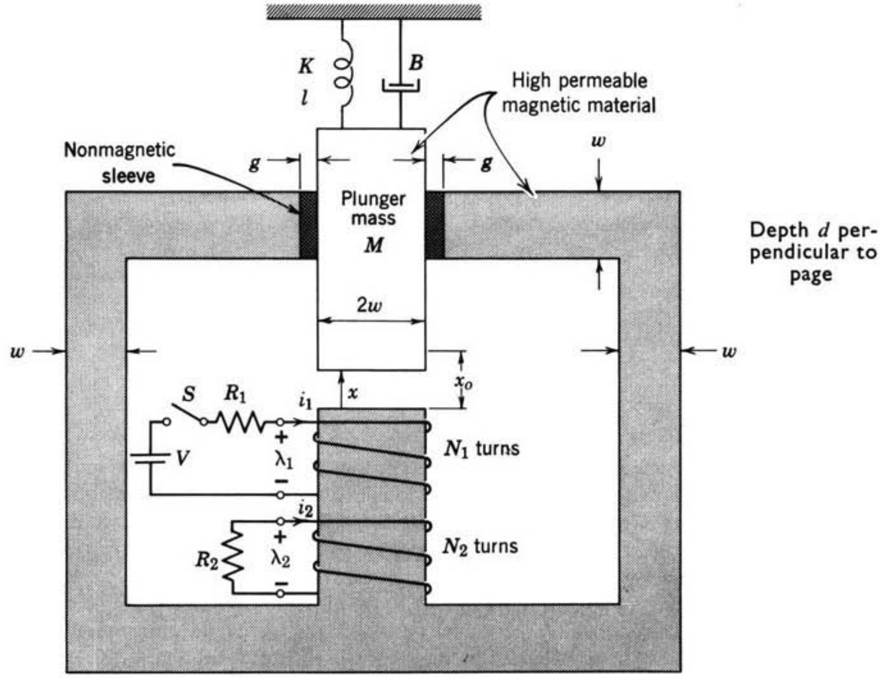


Fig. 5P.26

(c) Determine the transient in Φ during interval (1). Write an equation of force equilibrium for the plunger to determine the flux $\Phi = \Phi_1$ when interval (1) ends.

(d) Write an equation for Φ during interval (2). Assume the parameters

$$\frac{l}{x_o} = 2, \qquad \frac{x_o}{g} = 4, \qquad \frac{\Phi_o}{\Phi_1} = 10$$

and integrate the equation resulting from (c) to find $\Phi(t)$ in interval (2).

(e) Find the transient in Φ during interval (3).

(f) Sketch Φ and x as functions of time throughout the three intervals.