
 THE DISCRETE FOURIER SERIES

 Solution 8.1

$$\begin{aligned}\tilde{X}_1(k) &= \sum_{n=0}^{N-1} \tilde{x}(n) e^{-j\frac{2\pi}{N}nk} \\ &= 2 + e^{-j\frac{\pi}{2}k} + e^{-j\frac{\pi}{2}3k}\end{aligned}$$

$$e^{-j\frac{3\pi}{2}k} = e^{-jk(-\frac{3\pi}{2} + 2\pi)} = e^{j\frac{\pi}{2}k}$$

Therefore,

$$\tilde{X}_1(k) = 2 + e^{-j\frac{\pi}{2}k} + e^{j\frac{\pi}{2}k} = 2 \left[1 + \cos \frac{\pi k}{2} \right].$$

 Solution 8.2

$$\begin{aligned}\tilde{X}(k) &= \sum_{n=0}^{N-1} \tilde{x}(n) W_N^{kn} \\ \tilde{X}^*(k) &= \sum_{n=0}^{N-1} \tilde{x}^*(n) W_N^{-kn}\end{aligned}$$

or, since $\tilde{x}(n)$ is real,

$$\tilde{X}^*(k) = \sum_{n=0}^{N-1} \tilde{x}(n) W_N^{-kn}$$

Finally, substituting $-k$ for k

$$\tilde{X}^*(-k) = \sum_{n=0}^{N-1} \tilde{x}(n) W_N^{kn} = \tilde{X}(k)$$

Note, incidentally, that this is indeed satisfied for problem 8.1.

 Solution 8.3

If we show that $\tilde{X}(k)$ is real, then from problem 8.2 it follows that $\tilde{X}(k)$ is also even. Thus

$$\tilde{X}^*(k) = \sum_{n=0}^{N-1} \tilde{x}(n) W_N^{-kn}$$

Replacing n by $-n$ in the summation on the right-hand side

$$\tilde{X}^*(k) = \sum_{n=0}^{-N+1} \tilde{x}(-n) W_N^{kn}$$

or since $\tilde{x}(n)$ is even

$$\tilde{X}^*(k) = \sum_{n=0}^{-N+1} \tilde{x}(n) W_N^{kn}$$

Finally, since $\tilde{x}(n)$ is periodic the limits on the summation can be replaced by the interval 0 to N-1. Thus $\tilde{X}^*(k) = \tilde{X}(k)$, i.e. $\tilde{X}(k)$ is real.

Solution 8.4

(i) Since $\tilde{x}(n)$ is periodic with period 10, $\tilde{X}(k)$ is also periodic with period 10. Thus (i) is true.

(ii) Since $\tilde{x}(n)$ is real, $\tilde{X}^*(k) = \tilde{X}(-k)$. In order for the stated property to also be true, $\tilde{X}(k)$ must be real, which requires that $\tilde{x}(n)$ be even, which is not the case. Thus (ii) is not true.

(iii) $\tilde{X}(0) = \sum_{n=0}^{N-1} \tilde{x}(n) = 0$. Thus (iii) is true.

(iv) $\tilde{X}(k) e^{jk \frac{2\pi}{5}}$ is the Fourier series for $\tilde{x}(n+2)$. From the figure we note that $\tilde{x}(n+2)$ is not an even function. Thus $\tilde{X}(k) e^{jk \frac{2\pi}{5}}$ is not real. However, $\tilde{x}(n-2)$ is an even sequence and thus $\tilde{X}(k) e^{-jk \frac{2\pi}{5}}$ is real

Solution 8.5

See Figure S8.5-1 on next page.

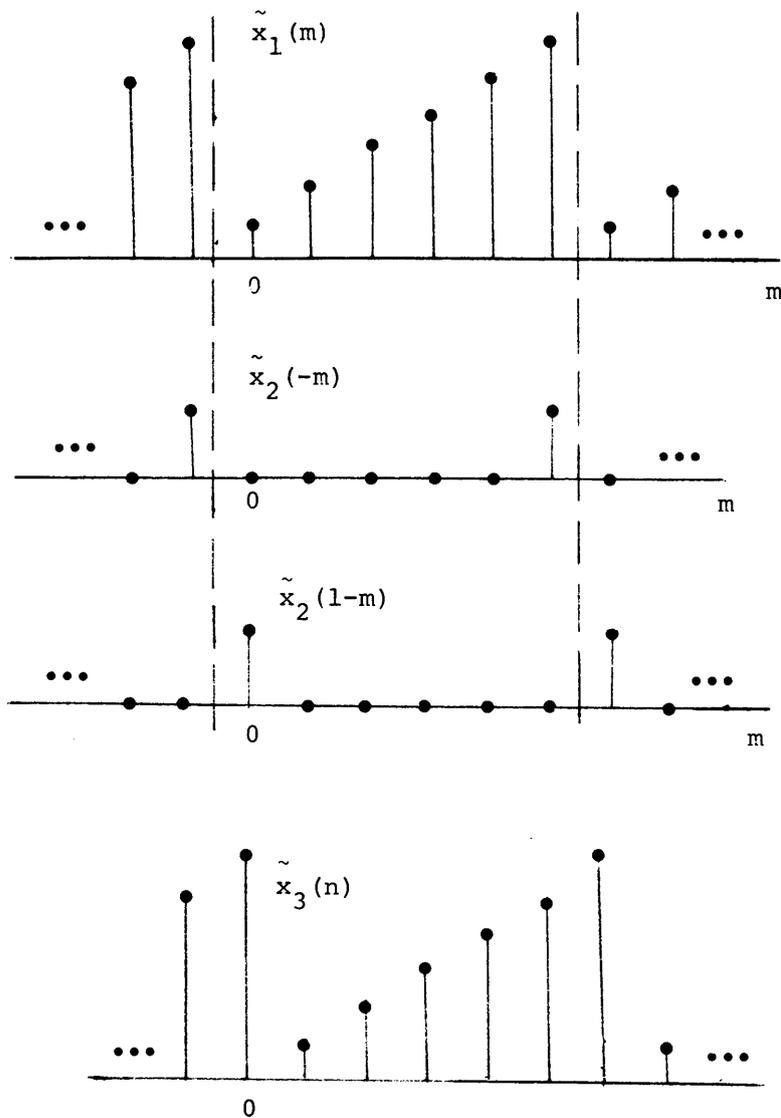


Figure S8.5-1

Solution 8.6

The Discrete Fourier series coefficients of $\tilde{X}(k)$ would be defined as

$$\tilde{Y}(n) = \sum_{k=0}^{N-1} \tilde{X}(k) w_N^{kn}$$

$\tilde{x}(n)$ is given by

$$\tilde{x}(n) = \frac{1}{N} \sum_{k=0}^{N-1} \tilde{X}(k) w_N^{-kn}$$

thus $\tilde{y}(n) = N \tilde{x}(-n)$.

Solution 8.7

(a) The time origin can be chosen such that all the $\tilde{X}(k)$ are real if $x(n)$ can be shifted to be an even function. It can for sequence (b) but not for the others.

(b) This requires that the time origin be chosen so that $\tilde{x}(n)$ is odd. This cannot be done for any of the sequences.

Solution 8.8

$$\begin{aligned}\tilde{X}_1(k) &= \sum_{n=0}^{N-1} \tilde{x}(n) W_N^k \\ \tilde{X}_2(k) &= \sum_{n=0}^{2N-1} \tilde{x}(n) W_{2N}^{kn} \\ &= \sum_{n=0}^{N-1} \tilde{x}(n) W_{2N}^{kn} + \sum_{n=0}^{N-1} \tilde{x}(n+N) W_{2N}^{k(n+N)}\end{aligned}$$

or, since $\tilde{x}(n)$ is periodic with period N and $W_{2N}^N = -1$

$$\begin{aligned}\tilde{X}_2(k) &= \sum_{n=0}^{N-1} \tilde{x}(n) W_{2N}^{kn} [1 + (-1)^k] \\ &= [1 + (-1)^k] \sum_{n=0}^{N-1} \tilde{x}(n) W_{2N}^{kn}\end{aligned}$$

Thus, for k odd, $\tilde{X}_2(k) = 0$. For k even, $W_{2N}^{kn} = W_N^{n(k/2)}$

and

$$\begin{aligned}\tilde{X}_2(k) &= 2 \sum_{n=0}^{N-1} \tilde{x}(n) W_N^{n(k/2)} \\ &= 2 \tilde{X}_1(k/2) \quad k \text{ even.}\end{aligned}$$

MIT OpenCourseWare
<http://ocw.mit.edu>

Resource: Digital Signal Processing
Prof. Alan V. Oppenheim

The following may not correspond to a particular course on MIT OpenCourseWare, but has been provided by the author as an individual learning resource.

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.