In December 1998, the Mars climate orbiter was launched in hopes of providing detailed information about the atmosphere of Mars. The launch went well. However the two teams that collaborated on the project worked in different units--one used the metric system, and the other the English system, and the two systems of units were never properly reconciled. So while the orbiter was intended to fall into orbit around Mars, instead it crash landed on the surface. So units are pretty important. This video is part of the Problem Solving video series. Problem-solving skills, in combination with an understanding of the natural and human-made world, are critical to the design and optimization of systems and processes. Hi my name is Ken Kamrin, and I am a mechanical engineering professor at MIT. Today I want to talk to you about using unit analysis in problem solving. This might seem simple, but it is a critical tool for validating your calculations. So pay attention. To understand this example, you should be familiar with the definition of work. After watching this video, you should be able to utilize and apply the key properties of unit analysis: when two quantities are multiplied, their units also multiply. all terms added, subtracted or equated must have the same units. You should also be able to explain how derivatives and integrals affect units. Before we start the main example, let's discuss how integration and differentiation affect units. The first question is: what are the units of dx? You should think of dx as a very small change in x or a "little bit of x", Recall that a little bit of ice cream, is still ice cream. Even if it is a very very small amount. So the units of dx are precisely the units of x. Let's do an example. Let's consider the position function x(t) with units of meters, where t is time, with units of seconds. Then velocity is the time derivative of position v = dx/dt, and has units of meters per second. Let's look at how the notation is consistent with our physical interpretation: dx has units of meters, and it is divided by dt, which has units of seconds. So dx over dt should have units of meters over seconds. Now let's look at acceleration, which has units of meters per second squared. We know that acceleration is the time derivative of velocity, which is the second derivative of position with respect to time. Let's see how this plays out with notation... We can also determine how integration affects units. Let's use the example of the integral of velocity v with respect to time t. We know physically that this is position, which has units of meters. Unit analysis also makes this clear because v has units of m/s, and dt has units of s, so when we multiply, the seconds cancel, and we end up with units of seconds It might be interesting to point out that the integral symbol doesn't affect units at all. Just like the differential did not have units, it is just a symbol that represents the limit of sums. Since every term in the sum has units of v times units of t, the sums, hence the integral sign doesn't affect units at all. What we've seen: given a function f(y), differentiating with respect to y divides the units of f by the units of y. Differentiating twice divides the units of f by the units of y squared. Similarly, integrating with respect to y multiplies the units off by the units of y In this example, we'll see how unit analysis can help us check our calculation of the work done by an applied force. A machine in a factory is programmed to apply a force to a 3.0kg object to move it back and forth in the horizontal direction. The position of the object as a function of time is given by the equation x equals 3.0t minus 4.0 t squared plus 1.0 t cubed, where x is measured in units of meters, and t in seconds. Find the work done on the object by the force from t=0 to t=2. Note that work is done when a force is applied over a distance on an object. Let x0 be the position of the object at time t = 0. Let x2 be the position at time t = 2. Then the amount of work done on the object is computed by the equation W = integral x2 F · dx. Let's start by figuring out the units of work. We do this by analyzing our equation, which tell us that the units of work are the units of force times the units of dx. If we forget what the units of force are, we can use unit analysis to figure it out. We know that F = ma, and acceleration is in units of meters per second squared, and mass has units of kilograms, So the unit of force, also known as the Newton, is equivalently written as kilogram-meters per second squared. So the unit of force, also known as the Newton, is equivalently written as kilogram-meters per second squared. Recall that the units of dx are the units of x, which is meters. Thus the unit of work is a Newton-meter. So now we know what units our answer needs to be in. So let's start solving our problem. In order to solve our problem, we need to know the force acting on our object. We are given the mass of the object, the position function for the object, and F=ma. Because we can determine acceleration as the second derivative of position with respect to time, we can use our given information to determine the force. Let's go ahead and solve this problem, perhaps incorrectly. See if you can catch my mistake. Alright, so here we have position as a function of time. So let's differentiate to get the velocity. Where I've left off the decimal points. And we'll go ahead and differential the velocity to get the acceleration. And finally to get the force, we go ahead with F=ma. So there is my m. And according to this, our expression for a is -8 + 6t. So I look at this and I think to myself, uhoh, this can't be right, because this doesn't have units of force. We know that force should have units of Newtons, but it doesn't look like this has the correct units. So this entices us to go back into our problem to figure out what went wrong. Looking back up at the first equation for position, I see immediately that something has gone wrong, because position is in units of meters, and each of these terms don't have units of meters. In fact they don't even agree with each other. This tells me there had to be units for each of these constants that somehow got swept under the rug. So let's be rigorous, and put those units back in. Here we need units of meters per second to cancel with the seconds from time, and leave us with units of meters. Note that this means something physical, it is the initial velocity. Let's go back into this term here, and see that I have a t squared, and I need something with units of meters per seconds squared if I am going to get from this term something with units of meters. And lastly, we have this term over here that has a t cubed. And likewise the missing unit must have units of meters per second cubed, which physically is the unit of jerk, or the rate of change of acceleration. Now every term has units of meters, and so this equation makes sense. You may want to pause the video here and find the units for the constants in the formulas for velocity and acceleration. Show that these units agree with the units you find by differentiating x(t) Now that you have checked the units of acceleration, let's put it back in the force formula, and see that everything works out ok. So we got F=ma. M as before equals 3 kilograms. And the acceleration written with correct units is negative 8 point 0 meters per second squared plus 6 point 0 t meters per second cubed. To see that this has the correct units, lets expand this formula. -24 kg m per s

squared yep, that's a Newton. + 18t kg m per s cubed. Just to check, note that while this doesn't have units of N, time has units of seconds, so this term is entirely in units of newtons as well. Now that we've verified the of Force is in units of Newtons. we can compute the work integral. That means we want to evaluate the integral: x0 to x2 of F dx. We have an expression for F, so let's go ahead and put that in. I'm going to go ahead and leave off the units for now. We leave it as an exercise for later that you will do that. What do we get? Integral x0tox2 minus 24 plus 18t dx. So we look at this and we realize that we actually have a slight problem. Because the force is written in terms of time, and we are integrating with respect to x. Fortunately this is easy to resolve. Just remember that a small change in x, dx is equivalently velocity times a small change in time. This is easy to see if you express velocity as dx dt, because you multiply by dt, and it is clear that this gives you a small change in x. So in your calculus class this little maneuver is called a change of variable. So let's go ahead and do that. This means our integral can be written as an integral over time, from time 0 to time t=2 of force times velocity times dt. We write F as ma, and we recall that acceleration is the time derivative of velocity. Now we see how to integrate this, and we find that the integral is m times velocity squared divided by 2 evaluated at the two time end points. Plugging in our formula for the velocity, and evaluating, we get that the answer is negative 12. I intentionally left off the units while doing this computation. Now I leave it as an exercise for you to use unit analysis to check that this integral does in fact give you 12 Newton-meters or 12 Joules, the units for work. In the example, we saw that when you are adding numbers with units, it is important for those numbers to have the same units if you want your quantity to have physical meaning. We also learned that the differential quantity dx has the same units as x. We also saw that it is helpful to use unit analysis to check your work at points during and after the computation. You can see now that unit analysis can be a useful part of your problem solving strategy. It is important to understand the properties of units, and how they are affected by mathematical operations. Checking units at the end of a computation useful to see if solution is reasonable. Sometimes, unit analysis can suggest a formula for an unknown quantity in terms of given information. But physical knowledge is necessary to validate solution completely.